Homework 7

1. The probability of a single point is zero, so it does not matter if we include or exclude single points in the interval.

2. The probability that a random variable is less than its minimum possible value must be zero.

   The probability that a random variable is less than its maximum possible value must be one.

   For $x_1 > x_2$, $F(x_1) \geq F(x_2)$. This is because of the following:

   $$F(x_1) = \int_{\infty}^{x_1} f(x)dx = \int_{\infty}^{x_2} f(x)dx + \int_{x_1}^{x_2} f(x)dx = F(x_2) + \int_{x_1}^{x_2} f(x)dx$$

   Because $\int_{x_1}^{x_2} f(x)dx \geq 0$ ($f(x) \geq 0$ for all x, so the integral must be greater than zero), $F(x_1)$ must be greater than or equal to $F(x_2)$.

   Now, we have shown that $F(x)$ is monotonically increasing. So, if it differentiable, the derivative is greater than or equal to zero for all x.

3. $F(t) = 1 - \exp(-\lambda t)$. Therefore, $f(t) = \lambda \exp(-\lambda t)$.

   $$ET = \int_0^{\infty} t\lambda \exp(-\lambda t)$$

   Integration by parts: assign $u = t$ and $dV = \lambda \exp(-\lambda t)dt$

   $$du = dt$$

   $$V = -\exp(-\lambda t)$$

   $$ET = -t\exp(-\lambda t)|_0^{\infty} - \frac{1}{\lambda}\exp(-\lambda t)\bigg|_0^{\infty} = \frac{1}{\lambda}$$

4.

$$N(t)/N(0) = \exp(-t)$$

$\exp(-1/2)$, $\exp(-1)$, and $\exp(-2)$ for 1/2, 1, and 2 weeks.

$1/2 = \exp(-t^*)$, so $t^* = \ln 2$ is the time where half the population is left.

5.

$$\frac{\mu}{\mu + \lambda}$$