# Derivation of No Treat-Test and Test-Treat Probability Thresholds

Michael Kohn 10/9/09

$B$ = cost of failing to treat a $D+$ individual
$C$ = cost of treating a $D-$ individual unnecessarily
$T$ = cost of test
$P$ = probability of $D+$

$p[-|D+]$ = probability of negative test given $D+$ = 1– sensitivity
$p[+|D-]$ = probability of positive test given $D-$ = 1– specificity

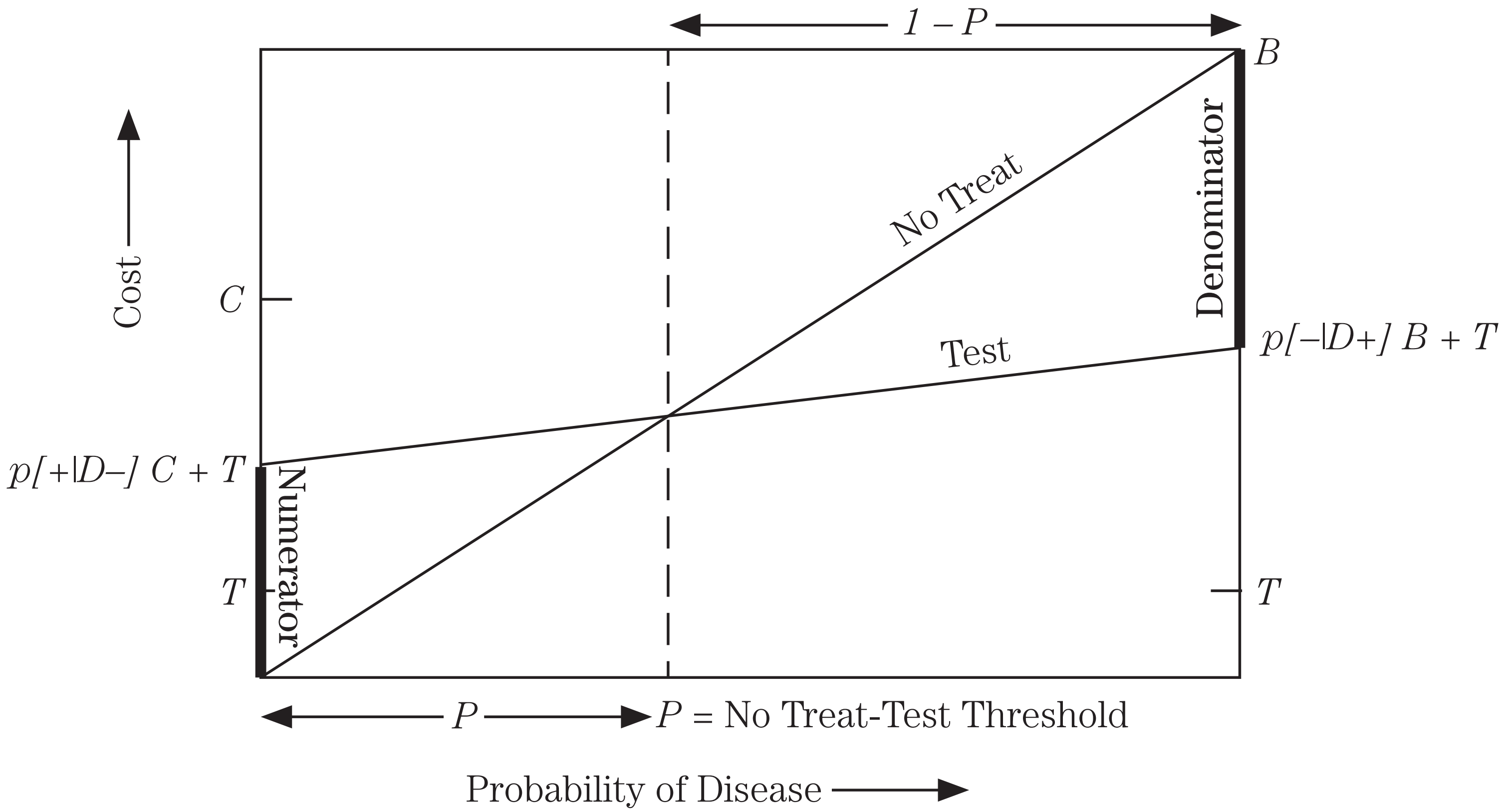
Expected Cost of No Treat strategy: $(P)B$

Expected Cost of Treat Strategy: $(1-P)C$

Expected Cost of Test Strategy:
$P\ (p[-|D+])\ B + (1-P)\ (p[+|D-\ ])\ C + T$

Reminder about odds and probability:
Convert odds to probability by adding the numerator to the denominator.
If threshold odds are C/B, then threshold probability is C/(B+C).

## No Treat-Test Threshold Geometry




Convince yourself that the ratio of the line labelled "**Numerator**" to the line labelled "**Denominator**" is equal to the threshold odds P / (1–P)

$$P / (1-P) = \frac{(p[+|D-])C + T}{B - (p[-|D+])B - T}$$

$$= \frac{(p[+|D-]\ C + T}{B\ (1 - p[-|D+]) - T}$$

Substitute $p[+|D+]$ *for* $1 - p[-|D+]$

$$= \frac{(p[+|D-])\ C + T}{(p[+|D+])\ B - T}$$

= No Treat-Test Threshold Odds
(add numerator to denominator for probability)

$$= \frac{(p[+|D-])\ C + T}{(p[+|D-])\ B + (p[+|D-])\ C}$$ = No Treat-Test Threshold Probability

# No Treat-Test Threshold
# Algebra

No Treat-Test threshold is where the expected cost of the "No Treat" strategy equals the expected cost of the "Test" strategy.

$$(P)(B) = P\ (p[-|D+])\ B + (1-P)\ (p[+|D-])\ C + T$$

substitute $(P)(T) + (1-P)T$ for $T$

$$(P)(B) = P\ (p[-|D+])\ B + (P)(T) + (1-P)\ (p[+|D-])\ C + (1-P)\ T$$

$$(P)(B) = \underbrace{P\ (p[-|D+]\ B + T)}_{\text{subtract this}} + (1-P)\ (p[+|D-]\ C + T)$$

$$(P)(B) - P\ (p[-|D+]\ B + T) = (1-P)\ (p[+|D-]\ C + T)$$

$$(P)[(B)\ (1- p[-|D+]\ ) - T] = (1-P)\ (p[+|D-]\ C + T)$$

substitute $p[+|D+]$ for $1-p[-|D+]$

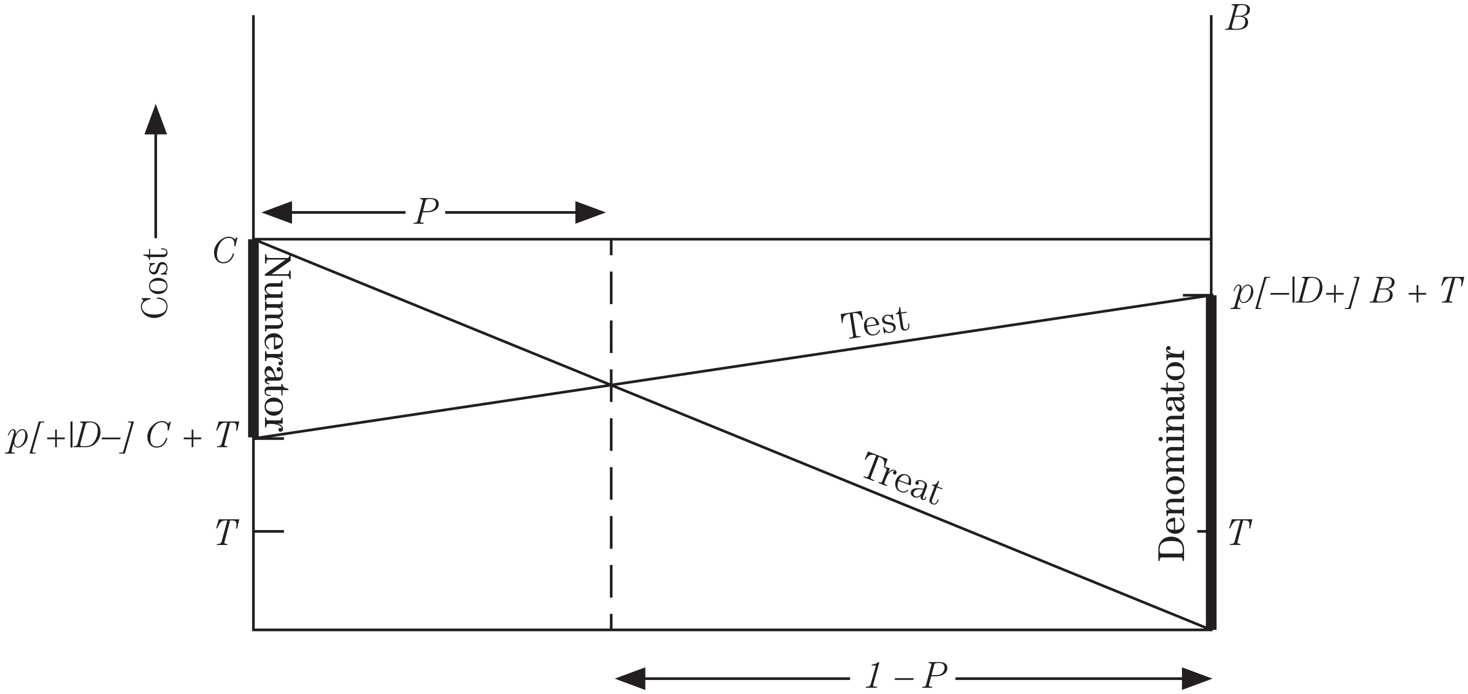
$$(P)\ [p[+|D+](B) - T] = (1-P)\ (p[+|D-]\ C + T)$$

$$\frac{P}{(1-P)} = \frac{p[+|D-]\ C + T}{p[+|D+]\ B - T}$$

This is threshold odds. To get threshold probability add the numerator to the denominator.

$$P = \frac{(p[+|D-])\ C + T}{(p[+|D+])\ B + (p[+|D-])\ C} = \text{No Treat-Test Threshold Probability}$$

## Test-Treat Threshold
## Geometry




Convince yourself that

$$P\,/\,(1\text{–}P) = \frac{C - (p[+|D-])\ C + T}{(p[-|D+])\ B + T}$$

$$= \frac{C\ (1 - p[+|D-] - T}{p[-|D+]\ B + T}$$

Substitute $p[-|D-]$ for $1 - p[+|D-]$

$$= \frac{C\ (p[-|D-]) - T}{(p[-\mid D+])\ B + T}$$

= Test-Treat Threshold Odds
(add numerator to denominator for probability)

$$P = \frac{(p[-|D-])\ C - T}{(p[-|D+])\ B + (p[-|D-])\ C} = \text{Test-Treat Threshold Probability}$$

# Test-Treat Threshold
## Algebra

The Test-Treat strategy is where the expected cost of the “Test” strategy equals the expected cost of the “Treat” strategy.

$$P\,(p[-|D+])\,B + (1-P)\,(p[+|D-])\,C + T = (1-P)\,C$$

substitute $(P)(T) + (1-P)T$ for $T$

$$P\,(p[-|D+])\,B + (P)(T) + (1-P)\,(p[+|D-])\,C + (1-P)\,T = (1-P)\,C$$

$$P\,(p[-|D+]\,B + T) + \underbrace{(1-P)\,(p[+|D-]\,C + T)}_{\text{subtract this}} = (1-P)\,C$$

$$P\,(p[-|D+]\,B + T) = (1-P)\,C - (1-P)\,(p[+|D-]\,C + T)$$

rearrange

$$P\,(p[-|D+]\,B + T) = (1-P)\,(1 - p[+|D-]) - (1 - P)\,T$$

substitute $p\,[-|D-]$ for $1 - p\,[+|D-]$

$$P\,(p[-|D+]\,B + T) = (1-P)\,(p[-|D-]\,C - T)$$

$$\frac{P}{(1-P)} = \frac{p[-|D-]\,C - T}{p[-|D+]\,B + T}$$

This is threshold odds. To get threshold probability add the numerator to the denominator.

$$P = \frac{p[-|D-]\,C - T}{p[-|D+]\,B + p[-|D-]\,C} = \text{Test-Treat Threshold Probability}$$