Lecture 8 2/26/2016

$$\begin{tikzcd} S \arrow[r, bend left, "\lambda(S,I)"] & I \arrow[l, bend left, "\gamma"] \end{tikzcd}$$

$$\frac{dS}{dt} = -\lambda(S,I)S + \gamma I$$

$$\frac{dI}{dt} = \lambda(S,I)\,S - \gamma I = \lambda(S,I)(N-I) - \gamma I$$

$$\frac{d(S+I)}{dt} = \frac{dN}{dt} = 0$$

$$\frac{dI}{dt} = \frac{\beta I}{N}(N-I) - \gamma I$$

$I = 0$ for all time is a solution

Disease free equilibrium

if I is large, $\frac{dI}{dt}$ is negative

$I^{(0)}$ index cases

$$\frac{dI^{(0)}}{dt} = -\gamma I^{(0)} \Rightarrow I^{(0)} = I^{(0)}(0)e^{-\gamma t}$$

constant hazard, exponential decay

cumulative incidence of new infectives?

$$\frac{dI^{(+)}}{dt} = \beta \frac{I^{(0)}}{N}(N - I^{(0)}) \approx \beta I^{(0)}$$

$$dI^{(+)} = \beta I^{(0)} dt$$

$$I^{(+)}(t) = \int_0^t \beta I^{(0)} dt = \beta I^{(0)}(0) \int_0^t e^{-\gamma t} dt$$

$$= \frac{\beta I^{(0)}(0)}{\gamma} (1 - e^{-\gamma t})$$

$$I^{(+)}(\infty) = \frac{\beta I^{(0)}(0)}{\gamma}$$

$$\frac{I^{(+)}(\infty)}{I^{(0)}(0)} = \frac{\beta}{\gamma}$$

Number of new cases ever caused by that first generation per number of first generation cases

$$\boxed{R_0 = \frac{\beta}{\gamma}}$$

$\frac{1}{\gamma}$ how long you have it X

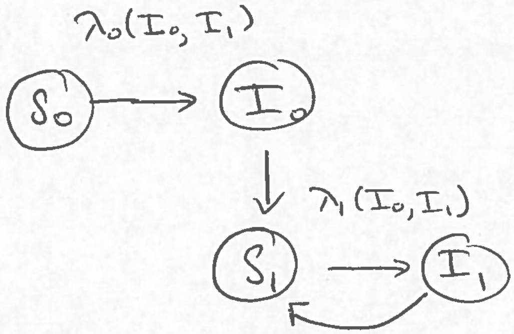
β how fast you make new cases

$\bar{S} = \frac{1}{R_0}$ equilibrium frxn of susceptibles (if $R_0 > 1$)

↳ at equilibrium each case can only replace itself

$$\bar{I} = 1 - \bar{S} = 1 - \frac{1}{R_0}$$

Immunity to Reinfection



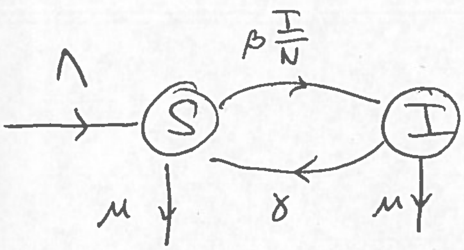
$\lambda_0(I_0, I_1) < \lambda_1(I_0, I_1)$

Write 4 differential equations corresponding to this problem for homework

$S_0 \to I_0 \to S_1 \to I_1 \to S_2 \to I_2 \to \ldots$

Demography - SIS w/ turnover

write equations
write R_0 for this model
for homework

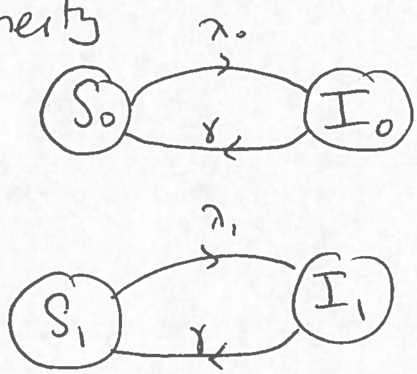


Flow out of $I = (\gamma + \mu) I$

Waiting time in I - exponential @ rate $\underline{\underline{\gamma + \mu}}$

Mean duration $\frac{1}{\gamma + \mu}$

Heterogeneity



λ_i

$\frac{\partial \lambda_i}{\partial I_j} \geq 0$ If you increase the infectives in any group, you increase (or keep the same) the risk in every group

$c_0,\ c_1$ # of unprotected partnerships in each group

$$\lambda_i = c_i \times \ldots$$

00 0 from 0

01 0 from 1

⋮ ⋮ etc.

$$\begin{cases} \lambda_0 = c_0 \left(p_{00} \dfrac{I_0}{N_0} + p_{01} \dfrac{I_1}{N_1} \right) \beta \\ \lambda_1 = c_1 \left(p_{10} \dfrac{I_0}{N_0} + p_{11} \dfrac{I_1}{N_1} \right) \beta \end{cases}$$

β probability of infection given infected partner

Constraints:

$$\begin{cases} p_{00} + p_{01} = 1 \\ p_{10} + p_{11} = 1 \end{cases} \qquad N_1 c_1 p_{10} = N_0 c_0 p_{01}.$$

Random mixing:
$$p_{00} = \frac{c_0 N_0}{c_0 N_0 + c_1 N_1}$$

Home work problem

$$p_{00} = \frac{c_0 N_0 + c_1 N_1 q}{c_0 N_0 + c_1 N_1}$$

- Find other p's and work out what q is in public health terms

- How many cases in group 0 can a group 1 infection cause? + other 3 partial R_0's.