

Alternative Binary Regression Models; Logistic Regression for Case-Control Studies

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Biostatistics 208

Alternative regression models for binary outcomes

As discussed previously, there are situations where an alternative model for a binary outcome may be appropriate:

- When risk is thought to be related to predictors in a fashion other than assumed in the logistic model (i.e. linear and additive in the log odds)
 - For example, relative risk and risk difference-based models
- When risk of the outcome varies with time of follow-up in a prospective study
 - Survival analysis methods may be more appropriate in such situations
- In cases where external information suggests a specific formulation
 - HIV transmission example discussed in section 5.5.1 of text

Example: relationship between probability of coronary heart disease age and BMI in the WCGS study (excluding the extremely low value of BMI noted in the previous lab)

Logistic regression via binreg:

```
. binreg chd69 age bmi if bmi>12 & bmi!=., or
```

Generalized linear models	No. of obs	=	3153
Optimization : MQL Fisher scoring	Residual df	=	3150
(IRLS EIM)	Scale parameter	=	1
Deviance = 1727.411122	(1/df) Deviance	=	.5483845
Pearson = 3155.69054	(1/df) Pearson	=	1.001807

Variance function: $V(u) = u(1-u)$ [Bernoulli]

Link function : $g(u) = \ln(u/(1-u))$ [Logit]

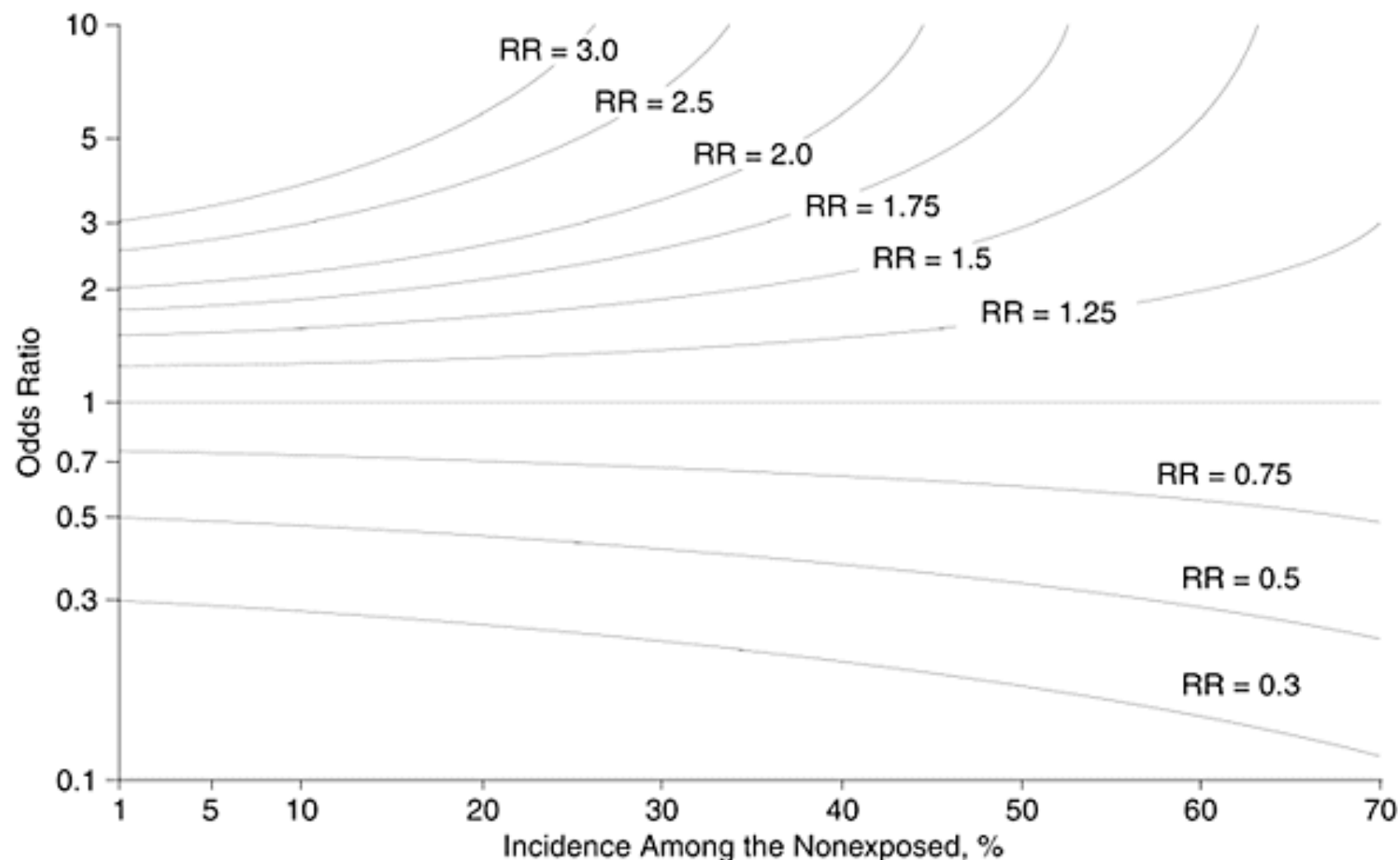
BIC = -23649.33

		EIM					
chd69		Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
age		1.076649	.0122104	6.51	0.000	1.052981	1.100849
bmi		1.08468	.0264501	3.33	0.001	1.034058	1.13778
_cons		.0003617	.0002961	-9.68	0.000	.0000727	.0017999

Note: The binreg comand in Stata allows fitting a number of alternative *generalized linear regression* models for binary outcomes, including *logit* (i.e. log odds), *log* (log probability), and *linear* (i.e. linear in probability).

Problem: Want to summarize age adjusted effect of BMI on CHD using a *relative risk* rather than an odds ratio

- *Why not use the logistic model and either assume that estimated ORs approximate RRs, or use the probability form of the model to estimate RRs?*
 - Odds ratios only approximate relative risks in the rare outcome setting (see *figure*)
 - Relative risks estimated using the probability form of the logistic model depend on other predictors in the model



Problem: Want to summarize age adjusted effect of BMI using a *relative risk* rather than an odds ratio

- **Solution:** Use a *log relative risk regression* model based on the log of the outcome probability:

$$\log P(\text{age}, \text{bmi}) = \beta_0 + \beta_1 \text{age} + \beta_2 \text{bmi}$$

- The coefficient β_2 is interpreted as the log relative risk associated with a unit change in bmi, holding age fixed.
- The coefficient β_1 is interpreted as the log relative risk associated with a unit change in age, holding bmi fixed.
- **Note:** recall that the coefficients for this model have to be estimated so that the left-hand side specifies a valid outcome probability (i.e. $0 \leq P \leq 1$, or equivalently $-\infty < \log P \leq 0$) for all values of bmi and age. This *constraint sometimes causes fitting problems* not encountered with the logistic model.
- Similar constraints and numerical difficulties arise in fitting additive risk difference models

Relative risk regression (continued)

- log relative risk models can be fitted in Stata using the `binreg` command:

```
. binreg chd69 age bmi if bmi>12 & bmi!=., rr iterate(100)
Generalized linear models                No. of obs      =      3153
Optimization      : MQL Fisher scoring    Residual df      =      3150
                  (IRLS EIM)              Scale parameter =          1
Deviance          =  1727.536638           (1/df) Deviance =  .5484243
Pearson           =  3149.490834           (1/df) Pearson  =  .9998384
Variance function: V(u) = u*(1-u)         [Bernoulli]
Link function     : g(u) = ln(u)           [Log]
```

BIC = -23649.21

chd69	EIM					
	Risk Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
age	1.068756	.0107595	6.61	0.000	1.047874	1.090053
bmi	1.073924	.0228021	3.36	0.001	1.03015	1.119558
_cons	.0005968	.0004258	-10.41	0.000	.0001474	.0024165

- Results quite similar to logistic model in this case
- Because of the constraint mentioned above, sometimes Stata will fail to estimate the parameters from a log rel. risk model. As shown in Lab 10, using the `iterate(100)` option stops estimation after 100 steps if convergence problems are encountered

Relative risk regression (continued)

- Approximate log relative risk regression for binary outcomes using the `poisson` regression command:

```
. poisson chd69 age bmi if bmi>12 & bmi!=., irr robust
```

```
Poisson regression                                Number of obs   =          3153
                                                    Wald chi2(2)    =          57.00
                                                    Prob > chi2     =          0.0000
Log pseudolikelihood = -876.93885                Pseudo R2      =          0.0270
```

chd69	IRR	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
age	1.068913	.0106896	6.66	0.000	1.048165	1.09007
bmi	1.075168	.0218344	3.57	0.000	1.033214	1.118825
_cons	.0005759	.0004083	-10.52	0.000	.0001435	.0023113

- This approach is generally **recommended** for models involving multiple predictors (especially those including continuous variables)
- Results very close to the estimates on the previous slide
- robust variance option required** to obtain appropriate standard error estimates

Reference: Zou G.A Modified Poisson Regression Approach to Prospective Studies with Binary Data. Amer J Epidemiol 2004; 159:702-6.

Example: coronary heart disease, age and BMI in the WCGS study (*continued*)

Problem: Want to summarize age adjusted effect of BMI using a *risk difference* (or excess risk) rather than an odds ratio

- *Solution:* **risk difference regression** model based directly on the outcome probability P :

$$P(\text{age}, \text{bmi}) = \beta_0 + \beta_1 \text{age} + \beta_2 \text{bmi}$$

- The coefficient β_2 is interpreted as the risk difference associated with a unit change in bmi , holding age fixed.
- The coefficient β_1 is interpreted as the risk difference associated with a unit change in age , holding bmi fixed.
- **Note:** model coefficients have to be estimated so that the left-hand side still defines a valid outcome probability (i.e. $0 \leq P \leq 1$) for all values of bmi and age . Fitting problems are common, especially for continuous predictors.

Risk difference regression (continued)

- Here is the `binreg` risk difference model fit of the last example:

```
. binreg chd69 age bmi if bmi>12 & bmi!=., rd iterate(100)
```

```
Iteration 1:    deviance = 1731.716
```

```
Iteration 2:    deviance = 1731.192
```

```
Iteration 3:    deviance = 1731.189
```

```
Iteration 4:    deviance = 1731.189
```

```
Iteration 5:    deviance = 1731.189
```

```
.
```

```
.
```

```
.
```

```
Iteration 95:   deviance = 1731.182
```

```
Iteration 96:   deviance = 1731.182
```

```
Iteration 97:   deviance = 1731.182
```

```
Iteration 98:   deviance = 1731.181
```

```
Iteration 99:   deviance = 1731.181
```

```
Iteration 100:  deviance = 1731.181
```

```
convergence not achieved
```

```
r(430);
```

- No convergence of the estimation process in 100 iterations (convergence is never achieved in this example, even if the iteration limit is increased to >1000).
- Model excluding BMI or using a categorical version converges (see next page):

Risk difference regression (continued)

- Using bmi recoded into 4 groups based on quartiles:

```
. xtile bmicat=bmi if bmi>12,nq(4)
. binreg chd69 age i.bmicat, rd
```

Generalized linear models	No. of obs	=	3153
Optimization : MQL Fisher scoring	Residual df	=	3148
(IRLS EIM)	Scale parameter	=	1
Deviance = 1730.073128	(1/df) Deviance	=	.5495785
Pearson = 3153.048459	(1/df) Pearson	=	1.001604

Variance function: $V(u) = u*(1-u)$	[Bernoulli]
Link function : $g(u) = u$	[Identity]
	BIC = -23630.56

		EIM				
chd69	Risk Diff.	Std. Err.	z	P> z	[95% Conf. Interval]	
age	.0051902	.000893	5.81	0.000	.00344	.0069405
bmicat						
2	.0170412	.0118722	1.44	0.151	-.0062278	.0403102
3	.0255155	.0121295	2.10	0.035	.001742	.049289
4	.0380856	.0129088	2.95	0.003	.0127848	.0633865
_cons	-.1784397	.0398784	-4.47	0.000	-.2565999	-.1002795

- Categorization leads to reasonable estimates, but a somewhat different interpretation

Conclusions about alternate binary regression approaches:

- Alternative models (e.g. relative risk / risk difference) may be preferable in some instances
 - requested by reviewers
 - scientific grounds
 - marked improvement in fit over the logistic
- Alternate models may raise numerical estimation problems that typically don't affect logistic regression
 - The Poisson regression approach illustrated on slide 7 is the recommended way to obtain estimated relative risks for binary outcomes and multiple predictors (*not valid for case-control studies*)
- The logistic model should generally be the default choice unless these other factors are compelling
 - recall that the logistic model can also be used as a basis for causal inference (e.g. for estimating average (marginal) causal effects, propensity scores, marginal structural models, etc.)

Alternatives to logistic regression for small samples & rare outcomes

As sample size shrinks:

- Reliability of inferences from standard methods (Wald (Z) test, 95% confidence intervals and even likelihood ratios diminishes
- difficulties in estimating coefficients become more common
- *Exact logistic regression* and *bootstrap methods* can both help
- What is a “small” sample?
- A related issue is *rare outcomes*, which can be an issue even in large samples
 - can cause estimation issues (hence the 10 outcomes per predictor rule of thumb)
 - frequently results in infinite OR estimates

Example: relationship between transmission of HIV to female partners of HIV infected men in CDC transmission study

Problem: Sample of $n = 31$ - calls validity of approx. inferences from `logistic` into question

`. logistic hivp sexd60 aids`

Logistic regression

Number of obs = 31
LR chi2(2) = 6.81
Prob > chi2 = 0.0331
Pseudo R2 = 0.2058

Log likelihood = -13.151565

hivp		Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
+-----+-----							
sexd60		7.081807	9.7318	1.42	0.154	.479099	104.6798
aids		12.80809	16.1979	2.02	0.044	1.073995	152.7449
+-----+-----							

`. exlogistic hivp sexd60 aids` (Note: `i.` notation doesn't work)

Exact logistic regression

Number of obs = 31
Model score = 6.582907
Pr >= score = 0.0258

hivp		Odds Ratio	Suff.	2*Pr(Suff.)	[95% Conf. Interval]	
+-----+-----						
1.sex60		5.403325	6	0.3047	.4320873	336.6659
1.aids		10.2224	3	0.1021	.7104864	589.9442
+-----+-----						

Modifications of logistic regression for rare outcomes

Even in large samples, outcomes may be rare

- can cause estimation issues for the logistic model (hence the 10 outcomes per predictor rule of thumb),
 - can lead to biased/infinite OR estimates (bootstrap not very helpful in these cases)
-
- When the exact logistic approach becomes computationally infeasible, the *penalized likelihood* version of the logistic model can be used to obtain more reliable inferences
 - In Stata, the downloadable `firthlogit` package can be used to fit penalized logistic models
 - *Note*: does not accept the `i.` syntax for categorical predictors and will not work with some post-estimation commands like `margins`

Reference: Heinze G, Schemper M. A solution to the problem of separation in logistic regression. *Stat Med*. 2002;21:2409-19.

Example: ICU data from homework #4

Problem: Sample of $n = 200$ with 40 outcomes. Model in question #3 involved 11 predictors:

```
. logistic sta i.crn i.typ i.can i.loc i.agecat i.syscat
```

note: 1.loc != 0 predicts success perfectly; 1.loc dropped and 5 obs not used

Logistic regression

Number of obs = 195

LR chi2(10) = 62.06

Prob > chi2 = 0.0000

Pseudo R2 = 0.3381

Log likelihood = -60.742305

sta	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
1.crn	2.840999	2.165964	1.37	0.171	.6375571	12.65969
1.typ	52.63315	69.20641	3.01	0.003	3.999717	692.611
1.can	13.62877	13.28441	2.68	0.007	2.017263	92.0769
loc						
1	1 (empty)					
2	16.38254	15.17629	3.02	0.003	2.665963	100.672
agecat						
2	4.240071	3.244213	1.89	0.059	.9464375	18.99566
3	5.588847	4.478942	2.15	0.032	1.161885	26.88322
4	13.20398	9.938337	3.43	0.001	3.020121	57.72781
syscat						
2	.7691673	.4733606	-0.43	0.670	.2302368	2.569608
3	1.059359	.6293668	0.10	0.923	.3306305	3.394246
4	.0768537	.0754065	-2.62	0.009	.0112329	.525821
_cons	.0011421	.0018182	-4.26	0.000	.0000504	.0258683

No inference about level 1 of loc

Example: ICU data from homework #4

Solution: Use `firthlogit`, after generating binary predictors for levels of `loc`, `agecat` & `syscat`:

```
. firthlogit sta crn typ can locc2 locc3 agec2 agec3 agec4 sysc2 sysc3 sysc4, or
```

initial: penalized log likelihood = -91.758995

... (additional iteration output omitted)

Iteration 5: penalized log likelihood = -56.752069

Penalized log likelihood = -56.752069

Number of obs = 200
Wald chi2(11) = 30.23
Prob > chi2 = 0.0015

sta	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
crn	2.585408	1.824617	1.35	0.178	.6483431	10.30987
typ	23.81719	26.04578	2.90	0.004	2.792809	203.1139
can	9.392172	8.038851	2.62	0.009	1.754747	50.27102
locc2	1622.537	3584.976	3.35	0.001	21.35382	123286
locc3	11.43597	9.426443	2.96	0.003	2.273238	57.53094
agec2	3.620715	2.560872	1.82	0.069	.9052191	14.48222
agec3	4.738263	3.503944	2.10	0.035	1.112131	20.1875
agec4	10.26244	7.1259	3.35	0.001	2.631508	40.02182
sysc2	.7950167	.4641699	-0.39	0.694	.2531645	2.496604
sysc3	1.059447	.5989035	0.10	0.919	.3498615	3.208204
sysc4	.1157282	.1005024	-2.48	0.013	.0210972	.634823
_cons	.0032221	.0043901	-4.21	0.000	.000223	.0465493

Estimated OR for level 1 of loc

Logistic regression for small samples/rare outcomes

Conclusion:

- exact logistic results preferred for small samples ($n < 50$), especially when standard approach fails to yield estimates
- penalized likelihood approach useful for problems with rare outcomes, and in cases where the exact approach is computationally infeasible
- bootstrap inference useful for small-moderate samples with adequate number of outcomes, and where exact methods are computationally infeasible
- conventional logistic regression (or the Poisson-based log relative risk model) preferred for moderate-large samples with adequate numbers of outcomes

Pooled logistic regression for grouped event time outcomes

- Application of the logistic regression model to survival (event time) outcomes
- For studies where observations of event times are limited to regular intervals (e.g. visit intervals in prospective studies)
- Data structured in “long” form, with one observation per person-visit, binary indicator of event occurrence (at most once / individual)
- Coefficients approximate relative hazards from Cox proportional hazards regression when outcomes are rare
- Advantages over Cox regression:
 - Can be applied to when info about event times limited to visit intervals
 - Accepts weights - used to fit *marginal structural models* for causal inference involving time-dependent confounders
- Disadvantages:
 - requires explicit model for the time dependence of outcome risk
 - often requires modeling assumptions about event occurrences and predictor values within observation intervals

Pooled logistic regression for grouped event time outcomes

- Time dependence of outcome risk is modeled by the **intercept** part of the model:

$$\log\left(\frac{P}{1-P}\right) = \beta_0(t) + \beta_1 x_1 + \dots + \beta_p x_p$$

- $\beta_0(t)$ represents the log outcome odds at time t , among individuals with all predictors set to zero
 - No time dependence: constant $\beta_0(t) = \beta_0$,
 - including time (t) as an untransformed predictor equivalent to assuming linearity of log outcome odds with increasing time
 - flexible dependence can be modeled by a cubic spline for t
- The pooled logistic is a **proportional odds model**
 - Approximates the **proportional hazards model** for rare events
 - Used for control of predictors acting as time-dependent confounder/mediators in causal inference (via incorporation of inverse probability weights) (Biostat 215)

References:

1. D'Agostino RB, Lee ML, Belanger AJ, Cupples LA, Anderson K, Kannel WB. Relation of pooled logistic regression to time dependent Cox regression analysis: the Framingham Heart Study. Stat Med. 1990;9:1501-15.
2. Hernán MA, Brumback B, Robins JM. Marginal structural models to estimate the causal effect of zidovudine on the survival of HIV-positive men. Epidemiology. 2000;11:561-70.

Pooled logistic regression for grouped event time outcomes

- **Example:** incident HSV-2 infection in the MIRA study

listing of data for two individuals (one with incident HSV-2):

```
. list id hsv2 mos agecat stihx newparts if id==28 | id==27
```

	id	hsv2	mos	agecat	stihx	newparts
202.	27	0	3	1	0	0
203.	27	0	6	1	0	0
204.	27	0	9	1	0	0
205.	27	0	12	1	0	0
206.	27	0	15	1	0	0
207.	27	0	18	1	0	0
208.	27	0	21	1	0	0
209.	27	0	24	1	0	0
210.	28	0	3	1	1	0
211.	28	0	6	1	1	0
212.	28	0	9	1	1	0
213.	28	0	12	1	1	1
214.	28	0	15	1	1	0
215.	28	1	18	1	1	0

time varying predictor

individuals with the outcome stop contributing data after interval of outcome occurrence

mos: months on study, **hsv2:** indicator of incident HSV-2 infection,
agecat: age at baseline, **stihx:** baseline STI history,
newparts: recent sexual partners (Y/N)

Pooled logistic regression for grouped event time outcomes

- **Example:** incident HSV-2 infection in the MIRA study

- **generate spline basis to allow for potentially nonlinear effects of time**

- **mkspline spl = mos, cubic nknots(3)**

- **logistic hsv2 spl* i.agecat i.stihx newparts**

Logistic regression

Number of obs = 6069

LR chi2(6) = 33.69

Prob > chi2 = 0.0000

Log likelihood = -509.18109

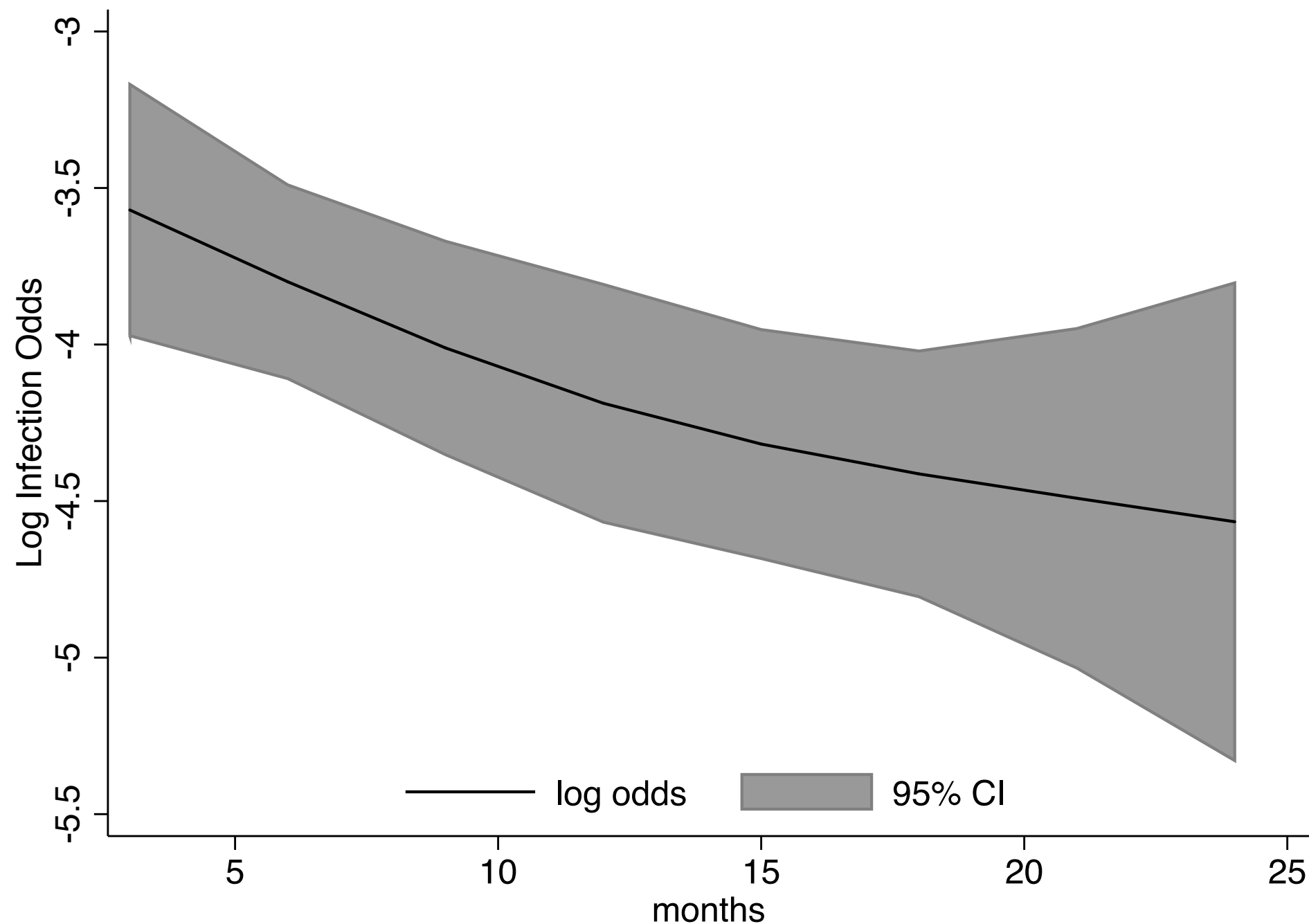
Pseudo R2 = 0.0320

hsv2	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
spl1	.9256021	.0379368	-1.89	0.059	.8541555	1.003025
spl2	1.035493	.0556198	0.65	0.516	.9320218	1.15045
agecat						
2	.6090384	.1369677	-2.20	0.027	.3919372	.946396
3	.5522162	.2213153	-1.48	0.138	.2517489	1.211297
1.stihx	1.92962	.4938946	2.57	0.010	1.168431	3.186695
newparts	1.826313	.4372661	2.52	0.012	1.142288	2.919945
_cons	.0354939	.0105938	-11.19	0.000	.0197741	.0637103

odds ratios for spl* predictors represent time variation (in months) in baseline HSV-2 risk; interest usually focuses on the other predictors in the model

Pooled logistic regression for grouped event time outcomes

- **Example:** Baseline log infection odds for HSV-2 example on previous slide (plotted using `xb1c` as in lab 8)



Linearity of log odds with months of follow-up appears reasonable

Cox regression for HSV-2 example

```
. stset mos hsv2, id(id)
. stcox i.agecat i.stihx newparts, efron
```

```
      failure _d:  hsv2
analysis time _t:  mos
           id:   id
```

Cox regression -- Efron method for ties

```
No. of subjects =          1000          Number of obs   =          6069
No. of failures =           104
Time at risk    =          18207

Log likelihood   =    -691.38351          LR chi2(4)       =          20.55
                                          Prob > chi2       =          0.0004
```

_t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----						
agecat						
2	.6131285	.1365595	-2.20	0.028	.3962479	.9487156
3	.5554717	.2209092	-1.48	0.139	.2547663	1.211105
1.stihx	1.918896	.4819664	2.59	0.009	1.172888	3.139399
newparts	1.793361	.4225623	2.48	0.013	1.130063	2.845987

- Results quite similar to pooled logistic regression results
- Modeling of the effect of time is *not required* for the Cox proportional hazards model (courtesy of the *proportional hazards assumption* and *partial likelihood*)
- Cox regression model doesn't accept weights (can't be used to control for time dependent confounder/mediators)

Logistic Regression for case-control data

Unmatched Case-Control Studies

Definition

Basic design principle: Sample individuals with outcome (cases) and without outcome (controls), determine exposure retrospectively.

- controls must be representative of target population
- quality of exposure information from cases & controls must be equally good

Advantages of case-control approach:

- appropriate for rare outcomes
- often require smaller samples; less expensive than comparable prospective studies
- odds ratios approximate relative risk (for rare outcomes)

Disadvantages:

- recall bias a factor in measuring exposures/risk factors
- temporal ordering of exposure & disease (i.e. causal links) not always clear

Example: Ile-et-Vilaine study of esophageal cancer

Sample of 200 adult males with esophageal cancer from regional hospitals in Ile-et-Vilaine (Brittany); sample of 778 adult male controls drawn from election records in same region.

```
. use esoph
(Ille-et-Vilaine case-control study of esophageal cancer)
. describe
Contains data from esoph.dta
  obs:          975                               Ille-et-Vilaine case-control
                                                    study of esophageal cancer
vars:           7                               7 Feb 2001 08:40
size:          31,200 (96.9% of memory free)
-----
  1. case      float  %9.0g                               Case status (1=case, 0=control)
  2. age       float  %9.0g
  3. agegp     float  %9.0g      agelab      Age group
  4. tob       float  %9.0g                               Tobacco consumption gm/day
  5. tobgp     float  %10.0g     toblab     Grouped tobacco consum.
  6. alc       float  %9.0g                               Alcohol consumption gm/day
  7. alcgp     float  %11.0g     alclab     Grouped alcohol consum.
-----
```

Example: Ile-et-Vilaine study of esophageal cancer

. tabulate case

Case status (1=case, 0=control)	Freq.	Percent	Cum.
0	775	79.49	79.49
1	200	20.51	100.00
Total	975	100.0	

. table tobgp

Grouped tobacco consum.	Freq.
0-9 gm/day	526
10-19	236
20-29	131
30+	82

. table alcgp

Grouped alcohol consum.	Freq.
0-39 gm/day	414
40-79	355
80-119	139
120+	67

. table agegp

Age group	Freq.
25-34	116
35-44	199
45-54	213
55-64	242
65-74	161
75+	44

Are odds ratios valid measures of association for the effect of predictors on the outcome in case-control studies?

Example: association between outcome (case/control status) and binary predictor for smoking in Ile-et-Vilaine study

Odds ratio for smoking status (outcome) by cases/control status (predictor):

```
. tabodds ditob case, or
```

case	Odds Ratio	chi2	P>chi2	[95% Conf. Interval]	
0	1.000000	.	.	.	.
1	10.407051	64.89	0.0000	5.119049	21.157585

```
Test of homogeneity (equal odds): chi2(1) = 64.89
```

```
Pr>chi2 = 0.0000
```

```
Score test for trend of odds: chi2(1) = 64.89
```

```
Pr>chi2 = 0.0000
```

Example: association between outcome (case/control status) and binary predictor for smoking in Ile-et-Vilaine study

Odds ratio for disease by smoking status:

`. tabodds case ditob, or`

ditob		Odds Ratio	chi2	P>chi2	[95% Conf. Interval]	
0-9 gm/~y		1.000000	.	.	.	.
10+ gm/~y		10.407051	64.89	0.0000	5.119049	21.157585

Test of homogeneity (equal odds): chi2(1) = 64.89
Pr>chi2 = 0.0000
Score test for trend of odds: chi2(1) = 64.89
Pr>chi2 = 0.0000

- odds ratios in last two slides are identical (i.e. due to symmetry between exposure risk in cases compared to controls and disease risk in exposed compared to unexposed when risk is measured using the OR)

Interpretation of odds ratios for case-control studies

		exposure		
		+	-	
outcome	+	a	b	a+b
	-	c	d	c+d
		a+c	b+d	n

- For any binary outcome and a single binary predictor:
- For **prospective** and **cross-sectional** studies, odds ratios are estimated with the odds of the outcome among those with and without the predictor:

$$\text{odds for exposed} = \frac{a / (a + c)}{c / (a + c)}, \text{ odds for unexposed} = \frac{b / (b + d)}{d / (b + d)}, \text{ OR} = \frac{\frac{a / (a + c)}{c / (a + c)}}{\frac{b / (b + d)}{d / (b + d)}} = \frac{ad}{bc}$$

- For **case-control** studies, odds ratios are estimated with the odds of exposure among those with and without the outcome:

$$\text{odds for outcomes} = \frac{a / (a + b)}{c / (a + b)}, \text{ odds for non-outcomes} = \frac{d / (c + d)}{d / (c + d)}, \text{ OR} = \frac{\frac{a / (a + b)}{c / (a + b)}}{\frac{d / (c + d)}{d / (c + d)}} = \frac{ad}{bc}$$

- ORs are the same under reversal of “roles” of outcome and predictor

Correspondence between odds ratios and relative risks

For rare outcomes, the OR approximates RR (see figure in slide 4)

- since the quality of this approximation isn't always known, it's good practice to call an odds ratio an OR

Summary odds ratios for case-control studies

Standard frequency table methods to estimate odds ratios for associations controlled for confounding variables also remain valid for case-control study data

Example: association between outcome (case/control status) and binary predictor for heavy alcohol consumption in Ile-et-Vilaine study controlled for age

```
. cc case dialc, by(agegp)
      Age group |          OR      [95% Conf. Interval]      M-H Weight
-----+-----
      25-34 |          .          0          .          0 (exact)
      35-44 |    5.046154    .9268664    24.86538    .6532663 (exact)
      45-54 |    5.665025    2.632894    12.16536    2.859155 (exact)
      55-64 |    6.088235    3.172432    11.70469    3.933884 (exact)
      65-74 |    2.580247    1.131489    5.857261    4.024845 (exact)
      75+  |          .    4.388738          .          0 (exact)
-----+-----
      Crude |    5.58042    3.897681    7.972556          (exact)
M-H combined |    5.079324    3.510674    7.348884
-----+-----
Test of homogeneity (B-D)      chi2(5) =      9.17  Pr>chi2 = 0.1025

      Test that combined OR = 1:
              Mantel-Haenszel chi2(1) =      83.59
                          Pr>chi2 =      0.0000
```

Definition of logistic regression model for unmatched case-control studies

If $P(x)$ is the probability of outcome occurrence among individuals with a single risk predictor at level x , the logistic regression model for a case-control sample is:

$$\log\left[\frac{P(x)}{1-P(x)}\right] = \beta_0^* + \beta_1 x$$

- β_0^* *Not interpretable* as the log odds of outcome occurrence among indiv. with $x = 0$. It carries, no information about this risk in the absence of additional population-level information about case & control sample selection probabilities:

$$\beta_0^* = \beta_0 + \log(\pi_1/\pi_0)$$

$$\pi_1 = \Pr[\text{sampled} | y = 1] \quad ; \quad \pi_0 = \Pr[\text{sampled} = 1 | y = 0]$$

- Even though study participants are sampled based on their outcome status, the conventional logistic model can be used to analyze data from case-control studies
 - the regression coefficients *retain the odds ratio interpretation* familiar from prospective and cross-sectional studies
 - the intercept coefficient *cannot* be related to outcome risk/odds (unless population sampling probabilities are known)
 - other regression models for binary outcomes *not appropriate* for case-control data

Example Ile-et-Vilaine study of esophageal cancer (continued)

Effect of alcohol and tobacco use controlled for age

. logistic case i.alcgp i.tobgp i.agegp

Logistic regression

Number of obs = 975

LR chi2(11) = 285.22

Prob > chi2 = 0.0000

Log likelihood = -352.13648

Pseudo R2 = 0.2882

case	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
+-----+						

Example: Ille-et-Vilaine study of esophageal cancer (*continued*)

Use of likelihood ratio test to evaluate independent effects of alcohol, tobacco and age:

```
* save results of previously fitted model
```

```
. estimates store M1
```

```
* fit model excluding tobacco effects
```

```
. quietly logistic case i.alcgp i.agegp
```

```
* test contribution of tobacco effects to joint model using lrtest
```

```
. lrtest M1
```

```
likelihood-ratio test
```

```
(Assumption: . nested in M1)
```

```
LR chi2(3) = 23.69
```

```
Prob > chi2 = 0.0000
```

```
* repeat for alcohol & age effects
```

```
. quietly logistic case i.tobgp i.agegp
```

```
. lrtest M1
```

```
likelihood-ratio test
```

```
(Assumption: . nested in M1)
```

```
LR chi2(3) = 127.52
```

```
Prob > chi2 = 0.0000
```

```
. quietly logistic case i.tobgp i.alcgp
```

```
. lrtest M1
```

```
likelihood-ratio test
```

```
(Assumption: . nested in M1)
```

```
LR chi2(5) = 126.52
```

```
Prob > chi2 = 0.0000
```

- **Note:** A complete analysis would also consider possible interaction and mediation, as well as considering alcohol and tobacco as continuous measures.

Matched Case-Control Studies

Basic design principle: Sample individuals with outcome (cases) and without outcome (controls). Pair each case with one or more controls according to levels of specified predictors. Determine exposure retrospectively.

- 1:1, or *matched pairs* design matches each case with exactly one control
- 1: m design matches m controls with each case
- *frequency matching* fixes ratio of cases/controls within strata formed by selected confounding variables

Advantages:

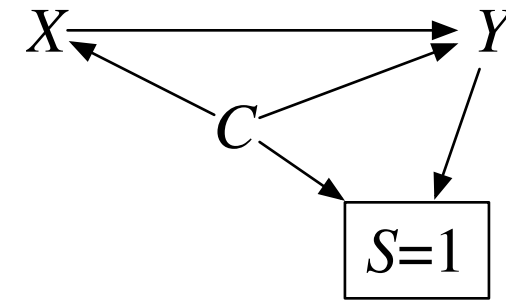
- can be more efficient than unmatched studies when confounding is effectively controlled
- applies to confounders that are hard to quantify (e.g. geographic location, familial traits)

Disadvantages:

- more complicated and expensive than unmatched designs
- inefficient when matching variables not strongly related to outcome/exposure
- may require a specialized matched analysis
- in many cases appropriate analysis of unmatched design yields same conclusions

Matched Case-Control Studies

Example DAG from a matched case-control study:



- Y : outcome
- X : exposure
- C : confounder
- S : sample inclusion indicator (the box indicates selection based on matching based on C)

Issues:

- Matching does not necessarily control confounding bias due to the matching variables
 - matched controls may not be representative of target population
- A “matched” analysis accounting only for matching (S), but ignoring adjustment for C can lead to bias (via the collider role of S)
- In general, matched or conventional analyses for matched studies should control for matching variables when possible

References

- Shahar E, Shahar DJ. Causal diagrams and the logic of matched case-control studies. Clin Epidemiol. 2012;4:137-44.
- Mansournia MAI, Hernán MA, Greenland S. Matched designs and causal diagrams. Int J Epidemiol. 2013 Jun;42(3):860-9.
- Pearce N. Analysis of matched case-control studies. BMJ. 2016;352

Example: matched case-control study of study of the association between use of estrogens and endometrial cancer (Mack, *et al*, 1976, NEJM)

```
. use endom
(Leisure World matched case-control study of endometrial cancer)
. describe
Contains data from endom.dta
  obs:          394
                                Leisure World matche
                                case-control study of
                                endometrial cancer
vars:           8
                                12 Feb 2001 13:17
-----
  1. set         float    %9.0g
  2. case        float    %9.0g
  3. age         float    %9.0g
  4. gall        float    %9.0g      vlab
  5. hyp         float    %9.0g      vlab
  6. ob          float    %9.0g      vlab
  7. est         float    %9.0g      vlab
  8. dose        float    %11.0g     doslab
                                Dose of conjugated estrogen
-----
. * n = 315 (4 controls matched to each case by age & marital status; 63 matched sets)
. list set case age est if _n<=10
      set      case      age      est
1.      1        1       74     Yes
2.      1        0       75      No
3.      1        0       74      No
4.      1        0       74      No
5.      1        0       75     Yes
6.      2        1       67     Yes
7.      2        0       67     Yes
8.      2        0       67      No
9.      2        0       67     Yes
10.     2        0       68     Yes
.* Note: the marital status variables is not available in this dataset
```

Example: matched case-control study of endometrial cancer (*continued*)

Frequency table methods for unmatched data can be used for single categorical predictors.

In Stata, the `mhodds` command estimates the Mantel-Haenszel odds ratio for the association between outcome and predictor controlling for the matched sets, and gives 95% confidence limits:

```
. mhodds case est set
```

```
Mantel-Haenszel estimate of the odds ratio  
Comparing est==1 vs est==0, controlling for set
```

```
Note: only 58 of the 63 strata formed in this analysis  
contribute information about the effect of the explanatory variable
```

Odds ratio	chi2(1)	P>chi2	[95% Conf. Interval]
8.461538	31.16	0.0000	3.437773 20.826746

- When additional predictors are involved, logistic regression is required.

Conditional logistic model for matched case-control studies

- Logistic regression model for matched case-control data is defined and applied similarly to the model for unmatched data. However, the version for matched data is based on a "conditional" model for the relationship between the outcome and predictors.
- Motivation for conditional analysis is to avoid estimating effects of matching variables by conditioning on them in estimation
- In cases where there are many matched sets characterized by unique values of the matching variables (e.g. twin studies), it may be infeasible to control for matching in a conventional logistic model
- In cases where matching is based on measured characteristics (e.g. age), conventional (unmatched) logistic regression controlling for matching variables will generally provide valid results

Example: matched case-control study of endometrial cancer (*continued*)

Effect of estrogen use on endometrial cancer risk estimated using the conditional logistic model

```
. clogit case est, group(set) or
```

Conditional (fixed-effects) logistic regression

Number of obs = 315

LR chi2(1) = 35.35

Prob > chi2 = 0.0000

Log likelihood = -83.72159

Pseudo R2 = 0.1743

case		Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
-----+-----						
est		7.954681	3.347525	4.93	0.000	3.486714 18.14802

- or option requests that odds ratios be printed rather than coefficients
- model does not include a constant term

Example: matched case-control study of endometrial cancer (*continued*)

Effect of estrogen use controlled for history of gall bladder disease:

```
. clogit case est gall, group(set) or
Conditional (fixed-effects) logistic regression      Number of obs      =           315
                                                    LR chi2(2)          =           45.05
                                                    Prob > chi2         =           0.0000
Log likelihood = -78.871308                        Pseudo R2           =           0.2221
```

case	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
est	8.287802	3.644927	4.81	0.000	3.500144 19.62424
gall	3.577465	1.469865	3.10	0.002	1.598984 8.003994

"Full" model for the joint effect of estrogen and history of gall bladder disease:

```
. clogit case i.est##i.gall, group(set) or
Conditional (fixed-effects) logistic regression      Number of obs      =           315
                                                    LR chi2(3)          =           49.33
                                                    Prob > chi2         =           0.0000
Log likelihood = -76.730576                        Pseudo R2           =           0.2432
```

case	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
1.est	14.88179	9.104216	4.41	0.000	4.486594 49.36211
1.gall	18.07166	15.95823	3.28	0.001	3.201415 102.0127
est#gall					
1 1	.1283818	.1277365	-2.06	0.039	.0182633 .902457

. * evidence for multiplicative interaction between est & gall

Example: matched case-control study of endometrial cancer (*continued*)

Use of `lincom` to get component odds ratios for estrogen use among women w/wo gall bladder disease:

```
. lincom 1.est + 1.est#1.gall, or
```

```
( 1) [case]1.est + [case]1.est#1.gall = 0
```

case		Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----							
(1)		1.910551	1.517452	0.82	0.415	.4028036	9.061997

```
. lincom 1.est, or
```

```
( 1) [case]1.est = 0
```

case		Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
-----+-----							
(1)		14.88179	9.104216	4.41	0.000	4.486594	49.36211

Note: the `or` option is required for odds ratios when `lincom` is used after `clogit`.

Example: matched case-control study of endometrial cancer (*continued*)

Compare to results from corresponding unmatched analysis:

```
. logistic case i.est##i.gall age
Logistic regression                                Number of obs      =          315
                                                    LR chi2(4)          =          49.91
                                                    Prob > chi2         =          0.0000
Log likelihood = -132.67224                        Pseudo R2          =          0.1583
```

case	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
est						
Yes	15.46892	9.492168	4.46	0.000	4.646637	51.49694
gall						
Yes	19.22229	16.29279	3.49	0.000	3.650294	101.2238
est#gall						
Yes#Yes	.1085787	.1024346	-2.35	0.019	.0170887	.6898893
age	1.013276	.0250903	0.53	0.594	.9652738	1.063664
_cons	.009936	.0186745	-2.45	0.014	.0002497	.395389

- Estimates fairly close to conditional logistic results in this case
- In this example, a conventional unmatched analysis might be preferred if the marital status matching variable was available and included in the model.

Conditional logistic model for matched case-control studies

Conclusions

- matching should be accounted for in analysis if it is used in study design
 - conventional unmatched analyses may be used in many cases, but matching variables should be included for adjustment
 - in cases where there are many matched sets, a conditional (matched) analysis may be needed
- inappropriate matching ("overmatching") can lead to inefficiency (i.e. higher variances) than corresponding unmatched designs