Biostat 200 Lecture 3

Today’s topics

- Review of some probability facts
- Check in on what you should have learned so far
- Probability distributions

From last lecture: Independence vs. mutual exclusivity

- Mutual exclusivity: $P(A \cap B) = 0$
  - A and B cannot occur together

- If A and B are independent:

  $P(B \mid A) = P(B)$

  $P(A \mid B) = P(A)$

From last lecture: Independence vs. mutual exclusivity

- Mutual exclusivity: $P(A \cap B) = 0$
  - A and B cannot occur together
  - Mutual exclusitivity gets us the additive rule

$$P(A \cup B) = P(A) + P(B) + P(A \cap B)$$
$$= P(A) + P(B)$$

– Independence gets us the multiplication rule
  - If A and B are independent, then

$$P(A \cap B) = P(A)P(B)$$

From last lecture: Independence vs. mutual exclusivity

- Examples of non-independence of an <u>event</u>
  - Repeat measurement of the same study participant
  - Measurements of persons within the same family or some other clustering unit
- We also talk about two <u>random</u> <u>variables</u> being independent, i.e. if one is related to the other. The chi-square test is often called a test of independence.

What you should have learned from the past 2 weeks

- Types of variables
- The ability to perform in Stata and understand:
  - Basic manipulation of data, opening and saving data sets and .do files, basic data cleaning
  - Basic summaries relevant to different types of variables
  - Basic graphical analyses of different types of variables
- Basic probability concepts, especially conditional probability, mutual exclusivity, and independence

Where we go from here

- Use probability concepts to discuss theoretical distributions
- Knowing (or assuming) that a variable follows a certain distribution, you can calculate the probability of observing a certain value for that variable
- Next week: We will examine the probability distribution of sample means (the normal distribution)
- Knowing the distribution of a sample mean allows us to calculate the probability of observing a particular sample mean
- We will extend these concepts to examine other things, such as: differences in means and proportions between two or more groups, regression slopes, etc. (hypothesis testing)

Probability distributions

- Variables whose outcome can occur by chance, i.e. are not fixed, are called <u>random variables</u>

- Probability distributions describe the possible values of the random variable

Why do we care about probability distributions?

- Many statistical tests (i.e. p-values) are based on probability distributions

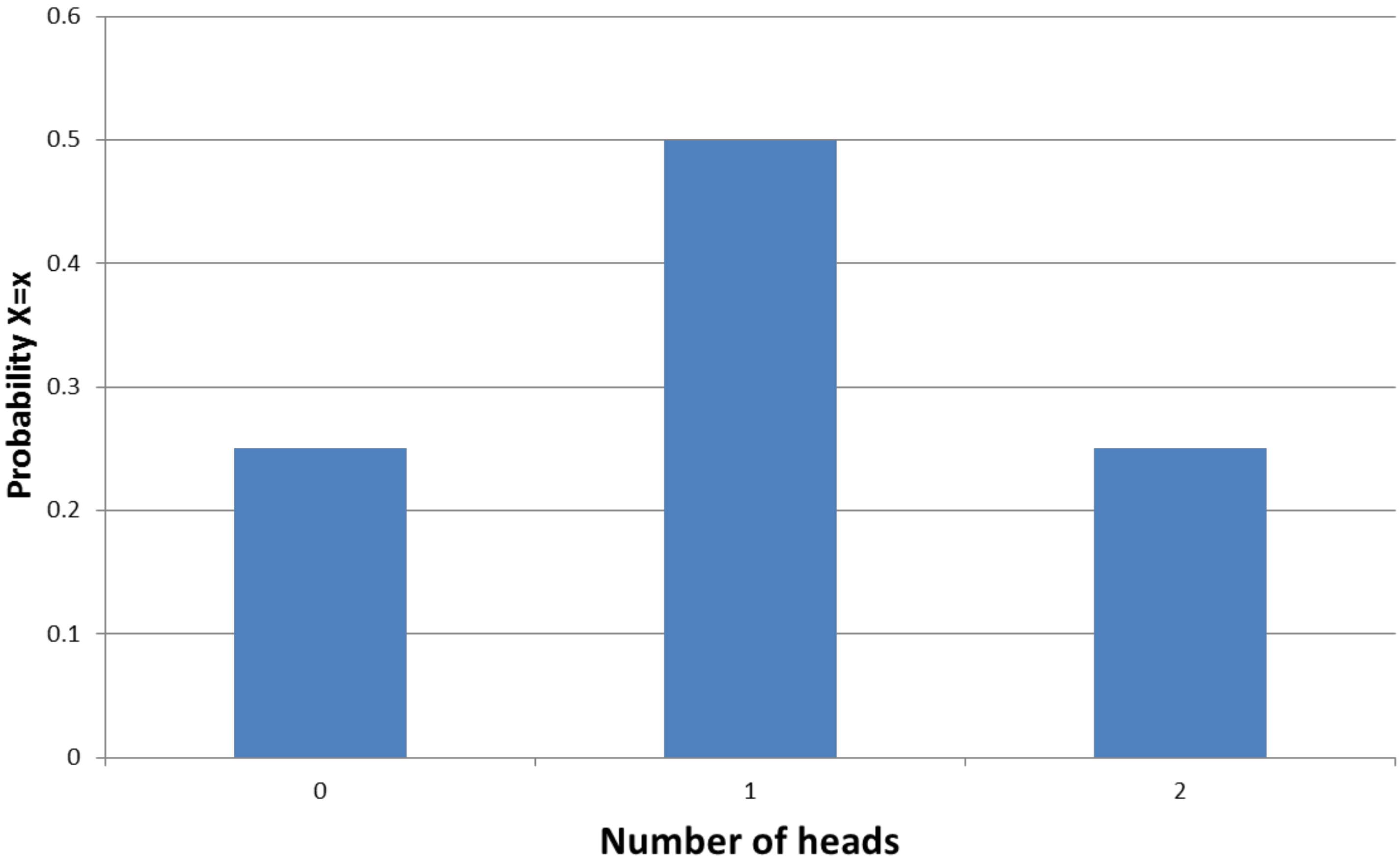
- For discrete (or categorical) random variables the probability distribution describes the probability of each possible value
- For example, consider the experiment in which you flip a coin 2 times and count the number of heads.
  - The possible outcomes of the experiment are: HH, TH, HT, TT.
  - You want to focus on the number of heads, which could be 0, 1, or 2. The probability of each outcome is:

Number of heads	Probability
0	.25
1	.5
2	.25

- The table looks similar to a frequency table of the data, but it is actually the theoretical distribution
- If you perform an infinite number of experiments, your data will look like this table

Number of heads	Relative frequency
0	.25
1	.5
2	.25

- The graphical representation of the probability distribution for tossing a coin 2 times is:



- Note that the probabilities add to 1. This is true of all probability distributions.

- This is a <u>theoretical probability distribution</u> based on our understanding of coin tossing
  - The probability of a head on each toss is .5
  - The probability of heads on the first toss is independent of the second toss
  - It’s actually the binomial distribution

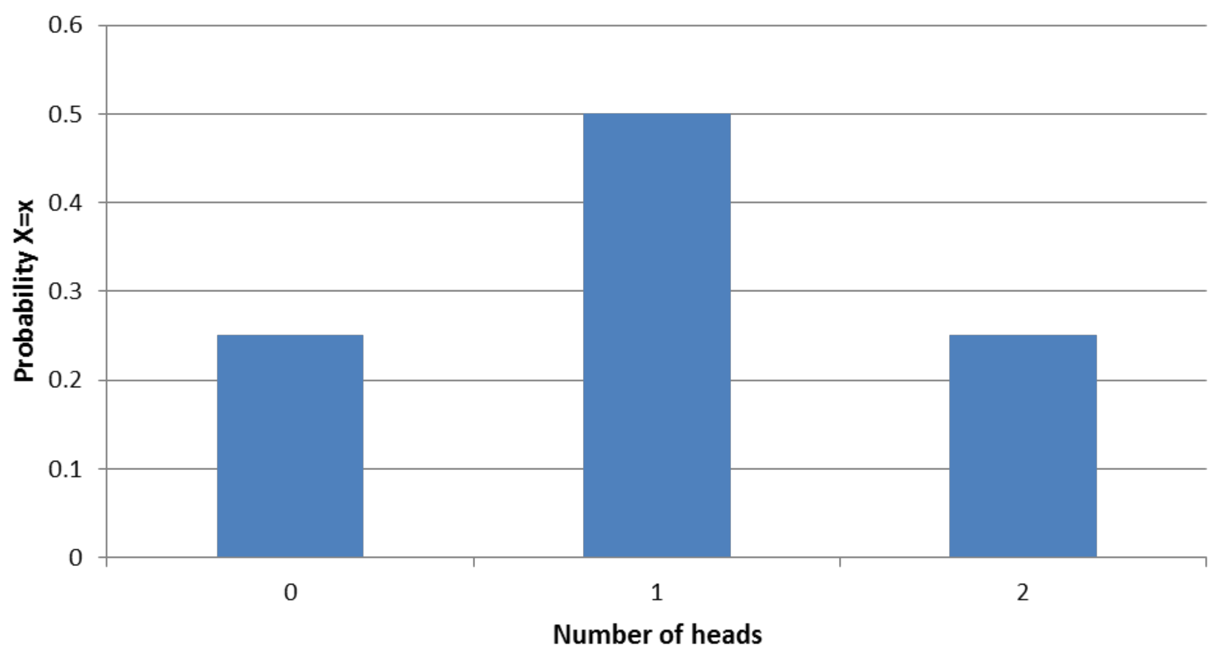
- We can write down a formula for the probability of observing a certain # of heads,
- This is written in general form $P(X=x)$
  - For 0 heads: $P(X=0)$
  - For 1 head: $P(X=1)$
  - For 2 heads: $P(X=2)$
  - For 1 or more heads: $P(X\geq 1)$

- We can use this theoretical distribution to make predictions about future experiments
- E.g. The probability that there will be at least 1 head in a trial of 2 coin tosses

$$P(X \geq 1) = P(X=1) + P(X=2)$$

(by what probability rule?)

$$= .5 + .25 = .75$$



- If you performed the experiment (i.e. 2 coin tosses) once, you'd get 0,1, or 2 heads
- Performing the experiment 10 times, I got: 2, 1, 1, 1, 1, 0, 0, 0, 1, 1

Number of heads	Frequency (%)
0	3 (30)
1	6 (60)
2	1 (10)

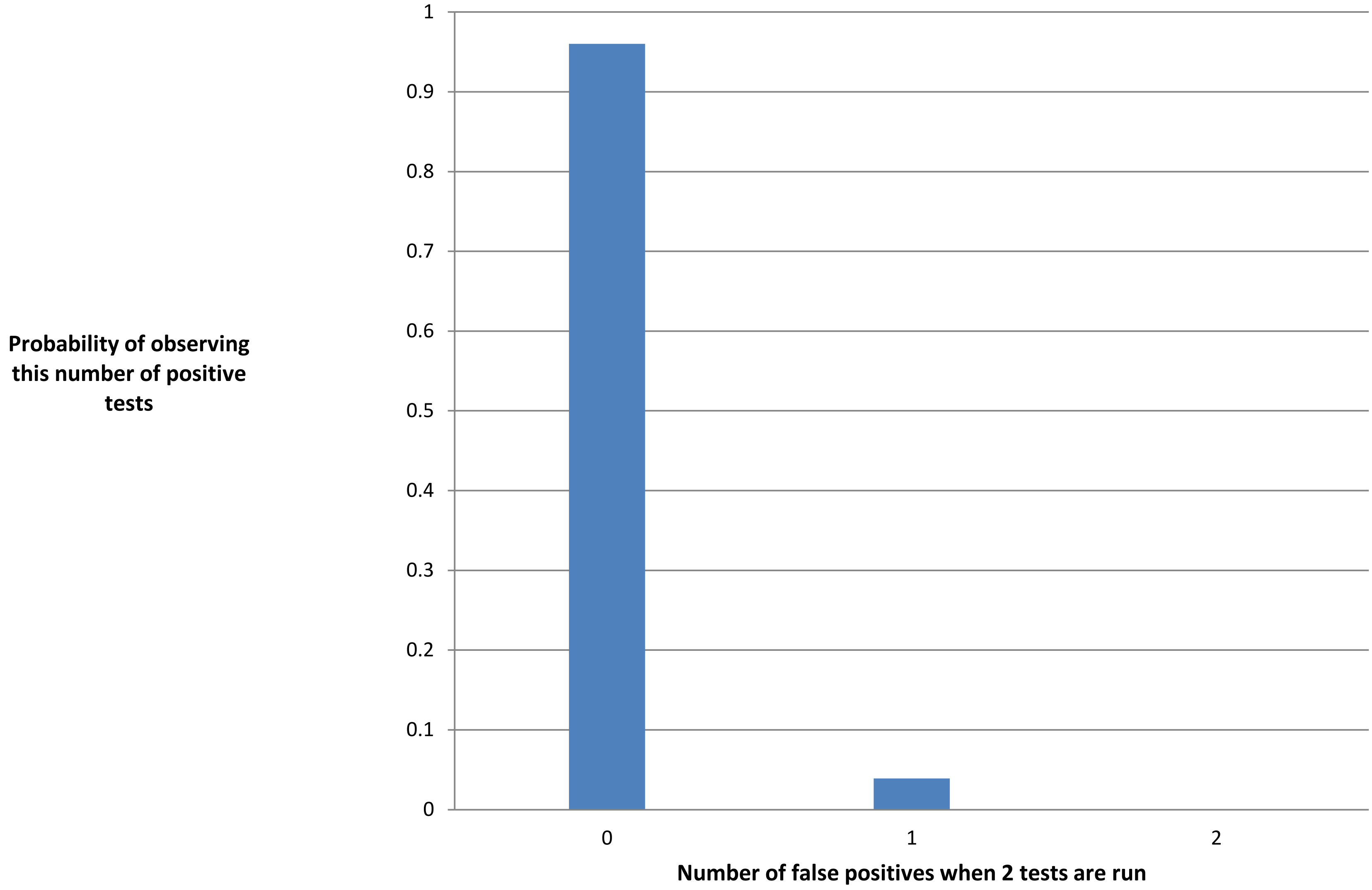
- What if we did the experiment 100 times?

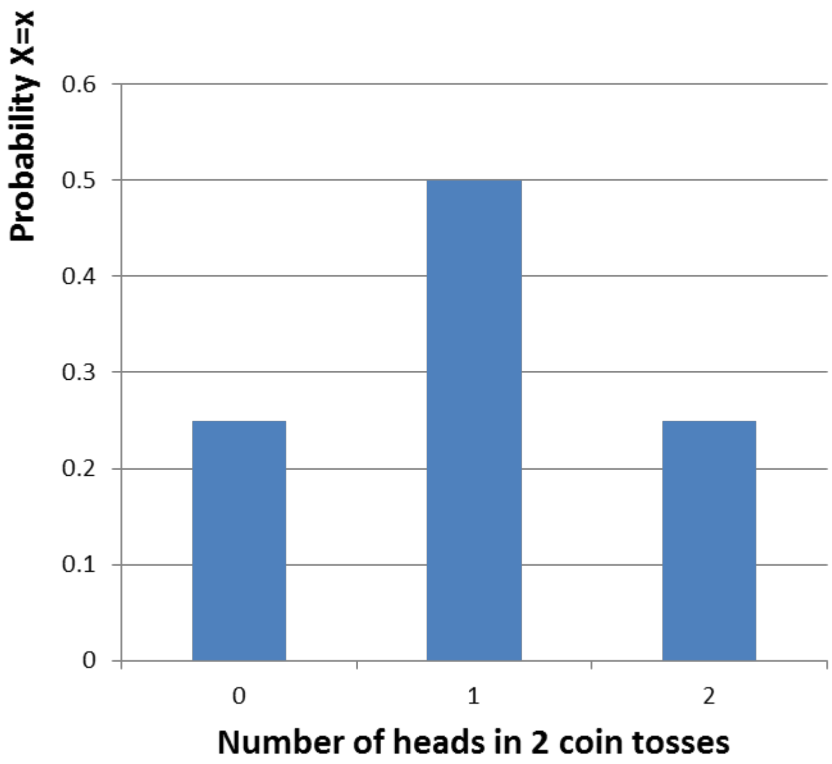
  1000 times? What would the frequency distribution for the outcomes look like?

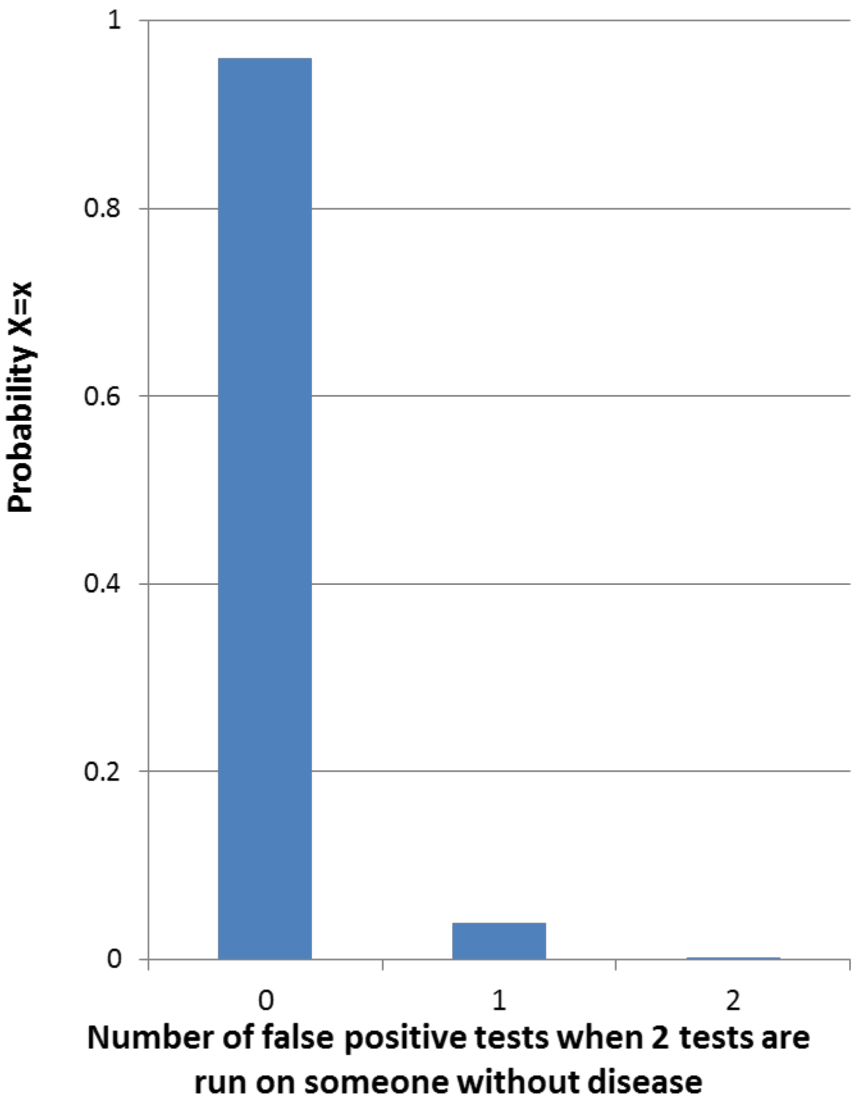
- This is the same situation as when we looked at two independent diagnostic tests with 2% false positive probability.
  - 2% of the time the test randomly gives a positive result when the true result is negative
- The possible outcomes were NegNeg, NegPos, PosNeg, PosPos
  - Calculating the probability 0, 1, or 2 positive tests:

Number of positive tests	Probability
0	$.98^2 = .9604$
1	.98*.02 *2= .0392
2	$.02^2$=.0004

- The graphical representation of the probability distribution for number of false positive tests is:







These both represent distributions that show the probability of 0, 1, or 2 events occurring in 2 independent trials. Why do they look so different?

Empirical Probability distributions

- Empirical probability distributions are based on real data
- They are usually based on a large sample or complete enumeration of a population
- The probabilities are calculated from the relative frequencies of the data
  - E.g. proportion of live births by age of mother in 1992

Probability distributions

- For <u>discrete (or categorical) variables</u> the probability distribution describes the probability of <u>each possible value</u>

- For <u>continuous variables</u>, the distribution describes the probability of a <u>*range* of values</u>

Bernoulli random variable

- If you have a variable that can take on one of two values with a constant probability p, then it is a Bernoulli random variable
- If the proportion of people in the population with a disease (the prevalence) is 15%, then when you randomly select one person, the probability that he/she has the disease is $P(Y=1)=p= 0.15$
- The probability that a randomly selected person does not have the disease is

$$P(Y=0)=1-p =0.85$$

Bernoulli distribution

- A Bernoulli random variable is said to follow the Bernoulli distribution
- p is the parameter that characterizes the distribution
- The Bernoulli distribution is a discrete distribution – the outcome is either 0 or 1
- It describes only one trial – so really is more theoretical than practical – it is the building block to describe the distribution of more than one trial

Binomial distribution

- Example: The proportion of people in the population with the disease (the prevalence) is 15%, then $P(Y=1)=0.15$ and $P(Y=0)=0.85$.
- One random draw will produce either 0 or 1 person with disease
- If we take a random sample of 5 people from this population, there will be 0,1,2,3,4, or 5 people with the disease.

If the probability of disease in each person is independent, then we can write down the probability of each of these outcomes even before we draw the sample.

For example, the probability that ALL of them will have the disease is P(X=5):

$$=P(X_1=1)^*\ P(X_2=1)^*\ P(X_3=1)^*\ P(X_4=1)^*\ P(X_5=1)$$

$$= 0.15 \times 0.15 \times 0.15 \times 0.15 \times 0.15 = 0.00008$$

by the multiplication rule for independence
$P(A \cap B)=P(A)P(B)$

The probability that NONE of them will have the disease is $P(X=0)$:

$$=P(X_1=0)*\ P(X_2=0)*\ P(X_3=0)*\ P(X_4=0)*\ P(X_5=0)$$

$$=0.85 \times 0.85 \times 0.85 \times 0.85 \times 0.85 = 0.444$$

The probability that exactly one person $P(X=1)$ has the disease

$$
\begin{aligned}
= \ & P(X_1=1) * P(\text{the other } 4=0) \\
& + P(X_2=1) * P(\text{the other } 4=0) \\
& + P(X_3=1) * P(\text{the other } 4=0) \\
& + P(X_4=1) * P(\text{the other } 4=0) \\
& + P(X_5=1) * P(\text{the other } 4=0)
\end{aligned}
$$

$$
\begin{aligned}
&= \mathbf{0.15} \times 0.85 \times 0.85 \times 0.85 \times 0.85 \\
&+ 0.85 \times \mathbf{0.15} \times 0.85 \times 0.85 \times 0.85 \\
&+ 0.85 \times 0.85 \times \mathbf{0.15} \times 0.85 \times 0.85 \\
&+ 0.85 \times 0.85 \times 0.85 \times \mathbf{0.15} \times 0.85 \\
&+ 0.85 \times 0.85 \times 0.85 \times 0.85 \times \mathbf{0.15}
\end{aligned}
$$

$$= 5 * .15 * .85^4$$

$$= 0.392$$

The probability that exactly two people $P(X=2)$ of 5 have the disease

= **0.15** x **0.15** x 0.85 x 0.85 x 0.85

+ **0.15** x 0.85 x **0.15** x 0.85 x 0.85

+ **0.15** x 0.85 x 0.85 x **0.15** x 0.85

+ **0.15** x 0.85 x 0.85 x 0.85 x **0.15**

+ 0.85 x **0.15** x **0.15** x 0.85 x 0.85

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+ 0.85 x 0.85 x **0.15** x **0.15** x 0.85

+ 0.85 x 0.85 x **0.15** x 0.85 x **0.15**

+ 0.85 x 0.85 x 0.85 x **0.15** x **0.15**

$= 10 * .15^2 * .85^3$

= 0.138

The probability that <u>no people</u> P(X=0) of 5 have the disease = .444

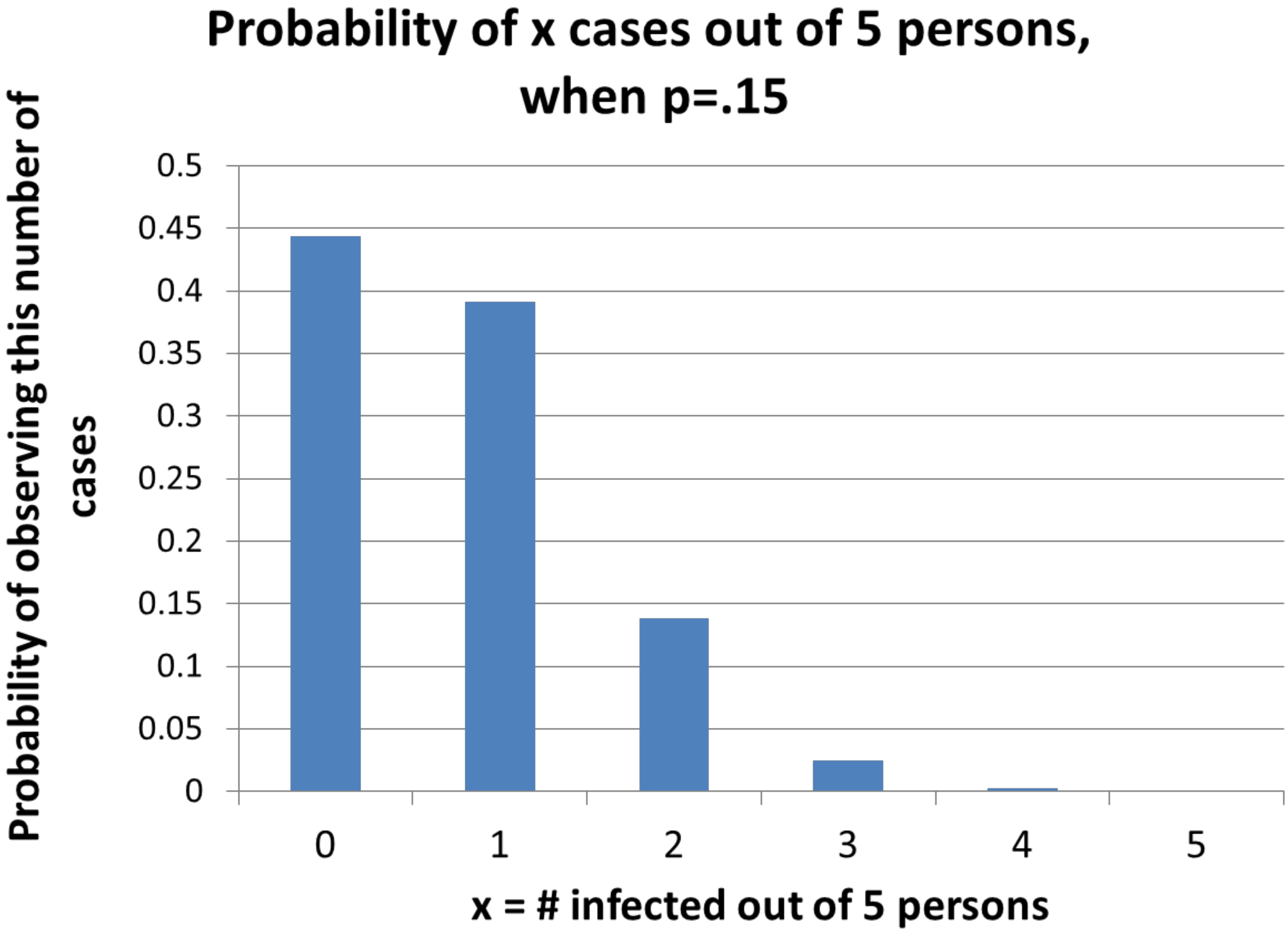
The probability that <u>exactly one person</u> P(X=1) of 5 has the disease = .392

The probability that <u>exactly two people</u> P(X=2) of 5 have the disease = .138

The probability that <u>exactly three people</u> P(X=3) of 5 have the disease = .024

The probability that <u>exactly four people</u> P(X=4) of 5 have the disease = .002

The probability that <u>exactly five people</u> P(X=5) of 5 have the disease = .00008



Probability of x cases out of 5 persons, when p=.15
Probability of observing this number of cases
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
x = # infected out of 5 persons

The probability that exactly one person $P(X=1)$ has the disease

$P(X=1; n=5, p=0.15)$

= 0.15 x 0.85 x 0.85 x 0.85 x 0.85

+ 0.85 x **0.15** x 0.85 x 0.85 x 0.85

+ 0.85 x 0.85 x **0.15** x 0.85 x 0.85

+ 0.85 x 0.85 x 0.85 x **0.15** x 0.85

+ 0.85 x 0.85 x 0.85 x 0.85 x **0.15**

= 0.392

$= 5 * .15^1 * .85^4$

$= 5 * p^1 * (1-p)^4$

5 is the number of different ways you could get one "success" in the 5 "trials"

Binomial distribution

This generalizes to: $$P(X=x)=\binom{n}{x}p^{x}(1-p)^{n-x}$$

- *p* is probability of “success” in each “trial”
- *n* is the number of “trials” (e.g., coin flips, persons assessed for disease status, etc.)
- *n* and *p* are the parameters of the binomial distribution, i.e. the values that summarize the distribution
- x is the number of “successes” (e.g. heads, numbers with the disease, etc.)

Binomial distribution

$$P(X = x) = \binom{n}{x} p^{x} (1 - p)^{n-x}$$

This is the formula for the binomial distribution

- Note that Stata uses the symbol k for x

Binomial distribution

- Assumptions:
  - There are a fixed number of trials *n*
  - Within each trial, there are two mutually exclusive outcomes (heads vs. tails, disease vs. no disease)
  - The outcomes of the *n* trials are independent
  - The probability of success *p* is constant for each trial

$\binom{n}{x}$ is called “n choose x” and is the number of different ways to get x successes in n trials

There are 5 ways that there could be 1 success in 5 trials

There are 10 ways there could be 2 successes in 5 trials

- The formula for n choose x is

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \quad \text{where } n! = n * \ldots * 3 * 2 * 1$$

5 choose 1 = 5! / (1! * 4!)
= (5*4*3*2*1) / (1*4*3*2*1)
= 5
5 choose 2 = 5! / (2! * 3!)
= (5*4*3*2*1) / (2*1*3*2*1)
= 5*4/2 = 10
5 choose 3 = 5! / (3! * 2!) = 10

In Stata: display comb(n,k)

```
. display comb(5,3)
10
```

$$\binom{n}{0} = 1$$

$$\binom{n}{1} = n$$

$$\binom{n}{n} = 1$$

- Ways to find binomial probabilities
  - The previous equations
  - Stata
    - Binomialp(n,k,p)
    - Binomialtail(n,k,p)
  - Table A.1 in the textbook

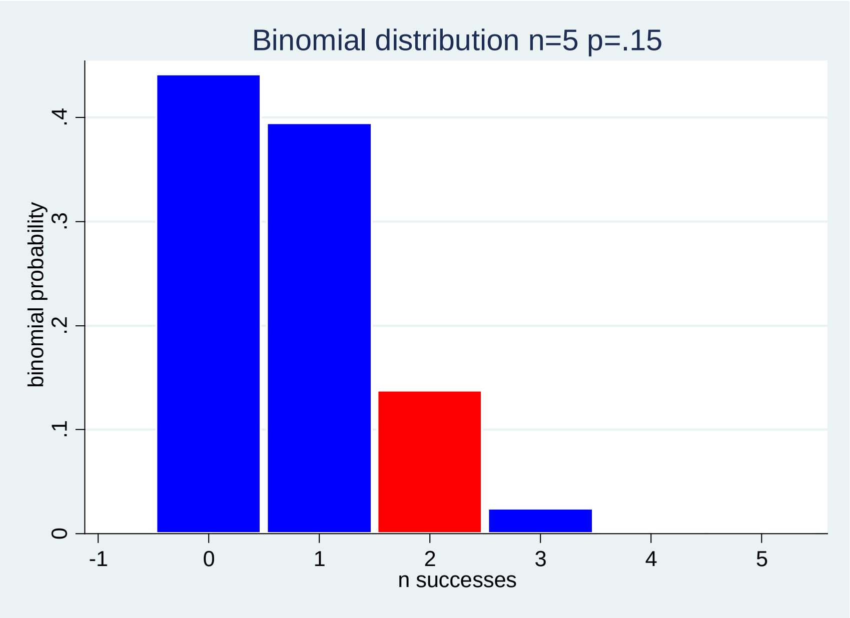
The binomial formula

What is the probability of exactly 2 cases of disease in a sample of n=5 where p=0.15?

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X = 2) = \binom{5}{2} .15^2 (.85)^3$$

```
. di comb(5,2)*.15^2*.85^3
.13817812
```

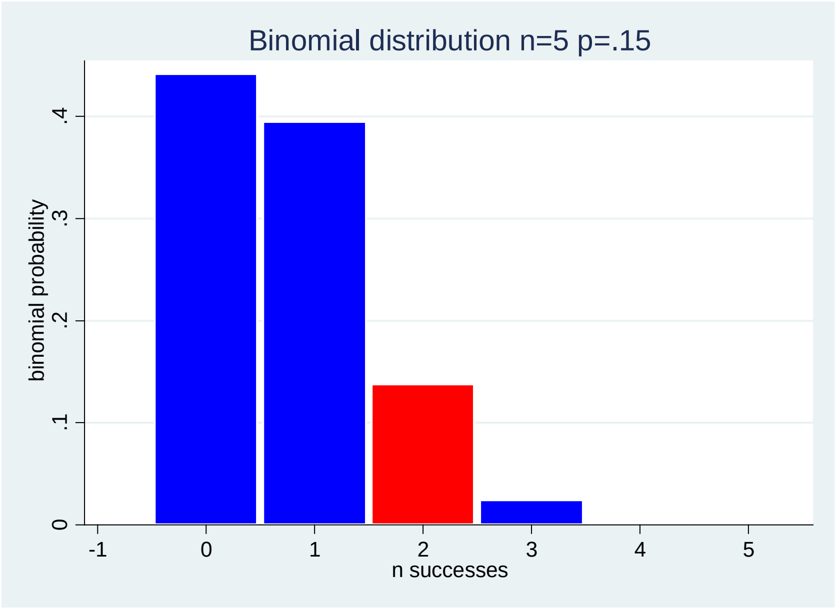


Stata binomialp()

What is the probability of exactly 2 cases of disease in a sample of n=5 where p=0.15?

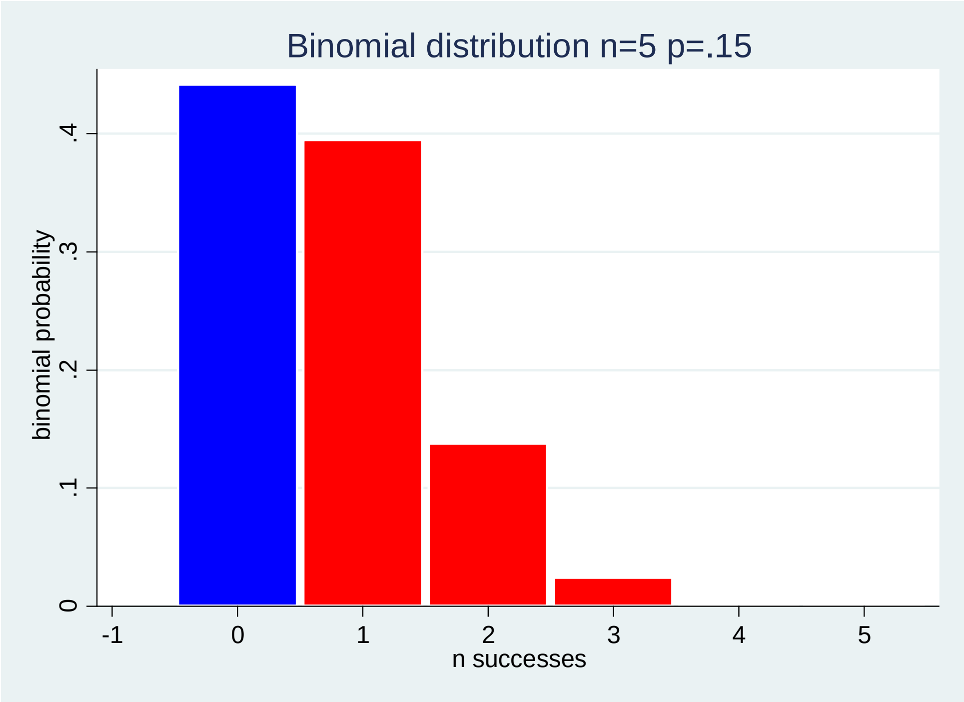
–Use binomialp(n,k,p) to get P(X=k) in n trials with probability of success in each trial=p

- `di binomialp(5,2,.15)`
  `.13817813`



What is the probability of 1 or more cases of disease in a sample of n=5 where p=0.15?

- We want P(X≥1)
- One way would be to calculate all the probabilities: P(X=1)+P(X=2)+ ... +P(X=5)
- But remember P(X≥1) = 1-P(X=0)



Binomial formula

What is the probability of 1 or more cases of disease in a sample of n=5 where p=0.15?

- $P(X \geq 1) = 1 - P(X=0)$

$$P(X=0) = \binom{5}{0}.15^0(.85)^5 = 1*.85^5$$

```
    di comb(5,0)*.85^5
.44370531
```

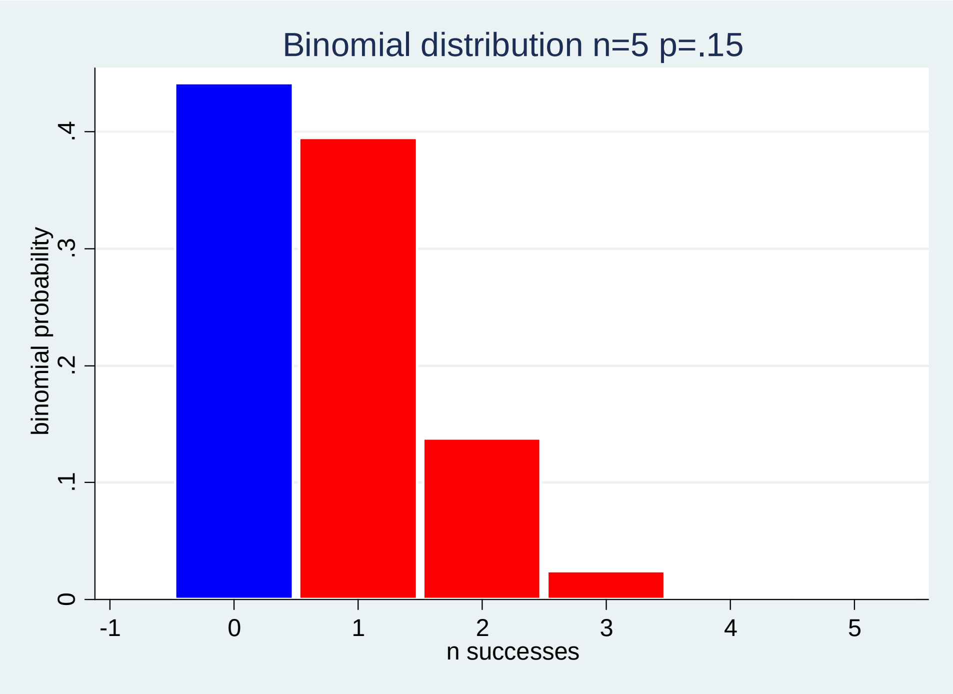
- So $1-P(X=0) = 1- 0.4437 = 0.5563$
- Or use

```
display 1-binomialp(5,0,.15)
.55629469
```

Stata binomialtail()

- The function <u>binomialtail(n,k,p)</u> gives us P(X≥k) so we can use it without manipulation
- `display binomialtail(5,1,.15)`

```
    .55629469
```



- Example. The probability that my husband will serve an ace in tennis at each point is .05. What is the probability that he will serve a perfect game, that is serve 4 aces in a row (assume each serve independent of the previous)?

$$P(X = 4) = \binom{4}{4} .05^4 (.95)^0 = 1 * .05^4$$

```
.di .05^4
6.250e-06
```

Or,

```
. di binomialp(4,4,.05)
6.250e-06
```

- Example. The probability that my husband will serve a double fault at each point is .25. Assuming each point is independent, what is the probability that he will serve 4 double faults in a row?

$$P(X = 4) = \binom{4}{4} .25^4 (.75)^0 = 1 * .25^4$$

```
. di .25^4
.00390625
```

- Or,

```
di binomialp(4,4,.25)
    .00390625
```

What is the probability that he will serve at least 1 double fault in a game that goes on for 6 points (again, assuming each point is independent)?

n= ?

p= ?

k= ?

$P(X \geq 1)$ = ?

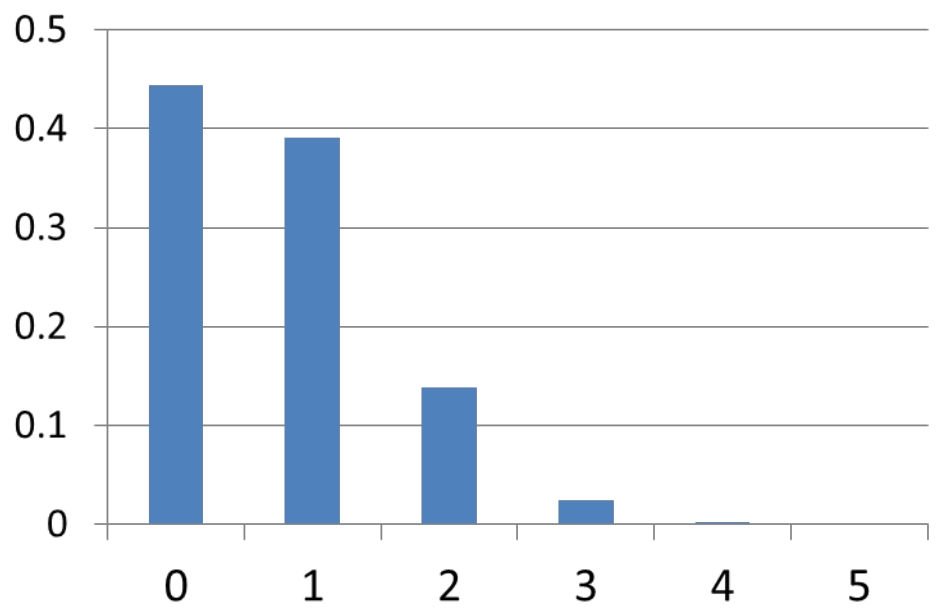
Means and Variances

- In Lecture 1 we learned how to calculate the sample mean, variance, and standard deviation
- Theoretical distributions have theoretical means, variances, and standard deviations
- For a discrete distribution, the mean of the distribution is $\sum_{i=0}^{n} x_i \, P(X = x_i)$ over all the possible values of x.

Binomial distribution

- The <u>mean</u> of a binomially distributed random variable X is *np*
- This means that over an large number of samples of size n with probability p of success, the mean number of successes ($\bar{x}$) over the samples will be approximately *np*
- In shorthand, *np* is the mean of the binomial distribution

- For our example with n=5 and p=P(disease)=.15 the mean of the distribution of the number of cases is 5*.15 = .75
  - If you took a sample of n=5, repeatedly, the mean number of cases over these samples would be 0.75
  - Does this fit with the graph of the distribution?

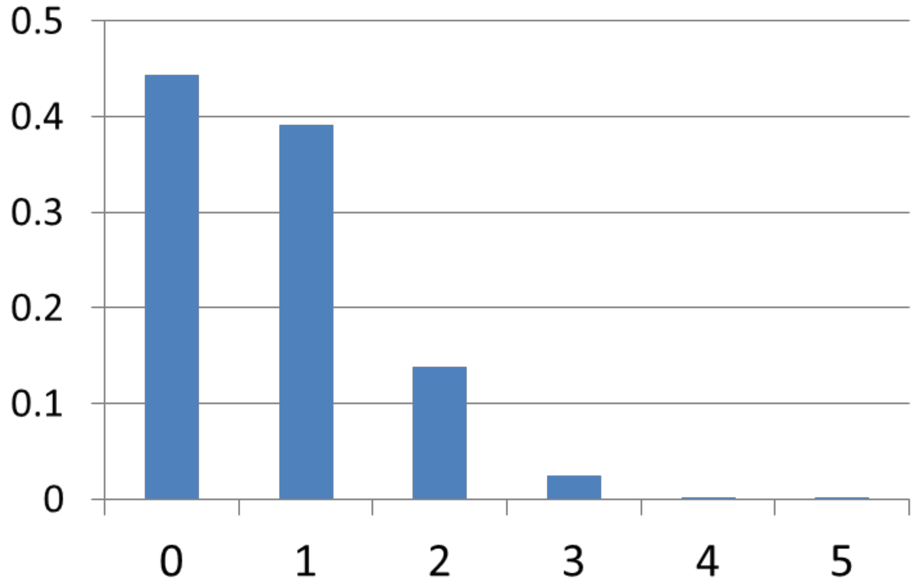


Binomial distribution

- The variance of a binomially distributed random variable X is *n*p*(1-p)*
- This means that over a large number of samples of size n, the *sample variance* of the X's, the number of successes, will be approximately *n*p*(1-p)*
- The standard deviation is

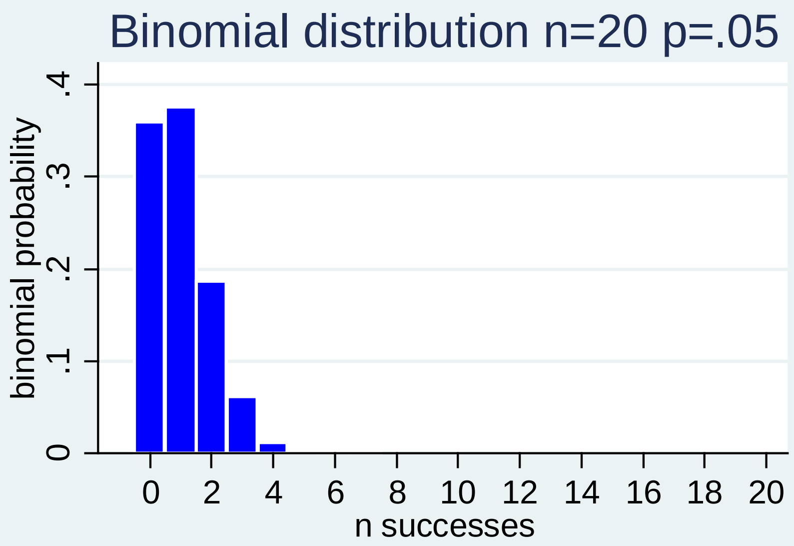
$$\sqrt{np(1-p)}$$

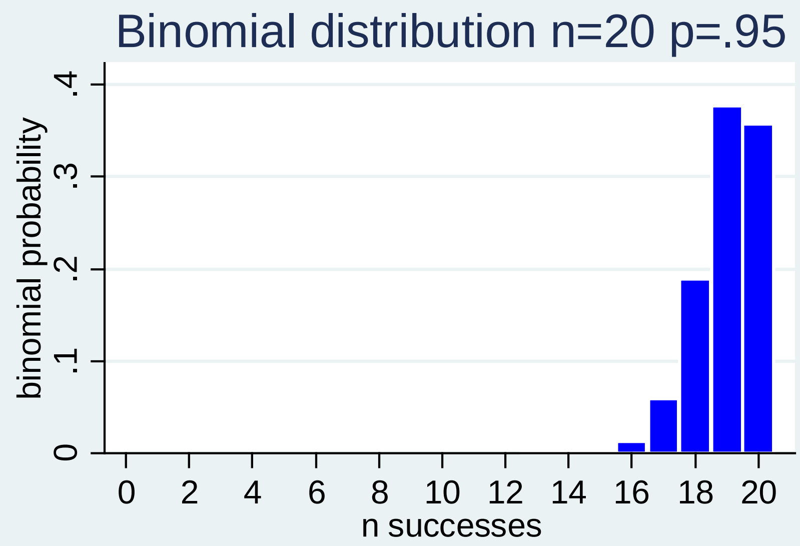
- For our example with n=5 and p=P(disease)=.15, the variance is n*p*(1-p) = 5*.15 *.85
  – If you took a sample of n=5, repeatedly, the sample variance in the number of cases over these samples would be this number

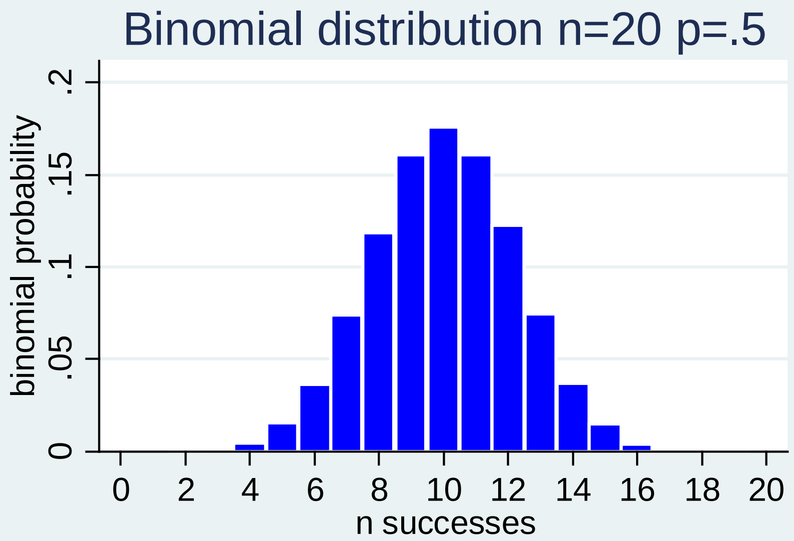


Binomial distribution

- Binomial mean = np
- Binomial variance= np(1-p)
  - Variance is largest when p=0.5, smaller when p closer to 0 or 1
  - The distribution is symmetric when p=0.5
  - The distribution is a mirror image for 1-p
    - E.g. the distribution for p=0.05 is the mirror image of the one for p=0.95


Binomial distribution n=20 p=.05
binomial probability
0
.1
.2
.3
.4
0
2
4
6
8
10
12
14
16
18
20
n successes


Binomial distribution n=20 p=.95
binomial probability
0
.1
.2
.3
.4
0
2
4
6
8
10
12
14
16
18
20
n successes


Binomial distribution n=20 p=.5
binomial probability
0
.05
.1
.15
.2
0
2
4
6
8
10
12
14
16
18
20
n successes

P(X=2) ?

P(X≥2) ?

Poisson distribution

- A discrete distribution to model rare events occurring in time or space
- Unlike the binomial distribution, which is based on a series of n trials, there is no theoretical limit to the number of events that can occur
- However, when n is large and p is small, it does act like the binomial
- The Poisson has only one parameter, λ, that is the mean number of events (and also the variance)

Normal distribution

- Used for <u>continuous variables</u> that cover the entire range, i.e. values can take on 1.432, -72.12
- Classic bell shaped curve
- Values can span from $-\infty$ to ∞
- Uni-modal and symmetric, so the mean is also equal to the median and mode

Normal distribution

- The probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{where } -\infty < x < \infty$$

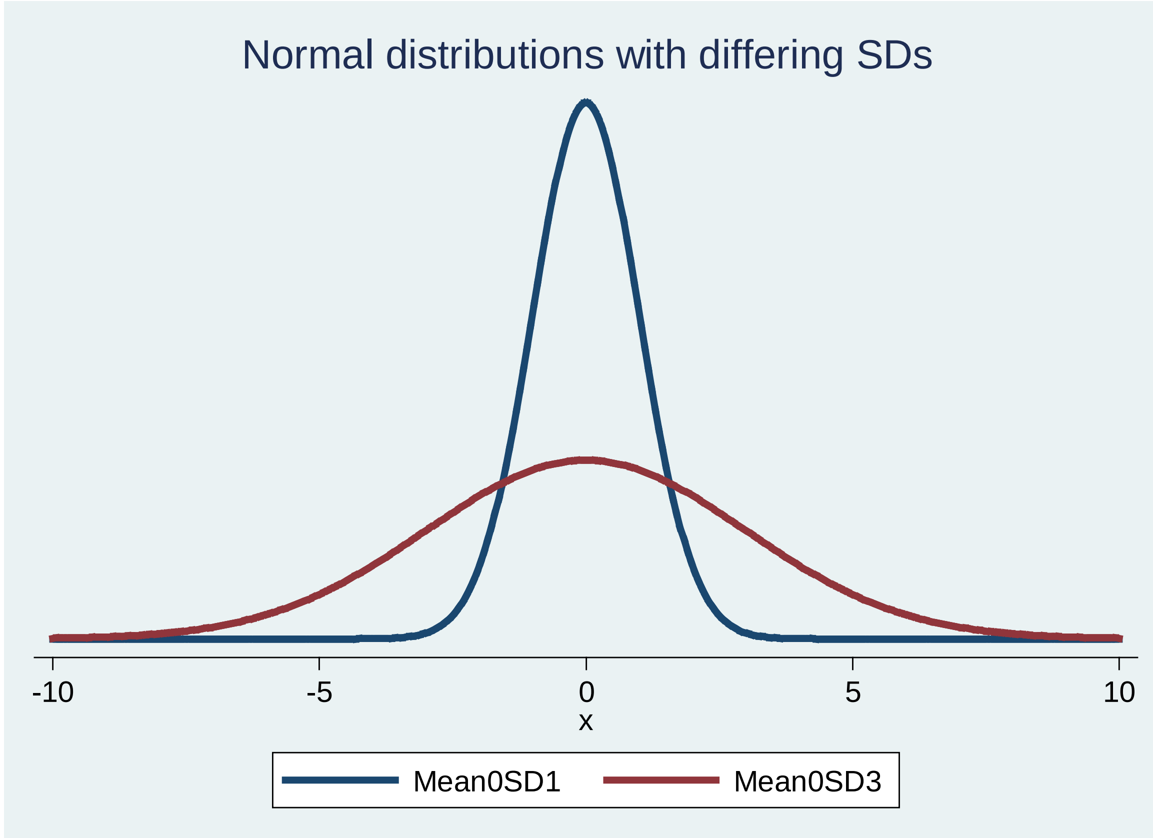
- μ is the mean and σ is the standard deviation of a normally distributed random variable
  - They are the parameters of the normal distribution
  - π is the constant that is approximately 3.14159

- Note that the left hand side of the equation is f(x) and not P(X=x)
- Why?
  - For a discrete distribution, the sum of the bars equals 1
  - For a continuous distribution, the area under the curve equals one
  - A continuous variable X can take on an infinite number of values, therefore P(X=x)=0

- If X has a normal distribution with mean μ and standard deviation σ we write

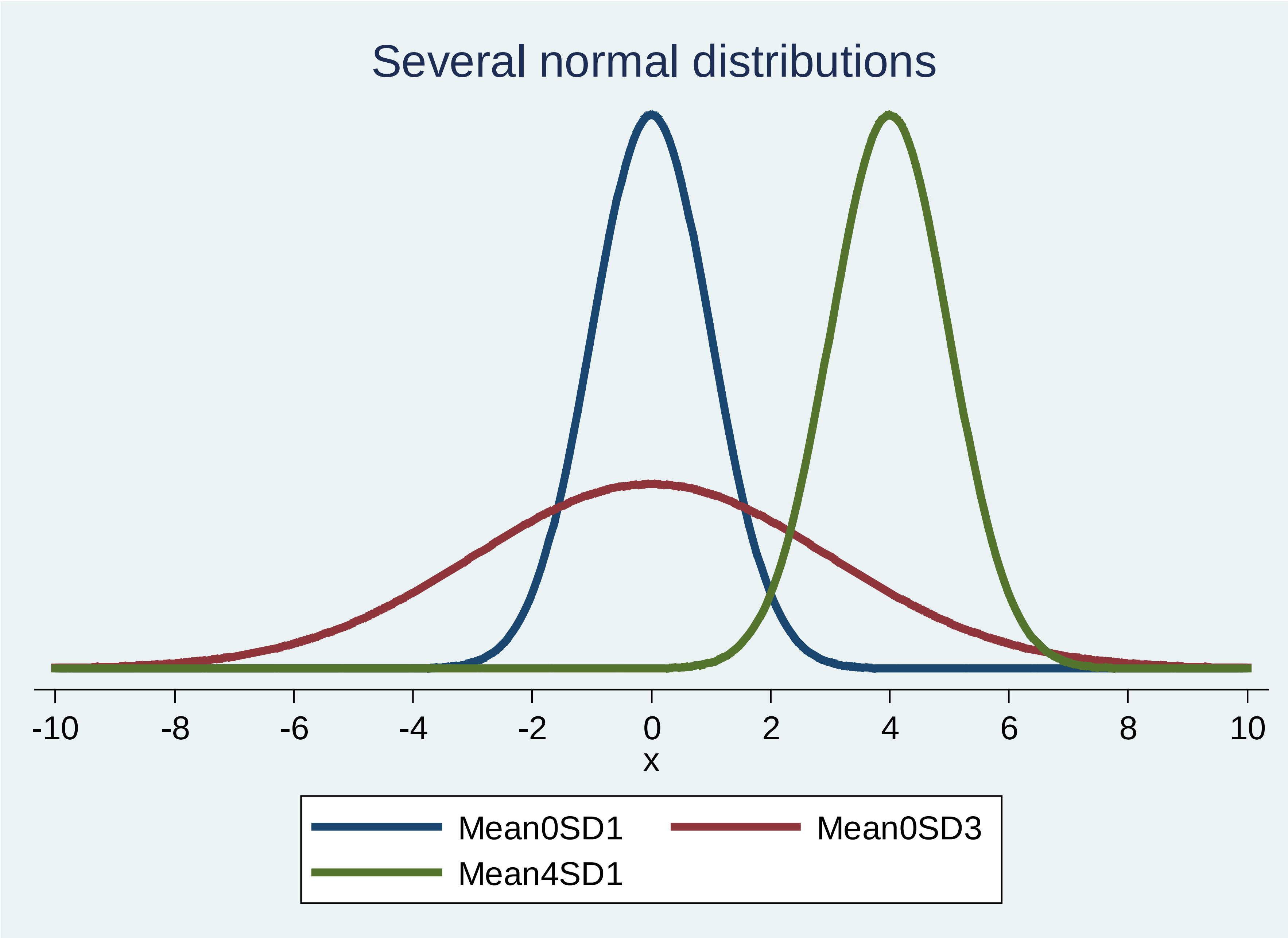
$$X \sim N(\mu, \sigma)$$

- Many variables are approximately normally distributed
- We can use the distribution to calculate probabilities associated with such variables



The standard deviation defines the amount of spread around the mean

- Small standard deviation – little spread around the mean (blue line SD=1)
- Large standard deviation – greater spread around the mean (red line SD=3)


Several normal distributions
-10
-8
-6
-4
-2
0
2
4
6
8
10
x
Mean0SD1
Mean0SD3
Mean4SD1

The Standard Normal Distribution

- μ and σ can take on an infinite number of values so we have an infinite number of curves
- For simplicity, we have a standard curve that we use as a reference

The Standard Normal Distribution

- This one curve has mean μ =0 and standard deviation σ =1 (and variance σ^2=1).
- Denoted N(0,1)
- The formula for the distribution simplifies to

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \text{ where } -\infty < x < \infty$$

The Standard Normal Distribution

- If X is a normally distributed random variable with mean μ and standard deviation σ then

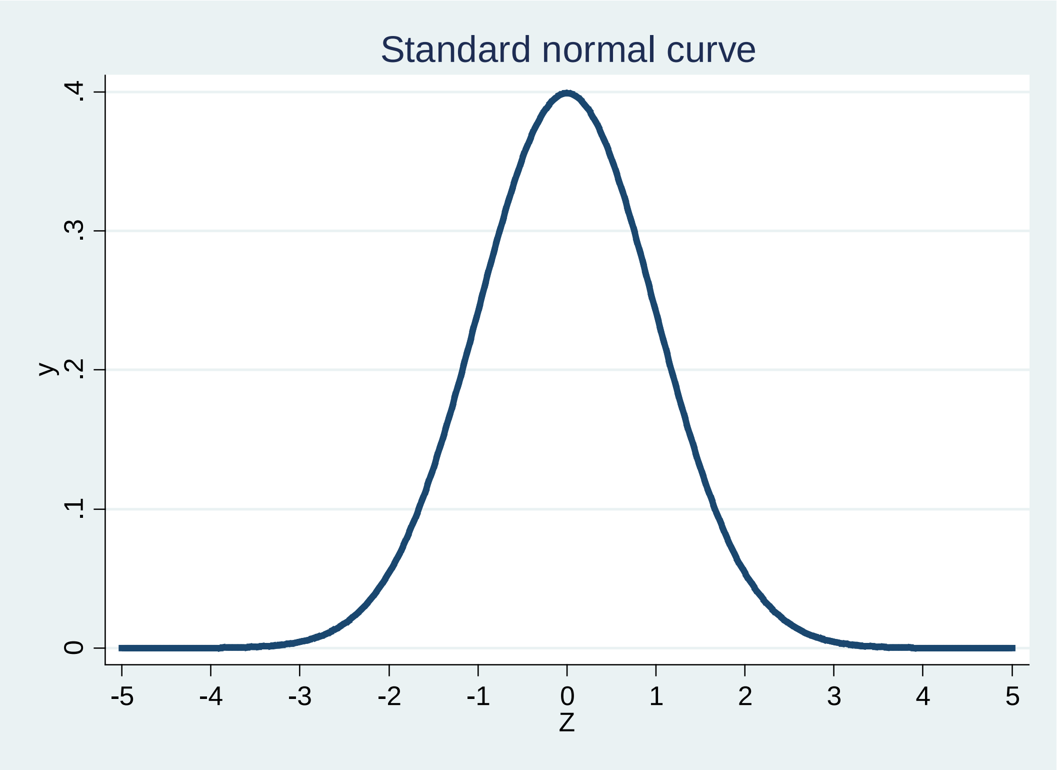
  $$Z = (X - \mu)/\sigma$$

  is a *standard* normal random variable

- That is, any normally distributed random variable with its mean subtracted off, divided by its standard deviation, is a standard normal random variable with mean=0 and standard deviation=1

The Standard Normal Distribution

- If $X \sim N(\mu, \sigma)$ then
- $Z = (X - \mu) / \sigma \sim N(0, 1)$



- We can use theoretical distributions to determine the probability of particular values of random variables

- For the binomial distribution, we added probabilities of the assumed distribution to calculate the probability of observing a certain number (k) of events (or more).

- Remember the probability of observing 1 or more disease cases in a sample of 5 was equal to

$$P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

- However, for a continuous variable, because there are an infinite number of values, we can't calculate P(X=x).

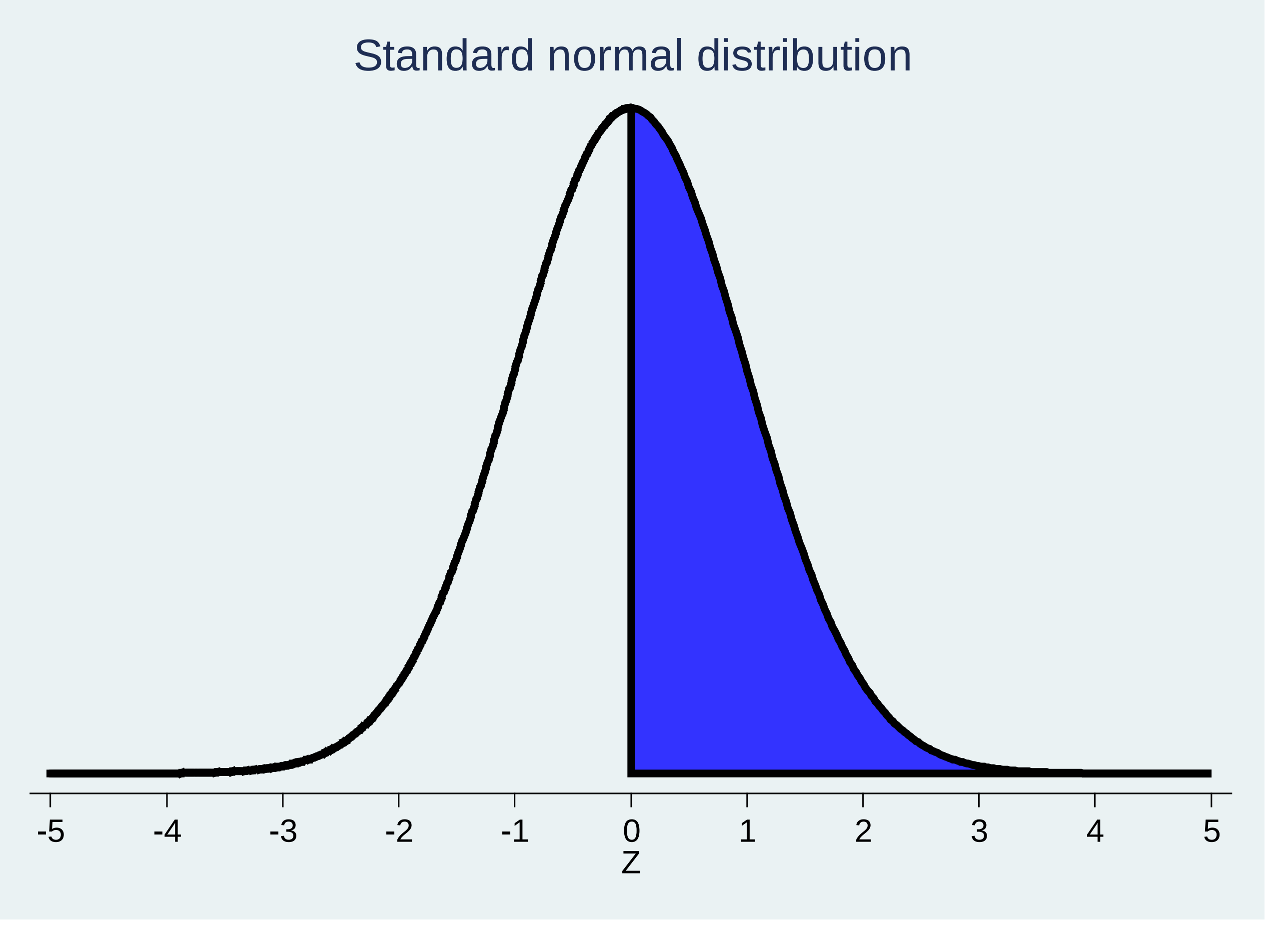
- However, we can calculate P(X ≥ x), which is the area under the normal curve from x to infinity

- The area under curves is calculated by taking the integral

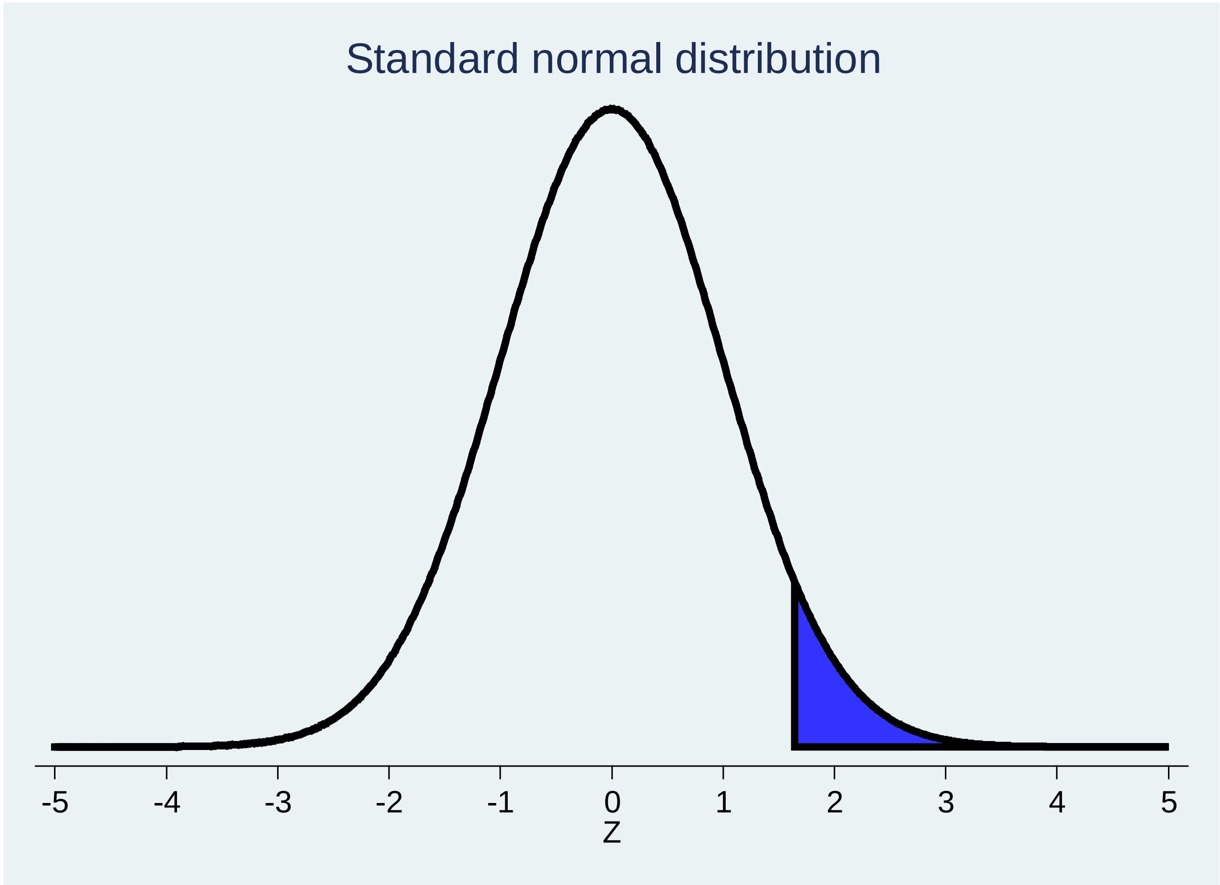
$$P(X \geq x) = \int_{x}^{\infty} f(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

For Z ~ N(0,1), P(Z≥0) = 0.50

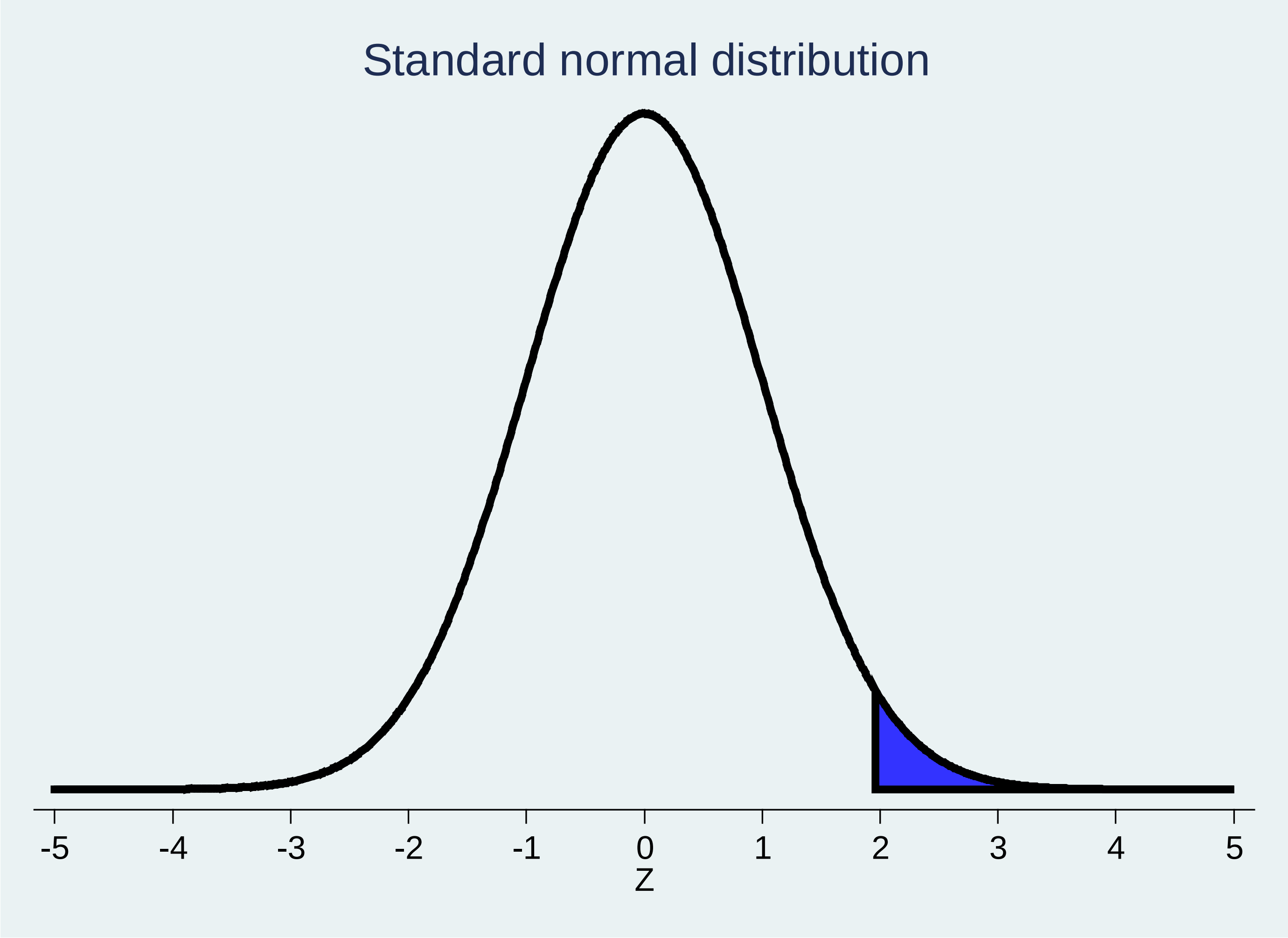
Remember that for the standard normal distribution that 0 is not only the mean but also the median of the standard normal distribution. Therefore the probability of observing the median value or more is automatically 0.5 .



For Z ~ N(0,1) $P(Z \geq 1.65) = 0.049$

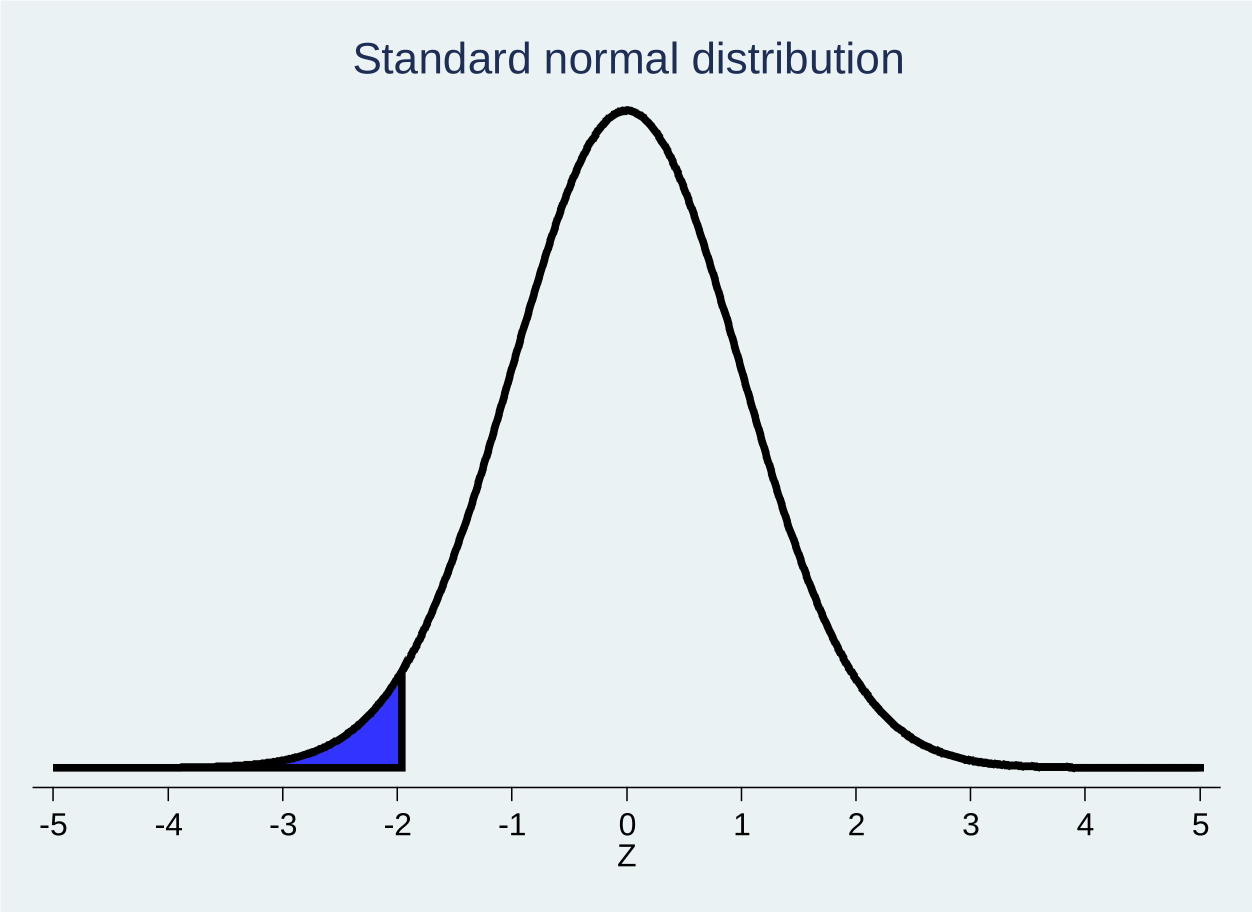


For Z ~ N(0,1) P(Z≥1.96) = 0.025



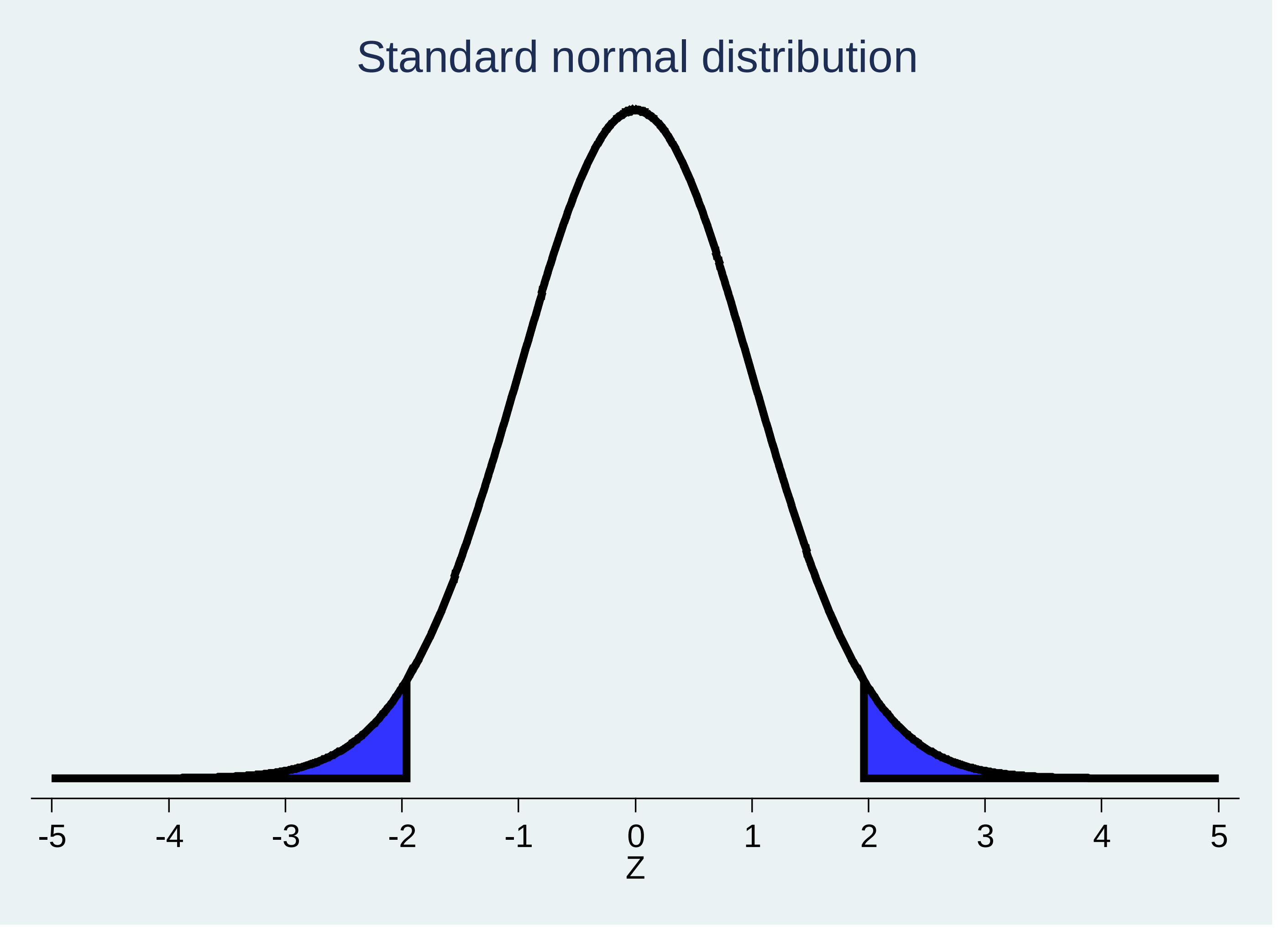
For Z ~ N(0,1) P(Z<-1.96) = 0.025

Z is symmetric



P(Z ≤ -1.96 or Z≥1.96) ?

P(-1.96 ≤ Z ≤ 1.96) ?

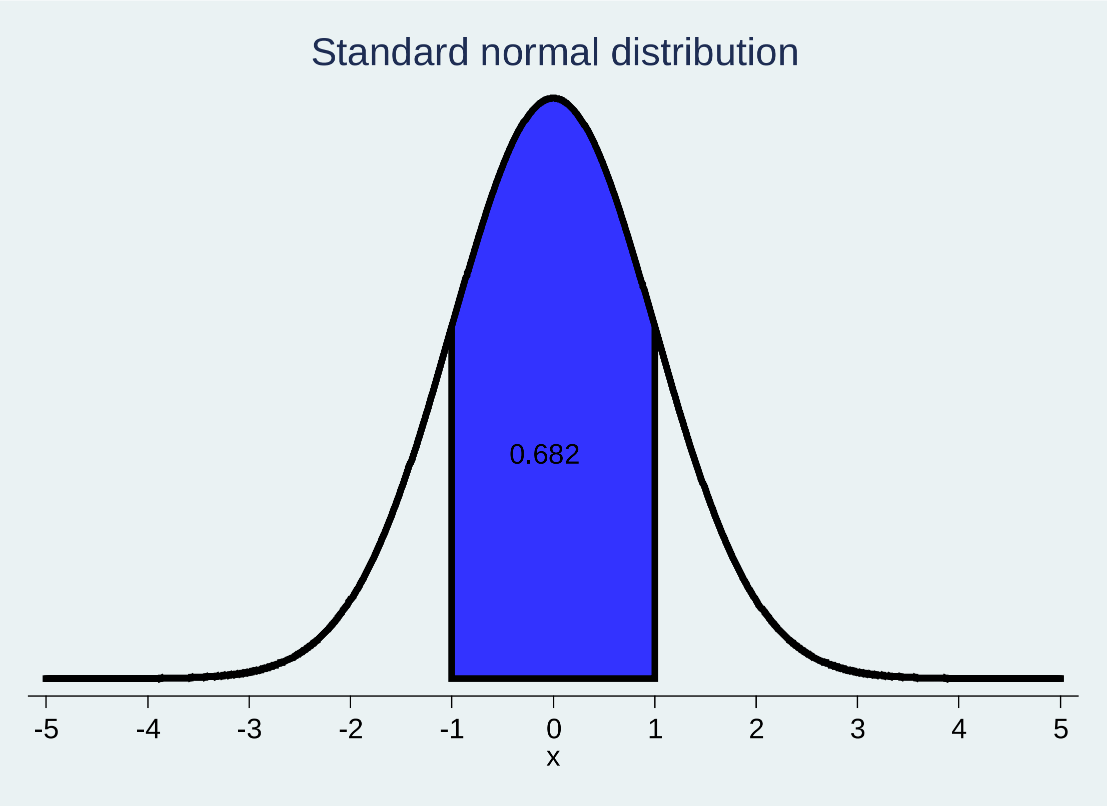


$P(\mu - 1\sigma \leq Z \leq \mu + 1\sigma)$

Remember $\mu=0$ and $\sigma=1$, so this is

$P(-1 < Z < 1) = 0.682$

Therefore, approximately 68.2% of the area of the standard normal is within 1 SD of the mean.

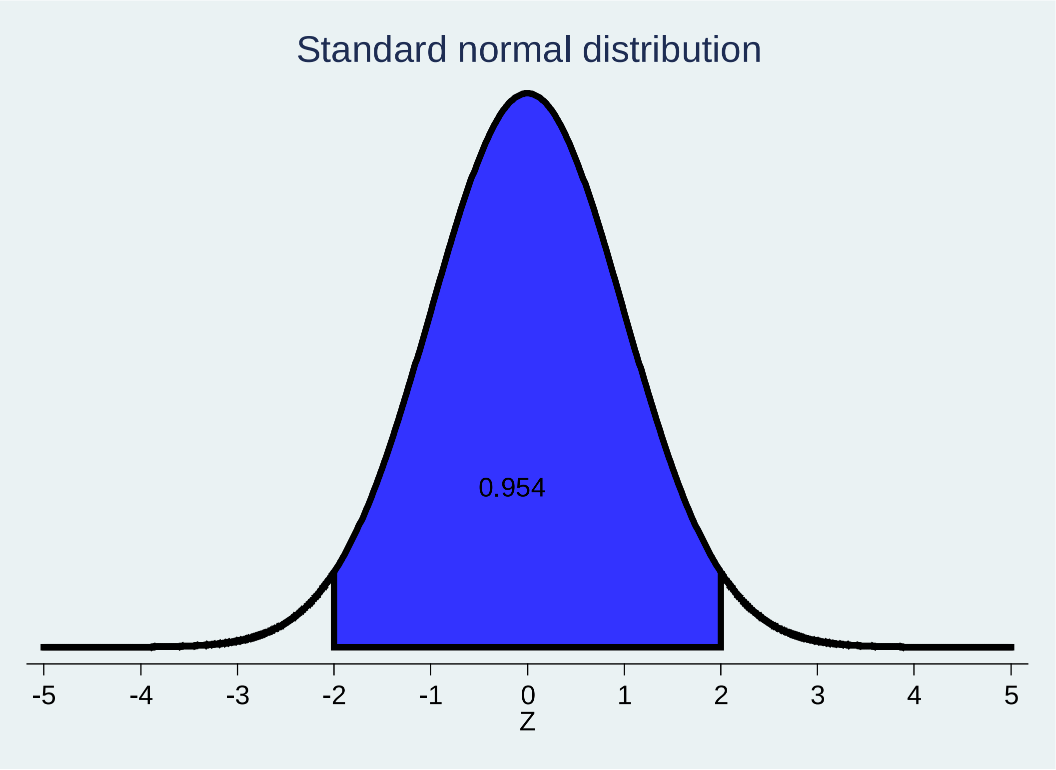


$P(\mu-2\sigma \leq Z \leq \mu+2\sigma)$

Remember $\mu=0$ and $\sigma=1$, so this is

$P(-2 < Z < 2) = 0.954$

Therefore, approximately 95.4% of the area of the standard normal is within 2 SD of the mean.



- Stata will calculate standard normal probabilities for you

- In Stata, the left portion of the curve P(Z<z) is calculated for you.

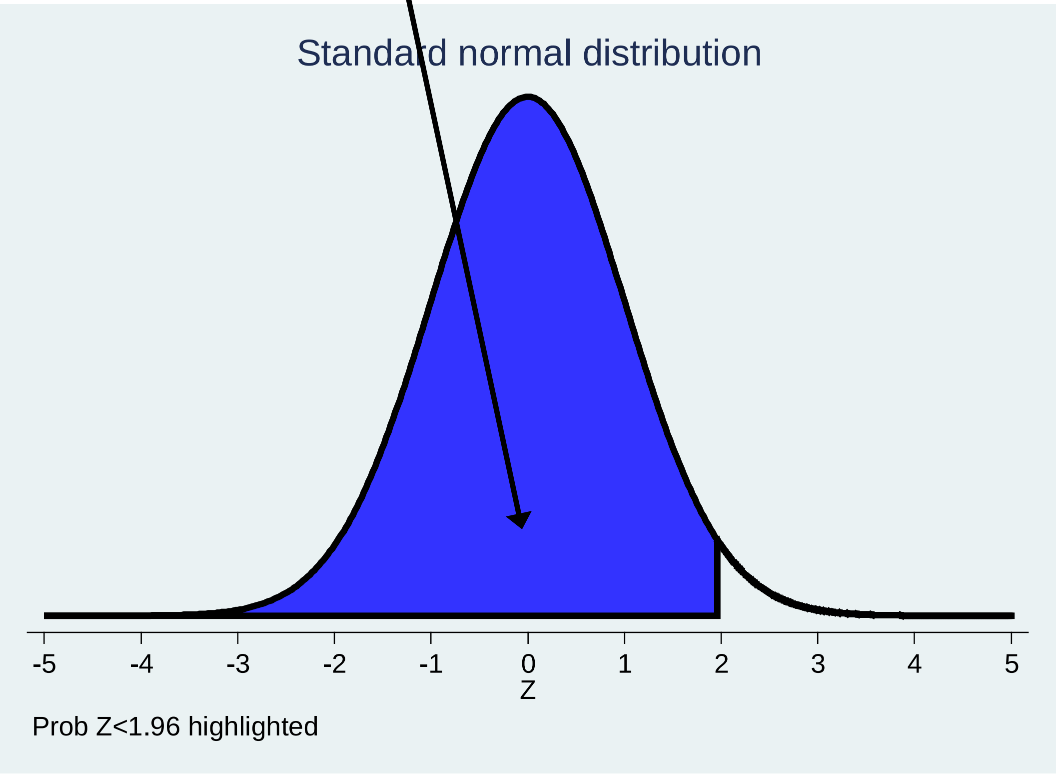
```
*** this gives you P(Z<1.96)
display normal(1.96)
.9750021
```

- If you want the right hand portion of the curve, P(Z≥z), you subtract your answer from 1

```
** this gives you P(Z<1.96)
display 1-normal(1.96)
.0249979
```

- If you want the middle:

```
 display normal(1.96)
             -normal(-1.96)
.95000421
```



Example

- X is a random variable representing the distribution of systolic blood pressure in 18-74 year old U.S. males

  ~N(129, 19.8)
- What is the proportion in the population with systolic blood pressure of over 150 mm Hg? P(X>150) ?

1. Convert the variable of interest to a standard normal variable N(0,1) to get the probability. I.e., subtract off the mean and divide by the SD.

   z=(150-129)/19.8 = 1.06 This is the z-score or z-statistic
2. Find P(Z>z) = P(Z>1.06) using Stata

```
. di 1-normal(1.06)
.1445723
```

Example

- The distribution of systolic blood pressure in 18-74 y.o. US males is ~N(129, 19.8)

- What is the upper 2.5% value for SBP in this population?

1. Find the z value that satisfies this, and then convert back to the original distribution.

   What is the value of z for which $P(Z \geq \mathbf{z}) = 0.025$?

   ```
   display invnormal(.975)
    1.959964
   ```

   Answer z=1.96

2. Transform back to the original units

   $z = 1.96 = (x-129)/19.8$

   $x = 1.96 * 19.8 + 129 = 167.8$ mm Hg

Example

- What is the lower 2.5% value for SBP?
- What is the value of z for which P(Z<z)=0.025?

1. Find the z value that satisfies this, and then convert back to the original distribution.

```
. display invnormal(.025)
-1.959964
```

z=-1.96

2. Transform back to the original units

   z=-1.96=(x-129)/19.8

   x=-1.96*19.8+129 = 90.2 mm HG

- So 95% of the population has SBP between 90.2 and 167.8

- So if you have a variable that is normally distributed and you know the mean and variance, you can find the values that comprise the middle 95% (or 99% or 90%) <u>of the population</u> Note that this is not a confidence interval
- For the middle 95%, the interval is

$$\mu-1.96*\sigma, \ \ \mu+1.96*\sigma$$

- For the middle 99%, the interval is

$$\mu-2.58*\sigma, \ \ \mu+2.58*\sigma$$

- Note that to include a higher %age, the interval gets wider!

Finding z values for probabilities in Stata

- To get the z value for P(Z<z) = p use

```
display invnormal(p)
```

E.g. what is the z value for P(Z<z) = 0.025

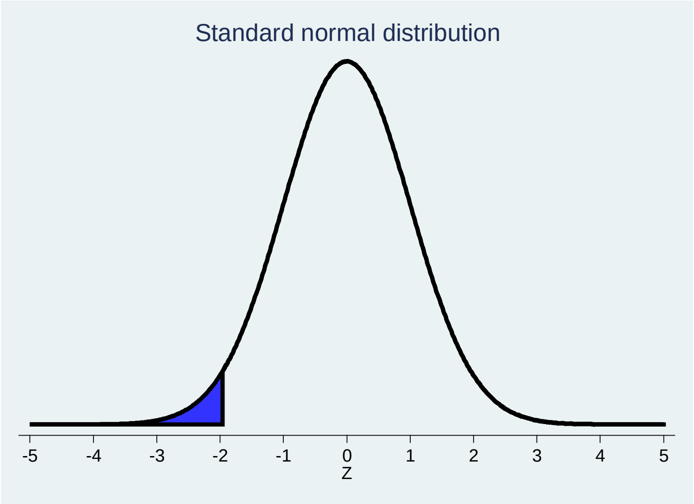
```
display invnormal(0.025)
 -1.959964
```

- To get the z value for P(Z≥z) = p use

```
display invnormal(1-p)
```

E.g. what is the z value for P(Z ≥ z) = 0.025

```
display invnormal(1-.025)
 1.959964
```



Key points

- For discrete probability distributions, you can calculate $P(X=x)$
- The binomial distribution gives the probability of the number of successes in n trials $P(X=x)$
- For continuous probability distributions, you can only calculate $P(X>x)$ or $P(X<x)$
- The normal distribution describes some continuous data – we'll see some very useful properties next week
- We transform to the standard normal distribution in order to work with the probabilities

For next time

- Read Pagano and Gauvreau
  - Chapter 7 (Review of today’s material)
  - Chapter 8, 9, and 14 (pages 324-329)