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On the Terminology and Dimensions of Incidence

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ABSTRACT. This paper is a review of the concept of incidence and the measures of incidence commonly used. The three components of incidence (observed event, population at risk, and observation period) and the four measures of incidence (incidence as number of events, as number of events per time unit, as number of events per unit of amount of observation, and as probability) are discussed. The terminology and dimensions of incidence measures used in epidemiology literature are reviewed. Ambiguities and uses of the same term for different purposes seem to suggest the need for standardized terminology. Incidence measures with length-based amount of observation are also briefly discussed. J CLIN EPIDEMIOL 50;8:891–897, 1997. © 1997 Elsevier Science Inc.

KEY WORDS. Incidence, nomenclature, terminology

INTRODUCTION

One of the most important aspects of modern science may well be the emergence and development of conceptual instruments to measure the frequency of contingent events. The use of these measures to estimate the likelihood of future occurrences of the event or to compare the chances of occurrence of events under different circumstances form an important part of the body of modern knowledge.

The concept of probability is a mathematical abstraction related to the measurement of the occurrence of events. Probability theory began to develop as a product of games of chance and was further elaborated mainly by physicists and mathematicians such as Euler, Laplace, or Kolmogorov. However, from its origins probability theory also had important links to health statistics and biometry through such authors as Petty, Quetelet, or Fisher. In fact, the synthesis of probability with statistics to form mathematical statistics at the beginning of the twentieth century has been attributed largely to the English Biometrics School [1].

The concept of incidence is also related to the measurement of the occurrence of events but it originated in epidemiology when epidemiology was fundamentally a non-mathematical field. Perhaps for this reason the concept of incidence has been in use for a long time with scant mathematical formalization. Even today, despite the fact that incidence is one of the most commonly used concepts in epidemiology, there is often a certain vagueness in its definitions and the terminology of measures of incidence is confusing. Neophytes may find it hard to understand, for instance, why incidence is sometimes defined as the count of new cases, and other times as a fraction whose denominator may be expressed as persons *or* as person-years. Not to mention what the difference is between incidence, incidence rate, incident risk, incidence density, or force of morbidity. As a well-known textbook of epidemiology [2] stated, "two different measures of disease occurrence may be calculated, both commonly (*and confusingly*) referred to as incidence rate" (emphasis added, J.A.T.G.). In addition, arguments regarding both the use of incidence measures and the terminology for these measures have appeared in the literature [3–6]. As a reflection of the common usage of the term in the epidemiological literature, even Last's *Dictionary* [7] also incurs in ambiguities in defining incidence measures. The entry for "incidence rate" states that the denominator is "the population at risk of experiencing the event during this period, *sometimes expressed* as person-time" (emphasis added, J.A.T.G.).

This paper attempts to analyze the concept of incidence, the measures of incidence commonly used, and the relationships between them. The discussion of these issues suggests the need for a standardized terminology.

CONCEPT OF INCIDENCE

Incidence is a set of measures used *to assess the dynamics of events* happening in a "population" or set of elements. Although usually defined as a measure of the occurrence of disease, incidence can be used to measure the occurrence of any kind of event, for example mitosis in cultured cells, volcano eruptions, vehicle crashes, pregnancy in women, malfunctions in machines or plants, marriages among sin-

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gles, or strikes in companies. Mortality measures are indeed no more than incidence measures where the event is death. Although predominantly used in epidemiology, the concept of incidence can be useful in many other fields, as a statistical tool to assess the occurrence of events. In fact, survival analysis—which is closely related to the concept of incidence—was originally developed in engineering to analyze data on mechanical failures.

The measures of incidence discussed in epidemiologic textbooks and articles vary. One, or more commonly two, measures are usually presented, for example "number of new cases" and "rate"; or "rate" and "risk"; or even "rate" and "cumulative incidence." However, from the dimensional point of view, i.e., considering the quantities and the units involved in the measure, there are actually *four* ways to measure incidence. The four incidence measures result from combining the three different components of incidence as will be subsequently explained.

THE THREE COMPONENTS OF INCIDENCE

Though usually unrecognized, and often not formally defined, three different components are required for incidence measures to be calculated: a population, an event, and an observation period.

The "population" is a set consisting of e elements (where e is a natural number or count) "at risk" of the event. In epidemiology the population elements are usually human beings. The observed set can be a well-defined or "closed" group as in cohort studies or clinical trials, where the number e of individuals "at risk" from some zero time is precisely known, and every element moves over the risk period. It can be also an "open" or "dynamic" group, for instance a large human population with elements continuously joining or leaving over calendar time [8]. In this case e has to be more or less approximately estimated, as when figures of disease incidence are computed for cities or nations.

The second component is a well-defined event that can supposedly happen to every element of the population and can be counted as occurring n times. Some kinds of events such as death or acute disorder are easy to detect and count but in other cases such as chronic diseases the time of occurrence of the event is often difficult to establish, which generates important difficulties or errors in counting the number of new cases [2–4,8]. Leaving these difficulties aside, two types of events can be considered: those that can happen only once to every element of the observed set, e.g., death to persons or fusion to bulb filaments; and those that can happen multiple times to every element, e.g., diarrheas to children or malfunctions to radiological sets. This second case poses complex problems for incidence measures and will not be considered here.

The third component is a period t (usually but not always an observation time) during which every element "at risk" of the event is observed. This observation time is loosely assessed when population incidences are estimated, but epidemiological studies with well-defined cohorts demand strict rules for accounting of this interval. When observation intervals are measured for individual elements, the interval ends when the element ceases to be observed (the element is then said to be "censored" in the epidemiologic jargon) or when the expected event occurs, since that element is no longer "at risk" of the event.

However, sometimes we are interested in measuring incidence of an event not along time but along a series of discrete events (for instance, cardiac arrests in a series of surgical operations) or along a different dimension (for instance, car crashes per distance run). In these cases incidence measures adopt peculiar characteristics that will be subsequently commented.

THE MEASURES OF INCIDENCE

From the aforementioned three basic components, four measures of incidence can be derived:

- the number of events, n;
- the number of events per time unit, n/t;
- the number of events per unit of amount of observation, $n/(e\,t)$; and
- the ratio of events to elements of the "at risk" population or proportion of elements of the "at risk" population in which the considered event actually happens, n/e.

Table 1 summarizes the mathematical definitions, the units, the range of variation, and the dimensions of these incidence measures.

Incidence as a Count of Events

The most basic measure of incidence is the number n of observed events. As a count, its units are the event itself: new *cases* of a disease, vehicle *crashes*, *suicides*, *crimes*.

Isolated from the number of elements at risk and the observation period this quantity appears to be a meaningless number. However, this kind of incidence measure is common in practice, for instance when the number of cases of AIDS "from the beginning of the epidemic" is reported for a nation, or when we are told the number of deaths due to traffic injuries in a country over the last weekend. In these examples of reported incidence counts there is an indirect reference to a "population at risk" and a period of observation, but the reference is only implicit and loose. In the epidemiological and biostatistical literature this count has sometimes been referred to as simply "incidence" or "incident cases" [7,9,10].

Incidence as Events per Unit of Time

The ratio of the number of events to the time of observation, i.e., the number of events per time unit, n/t, sometimes

TABLE 1. Mathematical definitions, units, ranges of variation and dimensions of incidence measures

Mathematical definition	Units	Range	Dimensional formula
n	Event units	0 to ∞	Dimensionless
n/t	Event units per time unit	0 to ∞	T^{-1}
$\frac{n}{e\,t}$	Event units per unit of amount of observation (amount of observation usually measured in element-time units)	0 to ∞	T^{-1}
n/e	Unitless	0 to 1	Dimensionless

Variables: n = number of events; e = number of elements of the observed population; t = observation interval, usually time (but can be length, see text).

has been called "incidence rate" [11,12], while other authors [13] referred to it as the "(instantaneous) rate (. . .) expressed as the number of new cases per year." This incidence measure is rarely used in the epidemiological literature but it is not so uncommon in social statistics, biometry, or physical sciences, where it is used, for instance, to assess the daily average number of suicides in a town, the number of truck accidents per month on a given highway, the average number of a given behavior in an animal colony, or the average count of disintegrations per minute in a radioactive substance. In all these examples, we are dealing with the λ parameter of the Poisson distribution, defined as the population mean number of occurrences per unit of time [14].

Incidence as Events per Unit of Amount of Observation (Events per Element-Time Unit)

A common way to measure incidence is to compute the number of events per time unit *and* per element of the observed population. The general formula for this type of incidence is

$$\frac{n}{e\,t}$$

in which the numerator is the number of events and the denominator the "amount of observation," with the "amount of observation" being equal to the product of the number of observed elements times the period of observation, $e \cdot t$. Therefore, incidence measured in events per element-time unit, amount-of-observation incidence or events per unit of amount of observation are equivalent terms.

There are different ways to compute this measure. In the simplest case, all the elements have been observed approximately during the same period and we choose not to consider a different observation time for the elements to which the event has occurred. In this case, there is an observation time t and a number e of observed elements and the amount of observation is simply the product $e \cdot t$. But very often in epidemiological studies we may wish to exclude from the denominator the observation time contributed by elements after they have experienced the event, or account for differential times of observation among different elements (for example, due to losses to follow up). If there are j different subgroups in the observed set of elements and every subgroup has been observed a time t_i, the amount of observation will be the summation of the products of the number e_i of elements in every subgroup times the period t_i during which that subgroup was observed. Therefore, the number of events per unit of amount of observation will be computed with this formula:

$$\frac{n}{\sum_{i=1}^{j} e_i\, t_i} \tag{i}$$

When every one of the e elements of the observed population has been observed a different period t_i, the former expression becomes

$$\frac{n}{\sum_{i=1}^{e} t_i} \tag{ii}$$

If we consider the size e of the at risk population as a continuous function of time t, the amount of observation will be the defined integral of the size of population at risk over the observation period (i.e., the area between t_0 and t_1 in Fig. 1) and the incidence measured in events per amount of observation will be theoretically given by

$$\frac{n}{\int_{t_0}^{t_1} \frac{de}{dt}} \tag{iii}$$

Apparently, expression (ii) is a fraction expressing events per time, as the denominator of the fraction appears to be a sum of times (in fact, that is the way in which it is presented by some authors [8,15]. But this is not really the case, since what we are summing up in the denominator in both

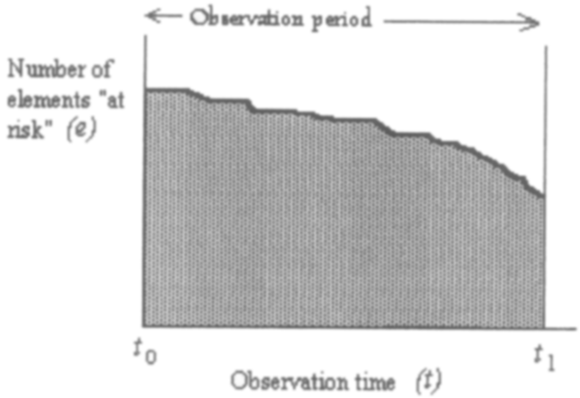

FIGURE 1. Amount of observation as an area determined by number of observed elements "at risk" and observation time.

expressions (ii) and (iii) are not time units but element-time units, be they person-years, cell-minutes, or worker-months. In fact, every area unit under the curve in Fig. 1 is a unit of amount of observation, not a time unit (which on the graph would be simply a length). Incidence measured as events per time unit (for instance accidents per month, mitosis per second, or suicides per year) conceptually differs from incidence measured as events per unit of amount of observation (accidents per worker-month, mitosis per second-cell, or suicides per person-year). While $n/(e\,t)$ takes into account the population size as well as time of observation, n/t only takes into account the time of observation.

Things become even more complex because exact time of occurrence of events or losses to follow up is often not precisely known. When estimating rates for large populations such as countries, cities, etc., even the number of elements in the population can be only grossly estimated. Assuming a homogeneous distribution of events and of losses to follow up during the interval in question, e is often estimated as the average population during the period of observation. The result of $n/(e\,t)$ expressed as yearly (or whatever) rates is usually called "incidence rate" [15,16], while measurements based on the exact computation of element-time (person-years) are referred to as "incidence density" [11], "force of morbidity" [11,15], "person-time rate" [17] or "person-time incidence rate" [7].

Regardless of the method of computation of the denominator, incidence based on events per unit of amount of observation is usually simply called "incidence" [7], or "incidence rate" [15,16], though is has also been called "relative rate" [13]. This is the most common measure of incidence found in vital or morbidity statistics, frequently expressed as deaths or cases per hundred thousand or per million persons, usually referred to one year. This kind of formulations tend to obscure the fact that the denominator of this incidence *par excellence* is not time or number of elements but rather time × number of elements, the product of both.

Incidence as Probability

The number of events in a given period per element of the "at risk" population, n/e, is equivalent to the proportion of elements of the population to which the event occurs in the defined interval. This is a dimensionless proportion varying between 0 and 1, which has been called "incidence" [18], "incident rate" [19,20], cohort incidence rate [8], "cumulative incidence rate" [7,8], "incidence proportion" [16,21], "cumulative incident rate or risk" [9], "cumulative incidence" [15], and "new (incident) events risk" [13]. "Risk" or "cumulative incidence rate" are probably the most used terms.

As this is an adimensional proportion, it can be considered the probability of the event happening to an element of the observed population over a given interval. However, as any event is only possible assuming nonoccurrence of other events that make the first impossible (disease is not possible if death comes first, mechanical failure of an electrical engine cannot be observed if electric power stops) we are actually dealing with a conditional probability [13]. If there are withdrawals or losses to follow up (censored elements) during the period of observation, the denominator can be corrected to reflect this. If the exact time of censoring is unknown, the withdrawals or losses can be assumed to be homogeneously distributed throughout the period of observation, and the formula n/e becomes $n/(e - 1/2\,w)$, where w is the number of censored elements [2]. This is what has been referred to as the actuarial method of risk calculation [22]. If the exact time of censoring is known, there are several ways to estimate the probability, as Elandt-Johnson [23] thoroughly discussed for mortality data. The more intuitive way is to assume that an element which was observed for only a fraction of the total observation period contributes only a fraction of an element to the denominator. This fraction is equal to t_i/t where t is the total observation time and t_i is the follow-up time of the i-th subject until censoring or follow up loss. Therefore, n/e now becomes

$$\frac{n}{e - w + \sum_{i=1}^{w} \frac{t_i}{t}} \qquad \text{(iv)}$$

The denominator of expression (iv) has been called the "effective number of initial exposed to risk" [23]. It is worthwhile mentioning that the denominator of this fraction is a corrected count of elements, not "person-years exposed" or "person-years of life experience" as claimed by Frost [24]. The Kaplan-Meier product-limit method [18] also provides a measure of incidence probability. If S is the probability of survival beyond a certain time, the incidence probability (of death or the event) will therefore be 1 − S. In other words, "the cumulative incidence function reduces to one minus the product limit survivor function estimator" [25]. Further discussion of this issue is beyond the scope of this paper.

TABLE 2. Different authors' terminology of incidence measures

	Incidence measured as			
Author	**Count of events [n]**	**Events per time unit [n/t]**	**Events per element-time unit [$n/(e\,t)$]**	**Probability [n/e]**
Last [7]	Incidence		Person-years incidence rate	Cumulative incidence rate
Freeman [11]		Incidence rate	Incidence density or force of morbidity	
Kelsey [16]			Incidence rate	Risk of probability of developing disease; cumulative incidence (rate)
Morgenstern[a]		(Instantaneous) rate	"Relative rate"; (relative) incidence rate	Risk (of incidence)
Rothman [15]			Incidence rate	Cumulative incidence
Selvin [26]			Incidence rate	Incidence proportion
Haberman [12,20]		Incidence rate		Incidence rate

[a]This is the terminology used by Morgenstern, Kleinbaum, and Kupper in an article on measures of disease incidences [13]. However, in the book *Epidemiologic Research: Principles and Quantitative Methods* [22] these authors restricted the discussion of incidence measures to what they called "rate" and "risk" (i.e., incidence in cases per person-year and incidence as a probability).

A different kind of incidence that is also a probability arises when the "interval" in which the event can happen is composed of discrete units that must be counted rather than continuously measured. This is the case for instance when measuring the risk of transmission of HIV per sexual intercourse, or the risk of cardiac arrest per intravenous injection of radiological contrast. These are ratios of two event counts that can be called incidences, though they are usually referred to as proportions or probabilities (perhaps because time is not involved), often expressed as a percentage.

Table 2 summarizes the commonly used terminology for the four incidence measures by different authors [7,11–13,15,16,20,22,26].

DIMENSIONS AND UNITS OF THE MEASURES OF INCIDENCE

The dimensional formula of a quantity is the expression of this quantity as a product of powers of the quantities considered as fundamental, usually mass (M), length (L), and time (T). For instance, as velocity is defined $V = L/T$, its dimensional formula reads $[V] = LT^{-1}$. Discrete variables measured as counts are dimensionless because they cannot be expressed in terms of fundamental magnitudes. However, dimensions should not be confused with units. For example, although counts are dimensionless, they do have units, be they cases (when counting diseases), persons (when counting populations), protons (when counting atomic numbers), etc. [27].

Incidence measured as a number n of events is therefore dimensionless. When measured as events per time unit, n/t, the dimensional formula of incidence is T^{-1}, as this type of incidence measure is a fraction in which the numerator is an adimensional count and the denominator is a time. When measured as events per unit of amount of observation, $n/(e\,t)$, the fraction representing incidence has an adimensional count in the numerator. The denominator is a product of a count times a time, so it is dimensionally a time, and the dimensional formula of the fraction is therefore T^{-1} too. That is the same as saying that both n/t and $n/(e\,t)$ are rates, in Elandt-Johnson's terminology [28]. Though these two incidence measures have the same dimensional formula, they are measured in different units (events per time versus events per element-time). Interestingly, other quantities such as the frequency of a radiowave (measured, for instance, in cycles per second [Hertz]) or the activity of a radionuclide (measured in nuclear transformations per second [Becquerel] also have T^{-1} as their dimensional formula.

When incidence is measured as n/e, which is a ratio of two counts, it is dimensionless as there are no dimensional quantities in either numerator or denominator.

THE NEED FOR A NEW TERMINOLOGY

The incidence measures commonly used receive a variety of names (Table 2). The term "incidence rate," which is assumed to refer to "a clearly defined concept" by many epidemiologists, can actually refer to several different measures. Twenty years ago Regina Elandt-Johnson [28] defined rate as a measure of change in an amount per unit change in another quantity. However, as has been pointed out by Rothman [15], her early effort to resist the incorrect use of the term "rate" was largely unsuccessful. Indeed "rate" is now widely used in public health to mean nothing more than proportion or even ratio, as can be inferred from such expressions as "prevalence rate" and "case fatality rate" (both are proportions) or "infant mortality rate" and "under-five mortality rate" (both are ratios). In a recently published epidemiology textbook [9], "rate" is defined as a measure "calculated by dividing the number of cases by the corresponding number of people in the population at risk," and even in Fleiss's [29] book on rates and proportions, ". . . the terms probability, relative frequency, proportion, and rate are used synonymously." This "incorrect" use of "rate" has been so common during the last 20 years that any effort to rescue its original meaning would be worthless. Table 2 reveals how inconsistently "rate" has been used by different authors even when defining incidence measures. Though most authors in epidemiology today use "incidence rate" in the same sense as Kelsey [17], Rothman [15], or Selvin [26] to refer to element-time (person-years) incidence, this use is not at all generalized. For instance, Freeman and Hutchison [11] use "incidence density" or "force of morbidity" to refer to person-time incidence and give the term "incidence rate" a sense completely opposite to that used by the most authors. Last [7] and others [9,19,20] use "rate" for almost any incidence measure, except for the count of new cases. Clearly "rate" now lacks specific meaning and it is best to avoid the term altogether when looking for precision.

Another term that has to be used carefully is "risk." Though Kleinbaum, Kupper, and Morgenstern [22] tried to reserve this term for the probability of disease, "risk" is now widely used to refer to the frequency of disease (or death) in general. Probably no epidemiologist would object to designating both incidence in cases per thousand person-years and "cumulative incidence" expressed as a percentage as measures of risk. Relative risks are computed as ratios of incidence probabilities (n/e) or person-year incidences ($n/[e\,t]$), so it does not seem appropriate to use "risk" to refer to only one of the incidence measures.

The term "cumulative incidence" is also confusing as it appears to suggest that something accumulates over from former periods (as in "cumulative frequency"). All these are good reasons to look for a new terminology.

The fraction n/e, which has been called "risk" by Morgenstern *et al.* [13] "incidence proportion" by Greenland [17] and Selvin [22], and "cumulative incidence" by Rothman [15] is from the conceptual point of view best understood as a probability, so it would be best named *incidence probability* or *probability of incidence*. No other incidence measure is a probability, so this term is not ambiguous.

Probably most epidemiologists would agree that in strict sense incidence is the number of events per element-time unit, $n/(e\,t)$. So it would be possible to admit only two incidence measures, incidence in the strict sense and incidence probability. Therefore neither the single count of events nor the λ parameter—the number of events per time unit—should be called incidence. However, as shown in Table 2, both have been called "incidence" by some authors and, in fact, the number of new cases in a loosely defined time period (usually one year) is often used as a synonym for incidence by practitioners in public health surveillance. "Incidence number" could perhaps be acceptable for the count of events (n). Considering that the idea of incidence as a measure of the occurrence of events over time is fairly widespread, the adjectives "absolute" and "relative" could be used to qualify this idea, according to whether it refers to the absolute number of events or to the number of events relative to the total population (as in statistics with "absolute" and "relative" frequency). In fact, Freeman and Hutchison [11] have stated that incidence has "two different forms, an absolute form and a population-based form." It thus seems logical to refer to one as "absolute" and the other as "relative" incidence. However this has an inconvenience, as "relative incidence" could be misunderstood to mean the ratio of two incidences (as "relative odds" is a ratio of two odds). For all these reasons, it would perhaps be best to reserve the term "incidence" for $n/(e\,t)$, i.e., the number of events per unit of amount of observation, and to qualify "incidence" with "probability," "number," or "absolute" when referring to n/e, n, and n/t, respectively. Table 3 outlines this proposed terminology.

INCIDENCE MEASURES WITH LENGTH-BASED AMOUNT OF OBSERVATION

At the beginning of this paper, incidence was defined as a set of measures to assess the *dynamics* of events occurring in a "population" or set of elements. One of the components

TABLE 3. A proposal for a standardized terminology of incidence measures

	Incidence measures denominations			
	Incidence number	**Absolute incidence**	**Incidence in the strict sense**	**Probability of incidence or incidence probability**
Definition	Number of events	Events per time unit	Events per unit of amount of observation (element-time unit)	Proportion of the population in which the event occurs
Mathematical definition	n	n/t	$\frac{n}{e\,t}$	n/e

defined for incidence measures be calculated was an interval t during which every element "at risk" of the event is observed." In physics, "dynamics" is related to motion, and this involves both time and space. The lapse or period for the observation of events does not have to be a time period, it can also be a distance. This kind of measure of incidence is very common in epidemiological studies of transport crashes and related deaths and injuries, where incidence is measured in number of events per time spent travelling or per unit distance of travel, and units such as events per passenger-hour, per vehicle-kilometer, per passenger-kilometer, or even per passenger-journey are utilized [30]. The study of injuries or crashes clearly illustrates the importance of defining the observed population. Expressing the incidence in terms of events per vehicle-kilometer, per passenger-kilometer, or per passenger-hour can reveal very different things in relation to risk factors or prevention opportunities [30].

When measuring length-based incidences, incidence in the strict sense will be given by the formula $n/(e\ l)$ where l is a length. Units will be events per passenger-kilometer or passenger-mile or vehicle-mile or whatever depending on the "population at risk" and the units we choose. It would also be possible to define absolute incidence as events per distance unit (n/l). There is no time involved in any of these two measures but these are rates in the Elandt-Johnson [28] sense of change in a quantity per unit change in another quantity. The dimensional formula of both $n/(e\ l)$ and n/l will be L^{-1}. In this context the incidence probability would also be a dimensionless proportion, defined as the proportion of vehicles or passengers (depending on what is the defined population "at risk") having a crash or an injury after traveling a given distance.

Formulas (i), (ii), and (iii) would be suitable (substituting l_i for t_i where l_i is the distance traveled by the group or element i) to express incidence per element-distance unit. Equally, *mutatis mutandis* formula (iv) would represent incidence as the probability associated with traveling a given distance.

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