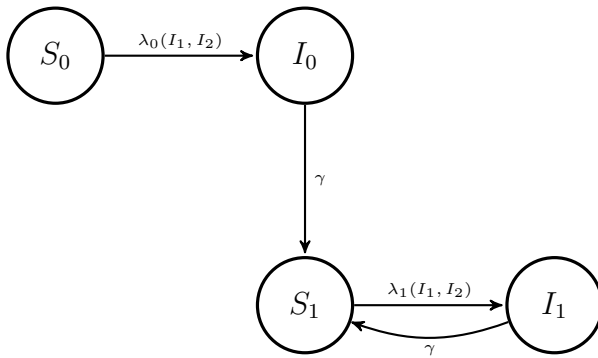


Homework 8

NB: Flow rates on compartmental model diagrams are per capita flow rates unless otherwise noted.

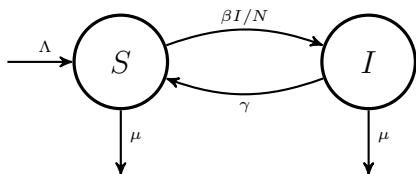
Exercise 1: Immunity to Reinfection 4pts

Suppose we want to model a disease where reinfection is possible, but individuals who have been previously infected have partial protection against reinfection. The compartmental model diagram below depicts such a model. Using this diagram, determine a $\lambda_0(I_1, I_2)$ and a $\lambda_1(I_1, I_2)$ that accomplishes partial immunity to reinfection and write the four differential equations corresponding to this model. Answers may vary. ■



Exercise 2: SIS Model with Death 3 pts

Write the differential equations and the R_0 for the model depicted below. [Λ is a constant inflow. You are not required to use the next generation method. You can write down R_0 and explain your reasoning.] ■



Exercise 3: SIS Model with Heterogeneity 6 pts



The model depicted above represents a heterogeneous-risk SIS model (e.g. STD model with a core group and a non-core group), where

$$\lambda_0 = c_0(p_{00}\frac{I_0}{N_0} + p_{01}\frac{I_1}{N_1})\beta$$

$$\lambda_1 = c_1(p_{10}\frac{I_0}{N_0} + p_{11}\frac{I_1}{N_1})\beta.$$

The model has the following constraints:

$$p_{00} + p_{01} = 1$$

$$p_{10} + p_{11} = 1$$

$$N_1 c_1 p_{10} = N_0 c_0 p_{01}$$

Suppose we parameterize p_{00} with a parameter $q \in [0, 1]$ as follows:

$$p_{00} = \frac{c_0 N_0 + c_1 N_1 q}{c_0 N_0 + c_1 N_1}$$

- Find p_{01} , p_{10} , and p_{11} in terms of c_0 , c_1 , N_0 , N_1 , and q .
- What is the parameter q in public health terms? (Hint: $q = 0$ corresponds to random mixing.)
- How many cases in group 0 can one group 0 infection cause in a fully susceptible population?
- How many cases in group 1 can one group 1 infection cause in a fully susceptible population?

- (e) How many cases in group 0 can one group 1 infection cause in a fully susceptible population?
- (f) How many cases in group 1 can one group 0 infection cause in a fully susceptible population?

■

Exercise 4 1 pt

How long did this assignment take you? ■