

The Relative Merits of Risk Ratios and Odds Ratios

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When a study outcome is rare in all strata used for an analysis, the odds ratio estimate of causal effects will approximate the risk ratio; therefore, odds ratios from most case-control studies can be interpreted as risk ratios. However, if a study outcome is common, the odds ratio will be further from 1 than the risk ratio. There is debate regarding the merits of risk ratios compared with odds ratios for the analysis of trials and cohort and cross-sectional studies with common outcomes. Odds ratios are conveniently symmetrical with regard to the outcome definition; the odds ratio for outcome Y is the inverse of the odds ratio for the outcome not Y. Risk ratios lack this symmetry, so it may be necessary to present 1 risk ratio for outcome Y and another for outcome not Y. Risk ratios, but not odds ratios, have a mathematical property called collapsibility; this means that the size of the risk ratio will not change if adjustment is made for a variable that is not a confounder. Because of collapsibility, the risk ratio, assuming no confounding, has a useful interpretation as the ratio change in average risk due to exposure among the exposed. Because odds ratios are not collapsible, they usually lack any interpretation either as the change in average odds or the average change in odds (the average odds ratio).

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For more than 20 years, there has been debate about the relative merits of risk ratios compared with odds ratios as estimates of causal associations between an exposure (such as smoking or medication for high blood pressure) and a binary outcome (such as death vs life). In this article, I discuss how these measures differ and review arguments for each. To limit my discussion, I will ignore other useful measures of association, such as the rate ratio, hazard ratio, risk difference, and rate difference.

AGREEMENTS REGARDING RISK RATIOS AND ODDS RATIOS

In a clinical trial or cohort study in which all subjects are followed up for the same time, the cumulative incidence (average

risk) of the outcome is $A/(A+B)$ among those exposed (**Table 1**) and $C/(C+D)$ among those not exposed; the corresponding odds are A/B and C/D , respectively. The risk ratio is therefore $[A/(A+B)]/[C/(C+D)]$ and the odds ratio is $(A/B)/(C/D)$. In the literature about these ratios, there are 2 areas where there is general agreement: if the outcome is rare, the odds ratio will approximate the risk ratio; and in most case-control studies, odds ratios will approximate risk ratios.

Approximation of Risk Ratios by Odds Ratios When Outcomes Are Rare

If the study outcome is rare among those exposed (Table 1), then A will be small relative to B, so the risk ($A/[A+B]$) will be close to the odds (A/B). Similarly, if the outcome is rare among those not exposed, then C will be small relative to D and $[C/(C+D)]$ will be close to (C/D) . If

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Table 1. Data From a Hypothetical Clinical Trial or Cohort Study^a

Exposed	Outcome		Risk of Outcome	Odds of Outcome
	Yes	No		
Yes	A	B	$A/(A+B)$	A/B
No	C	D	$C/(C+D)$	C/D

^aA, B, C, and D are counts according to exposure and outcome. Formulae for risk ratios and odds ratios (e, exposed; ne, not exposed; R, individual risks for outcome=1): Count A=sum of risks for outcome=1 if exposed=sum_e(R_e). Count B=sum of risks for outcome=0 if exposed=sum_e(1-R_e). Count C=sum of risks for outcome=1 if not exposed=sum_{ne}(R_{ne}). Count D=sum of risks for outcome=0 if not exposed=sum_{ne}(1-R_{ne}). Ratio change in cumulative incidence risk= $[A/(A+B)] \div [C/(C+D)] = \text{sum}_e(R_e)/(A+B) \div \text{sum}_{ne}(R_{ne})/(C+D) = \text{sum}_e(R_e)/(A+B) \div \text{sum}_{ne}(R_{ne})/(A+B) = \text{ratio change in average risk due to exposure for the exposed} = \text{average risk ratio} = \text{sum}_e(R_e/R_{ne})/(A+B)$. Ratio change in cumulative incidence odds= $(A/B) \div (C/D) = \text{sum}_e(R_e)/\text{sum}_e(1-R_e) \div \text{sum}_{ne}(R_{ne})/\text{sum}_{ne}(1-R_{ne}) = \text{sum}_e(R_e)/\text{sum}_e(1-R_e) \div \text{sum}_{ne}(R_{ne})/\text{sum}_{ne}(1-R_{ne}) = \text{ratio change in average odds due to exposure for the exposed} = \text{sum}_e[R_e/(1-R_e)]/(A+B) \div \text{sum}_{ne}[R_{ne}/(1-R_{ne})]/(C+D) = \text{average ratio change in odds due to exposure for the exposed} = \text{average odds ratio} = \text{sum}_e[R_e/(1-R_e)] \div \text{sum}_{ne}[R_{ne}/(1-R_{ne})]$.

Table 2. Hypothetical Data for a Trial of Drug X

Treatment	Outcome, No.		Risk of Death	Odds of Death
	Died	Survived		
Drug X	25	75	$25/(25+75) = .25$	$25/75 = 0.33$
Placebo	50	50	$50/(50+50) = .50$	$50/50 = 1.00$

the outcome is rare in both exposed and unexposed persons, the odds ratio ($[A/B]/[C/D]$) will approximate the risk ratio ($[A/(A+B)]/[C/(C+D)]$).

However, when the study outcome is common and the risk ratio is not close to 1, the odds ratio will be further from 1 compared with the risk ratio. If the risk ratio is greater than 1, the odds ratio will be greater still, and if the risk ratio is smaller than 1, the odds ratio will be even smaller. A hypothetical randomized trial of drug X is presented in **Table 2**; the risk ratio for death among patients given X, compared with those given placebo, is $.25/.50 = 0.5$. The corresponding odds ratio is $0.33/1 = 0.33$, which is further from 1.

Figure 1 shows the relationship between odds and risk ratios according to 4 levels of risk among unexposed persons; **Figure 2** uses 4 risk levels for exposed persons; and **Figure 3** shows 4 levels of average risk for both unexposed and exposed subjects. When the outcome risk is .01 or less, odds ratios and risk ratios agree well for risk ratio values ranging from 0.1 to 10 in all 3 figures. When cumulative incidence is .10, the odds ratio is within 10% of the risk ratio for risk ratios ranging from 0.1 to 1.8 in Figure 1, from 0.55 to 10 in Figure 2, and from 0.4 to 2.5 in Figure 3. When cumulative incidence is .25 or greater, the odds ratio differs notably from most of the risk ratios in all 3 figures.

Even if the outcome is uncommon among all persons in a study, the odds ratio may not approximate the risk ratio well if adjustment is made for potential confounding variables and the outcome is not rare in some expo-

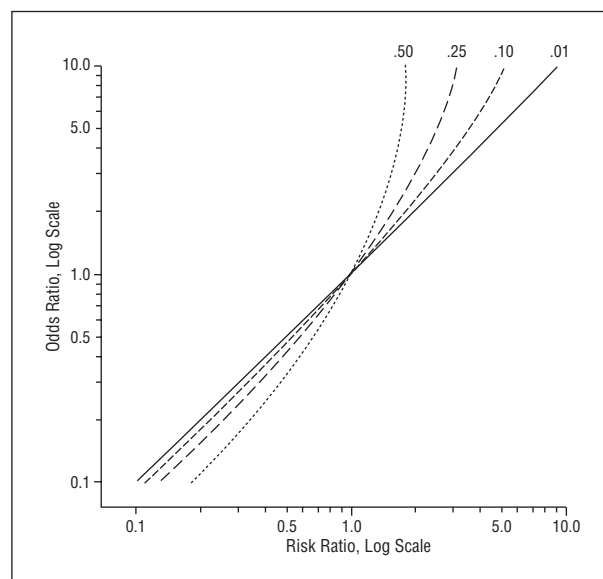


Figure 1. Relationship of the odds ratio to the risk ratio according to 4 levels of outcome risk (cumulative incidence) for unexposed subjects: .01, .10, .25, and .50.

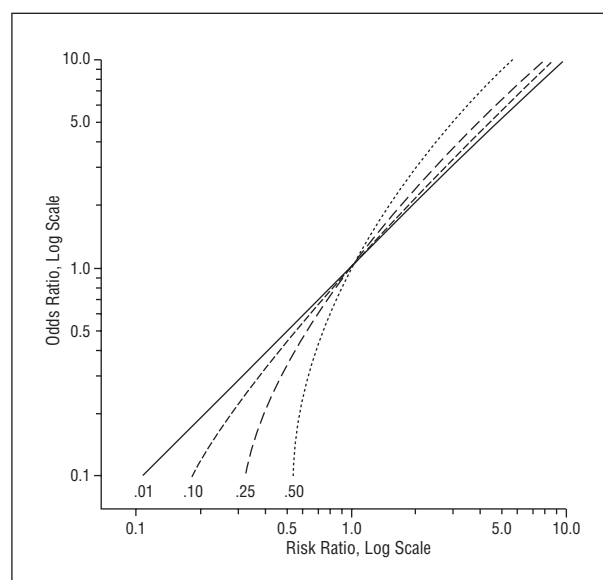


Figure 2. Relationship of the odds ratio to the risk ratio according to 4 levels of outcome risk (cumulative incidence) for exposed subjects: .01, .10, .25, and .50.

sure subgroups formed by levels of the confounding variable.¹ For example, in hypothetical data for a cohort study of traffic crashes (**Table 3**), death was uncommon among those wearing a seat belt (risk = $150/5500 = .027$) and among those not wearing a seat belt (risk = $300/5500 = .055$). The crude (unadjusted) odds ratio for death among belted occupants compared with unbelted occupants is 0.49, which is close to the risk ratio of 0.50. However, 375 of the 450 deaths (83%) were in high-speed crashes, in which 25% of those wearing seat belts and 50% of those not wearing seat belts died. Using logistic regression to adjust for speed, the adjusted odds ratio is 0.36; this does not approximate the adjusted risk ratio of 0.5 well. Estimates such as those in Figures 1, 2, and 3 should be used with caution because they fail to account for the possibility that out-

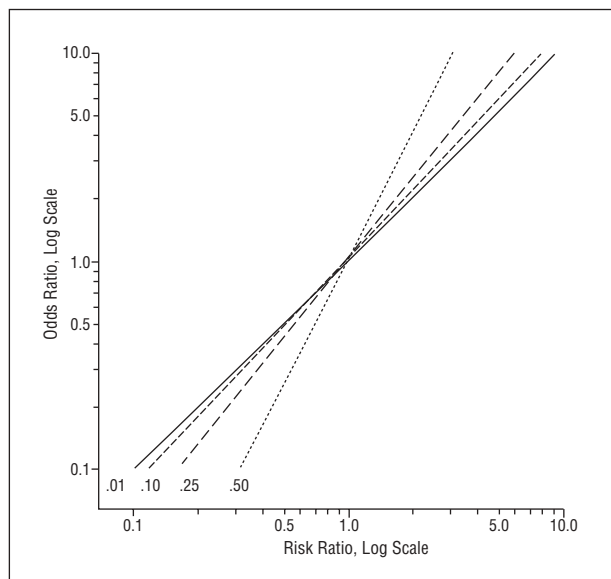


Figure 3. Relationship of the odds ratio to the risk ratio according to 4 levels of outcome risk (cumulative incidence) for an average exposed and unexposed subject: .01, .10, .25, and .50. Estimates assume the number of exposed subjects is equal to the number unexposed.

comes might be common in some exposure subgroups that contribute a notable portion of the outcomes.

Approximation of Risk Ratios by Odds Ratios in Most Case-Control Studies

Case-control studies are typically (but not always) used when outcomes are rare in the population from which study subjects are sampled. Outcome risks and odds often cannot be estimated directly from case-control data, because the sampling proportions of cases and controls may be unknown. However, the odds ratio for the outcome, $(A/B)/(C/D)$ in Table 1, can be rewritten as $(A/C)/(B/D)$, which is the odds of exposure among the selected cases (persons with the outcome) divided by the odds of exposure among the selected controls (persons without the outcome). This ratio can be estimated from case-control data and it will approximate the risk ratio in the population from which the cases and controls were sampled when outcomes are rare in that population. This insight was described in 1951² and contributed to the usefulness of case-control studies.

For completeness, I note that there are case-control designs in which controls are sampled at the time each case outcome occurs. In this design, the odds ratio will estimate the incidence rate ratio even when outcomes are common; no rare outcome assumption is needed.³⁻⁵

ARGUMENTS REGARDING RISK RATIOS AND ODDS RATIOS

There is debate regarding the merits of risk ratios compared with odds ratios for the analysis of controlled trials, cohort studies, and cross-sectional studies with common outcomes. I will discuss 4 arguments that have been used to advocate for one ratio or the other.

Table 3. Hypothetical Cohort Study of Seat Belt Use and Death in a Traffic Crash

Vehicle Crash Speed	Seat Belt Used	Outcome, No.		Risk	Risk Ratio	Odds	Odds Ratio
		Died	Survived				
Low	Yes	25	4975	.005	0.50	0.005	0.50
	No	50	4950	.010		0.010	
High	Yes	125	375	.250	0.50	0.333	0.33
	No	250	250	.500		1.000	
Total		450	10 550	.041		0.043	

Ease of Interpretation of Risk Ratios by Clinicians

Some argue that risk ratios should be preferred because they are more easily understood by clinicians.^{6,7} However, if odds ratios were otherwise superior, a better solution might be to use odds ratios and remedy any deficiency in clinician education.

Symmetry of Odds Ratios Regarding Outcome Definitions

Some authors prefer odds ratios because they are symmetrical with regard to the outcome definition. Imagine a hypothetical trial of drug X and outcome Y (Y=death in Table 2). There is symmetry for both the odds and risk ratios with regard to the definition of the exposure: both ratio estimates for treatment with X compared with no X are the inverse of the ratio estimates for no X compared with treatment with X.

However, if we change the definition of the outcome from the occurrence of Y to no occurrence of Y, only the odds ratio is symmetrical. The odds ratio for Y among those treated with X compared with those who did not get X is $(A/B)/(C/D) = (25/75)/(50/50) = (1/3)/(1) = 0.33$ (Table 2). The odds ratio for no occurrence of Y among those treated with drug X compared with those who did not get X is $(B/A)/(D/C) = (75/25)/(50/50) = 3/1 = 3$. These odds ratios are simply the reciprocal of each other. The corresponding risk ratios are $[A/(A+B)]/[C/(C+D)] = (25/100)/(50/100) = 0.5$ and $[B/(A+B)]/[D/(C+D)] = (75/100)/(50/100) = 1.5$; these risk ratios are not reciprocal. The symmetry property of the odds ratio is attractive because 1 odds ratio can summarize the association of X with Y, and the choice between outcome Y and outcome not Y is unimportant.⁸⁻¹²

If outcome events are rare, the odds ratio and the risk ratio for rare outcomes will be similar. The odds ratio for no event will be the inverse of the odds ratio for the event. The risk ratio for no event will necessarily be close to 1 (**Figure 4**) and therefore of little interest. Thus, when outcome events are rare, the symmetry issue is typically not important.

Constancy of Odds Ratios for Common Outcomes

Some authors prefer odds ratios because they believe a constant (homogenous) odds ratio may be more plausible than a constant risk ratio when outcomes are common. Risks range from 0 to 1. Risk ratios greater than 1

have an upper limit constrained by the risk when not exposed. For example, if the risk when not exposed is .5, the risk ratio when exposed cannot exceed 2: $.5 \times 2 = 1$. In a population with an average risk ratio of 2 for outcome Y among those exposed to X, assuming that the risk for Y if not exposed to X varies from .1 to .9, the average risk ratio must be less than 2 for those with risks greater than .5 when not exposed. Because the average risk ratio for the entire population is 2, the average risk ratio must be more than 2 for those with risks less than .5 when not exposed. Therefore, a risk ratio of 2 cannot be constant (homogeneous) for all individuals in a population if risk when not exposed is sometimes greater than .5. More generally, if the average risk ratio is greater than 1 in a population, the individual risk ratios cannot be constant (homogeneous) for all persons if any of them have risks when not exposed that exceed $1/\text{average risk ratio}$.

Odds range from 0 to infinity. Odds ratios greater than 1 have no upper limit, regardless of the outcome odds for persons not exposed. If we multiply any unexposed outcome odds by an exposure odds ratio greater than 1 and convert the resulting odds when exposed to a risk, that risk will fall between 0 and 1.^{8,11} Thus, it is always hypothetically possible for an odds ratio to be constant for all individuals in a population.¹²⁻¹⁴

Rebuttals to Odds Ratio Constancy Argument

Possibility of Constancy for Risk Ratios Less Than 1. For both risk and odds, the lower limit is 0. For any level of risk or odds under no exposure, multiplication by a risk or odds ratio less than 1 will produce a risk or odds given exposure that is possible: 0 to 1 for risks and 0 to infinity for odds. Thus, a constant risk or odds ratio is possible for ratios less than 1. If the risk ratio comparing exposed persons with those not exposed is greater than 1, the ratio can be inverted to be less than 1 by comparing persons not exposed with those exposed. Therefore, a constant risk ratio less than 1 is hypothetically possible. This argument has been used to rebut the criticism of the risk ratio in the previous argument.¹⁵

Constancy of Risk or Odds Ratios Not Necessary for Inference. It is not necessary that the risk or odds ratio be constant. We can use a ratio to make causal inferences or decisions about public and personal health if the ratio has an interpretation as an average effect: either (1) the ratio change in average risk or odds or (2) the average ratio change in risk or odds. The ratio change in average risk (or odds) is not the same as the average ratio change in risk (or odds) (Table 1).

An estimate of the change in average risk is useful, though it may not estimate the change expected for every individual or even for any particular individual. For example, if research suggests that wearing a seat belt in a crash reduces the average risk of death by 50%, people could use this information to make a decision about seat belt use, although this estimate might not apply to them in a given crash. If we had evidence that the average ratio change in risk varied with levels of another factor, say age, the best choice might be to estimate several risk ra-

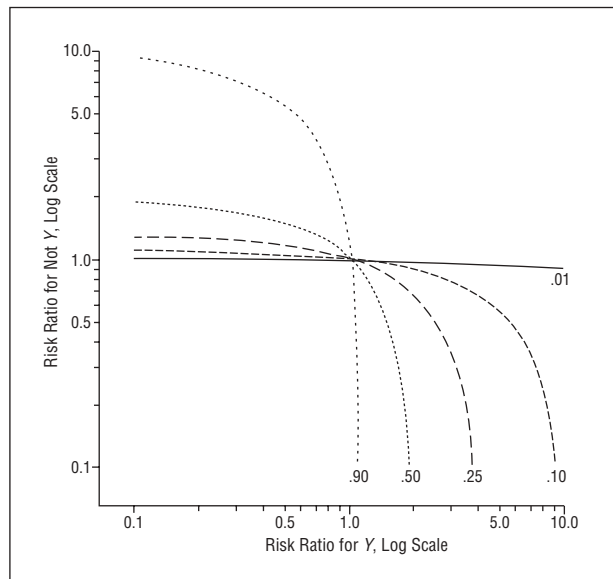


Figure 4. Relationship of the risk ratio for outcome Y to the risk ratio for outcome not Y, according to 5 levels of outcome risk (cumulative incidence) for unexposed subjects: .01, .10, .25, .50, and .90.

tios, each an estimate of the change in average risk within a particular subgroup.

Lack of Collapsibility—A Barrier to Estimating Odds Ratios That Can Be Interpreted as Averages. It has been known for years that risk ratios are collapsible while odds ratios are not,^{1,11,16-18} but this is not mentioned in much of the literature about these ratios. Some readers may find this topic unfamiliar, technical, and even counterintuitive. To simplify my discussion, I will assume that there is no bias due to confounding. Had exposure not occurred, the average risk for the outcome would have been the same in the exposed group as in the unexposed group.

Collapsibility means that in the absence of confounding, a weighted average of stratum-specific ratios (eg, using Mantel-Haenszel methods¹⁹) will equal the ratio from a single 2×2 table of the pooled (collapsed) counts from the stratum-specific tables. This means that a crude (unadjusted) ratio will not change if we adjust for a variable that is not a confounder.^{18,20}

I created hypothetical data for 2 randomized controlled trials (**Table 4** and **Table 5**). In both trials, the risk for the outcome, aside from exposure to treatment, falls into just 2 categories. There is no confounding, as the average risk without exposure is the same in each trial arm. Individual risks without exposure would not be known in an actual trial; if they were knowable, we would not need a control group. The key difference between the tables is that the risk ratio is constant for all persons in Table 4, whereas the odds ratio is constant in Table 5.

In Table 4, the risk ratio due to exposure is 3.00 for all exposed persons, regardless of outcome risk (.05 or .25) if not exposed. In a real trial, we could observe only the data in the last column, where the risk ratio is 3.00 for males, females, and all persons. The risk ratio of 3.00 for all persons has 3 interpretations¹: (1) the ratio change in risk due to exposure for the exposed group, (2) the ratio change in the average risk due to exposure among

Table 4. Hypothetical Randomized Trial of Treatment X and Outcome Y^a

	Low-Risk Subjects		High-Risk Subjects		All Subjects	
	Y	No Y	Y	No Y	Y	No Y
Males						
Treatment X, No.						
Yes	15	85	225	75	240	160
No	5	95	75	225	80	320
Risk ratio	3.00		3.00		3.00	
Odds ratio	3.35		9.00		6.00	
Females						
Treatment X, No.						
Yes	45	255	75	25	120	280
No	15	285	25	75	40	360
Risk ratio	3.00		3.00		3.00	
Odds ratio	3.35		9.00		3.86	
All Persons						
Treatment X, No.						
Yes	60	340	300	100	360	440
No	20	380	100	300	120	680
Risk ratio	3.00		3.00		3.00	
Odds ratio	3.35		9.00		4.64	

^aThe risk for outcome Y, aside from exposure, is either low (.05) or high (.25). In this table, risk ratios are the same for all subjects. Ratio change in average risk due to exposure among the exposed = $[(.05 \times (15 + 85 + 45 + 255) \times 3.00] + [.25 \times (225 + 75 + 75 + 25) \times 3.00] \div [(.05 \times (15 + 85 + 45 + 255)) + [.25 \times (225 + 75 + 75 + 25)]] = 3.00$. Average ratio change in risk for all exposed individuals (average risk ratio) = $[(3.00 \times (15 + 85)) + [3.00 \times (225 + 75)] + [3.00 \times (45 + 255)] \div [(15 + 85 + 225 + 75 + 45 + 255 + 75 + 25)] = 3.00$. Ratio change in average odds due to exposure among the exposed = $[(.05/.95) \times (15 + 85 + 45 + 255) \times 3.35] + [(.25/.75) \times (225 + 75 + 75 + 25) \times 9.00] \div [(.05/.95) \times (15 + 85 + 45 + 255)] + [(.25/.75) \times (225 + 75 + 75 + 25)] = 8.23$. Average ratio change in odds for all exposed individuals (average odds ratio) = $[(3.35 \times (15 + 85)) + [9.00 \times (225 + 75)] \div [(15 + 85 + 225 + 75 + 45 + 255 + 75 + 25)] = 6.18$.

Table 5. Hypothetical Randomized Trial of Treatment X2 and Outcome Y2^a

	Low-Risk Subjects		High-Risk Subjects		All Subjects	
	Y2	No Y2	Y2	No Y2	Y2	No Y2
Males						
Treatment X2, No.						
Yes	51	51	288	18	339	69
No	20	80	240	60	260	140
Risk ratio	2.50		1.18		1.28	
Odds ratio	4.00		4.00		2.65	
Females						
Treatment X2, No.						
Yes	153	153	96	6	249	159
No	60	240	80	20	140	260
Risk ratio	2.50		1.18		1.74	
Odds ratio	4.00		4.00		2.91	
All Persons						
Treatment X2, No.						
Yes	204	204	384	24	588	228
No	80	320	320	80	400	400
Risk ratio	2.50		1.18		1.44	
Odds ratio	4.00		4.00		2.58	

^aThe risk for outcome Y2, aside from exposure, is either low (.2) or high (.8). In this table, the odds ratio is the same for all subjects. Ratio change in average risk due to exposure among the exposed = $[(.2 \times (51 + 51 + 153 + 153) \times 2.50] + [.8 \times (288 + 18 + 96 + 6) \times 1.18] \div [(.2 \times (51 + 51 + 153 + 153)) + [.8 \times (288 + 18 + 96 + 6)]] = 1.44$. Average ratio change in risk for all exposed individuals (average risk ratio) = $[(2.50 \times (51 + 51)) + [1.18 \times (288 + 18)] + [2.50 \times (153 + 153)] \div [(51 + 51 + 288 + 18 + 153 + 153 + 96 + 6)] = 1.84$. Ratio change in average odds due to exposure among the exposed = $[(.2/.8) \times (51 + 51 + 153 + 153) \times 4.00] + [(.8/.2) \times (288 + 18 + 96 + 6) \times 4.00] \div [(.2/.8) \times (51 + 51 + 153 + 153) + (.8/.2) \times (288 + 18 + 96 + 6)] = 4.00$. Average ratio change in odds for all exposed individuals (average odds ratio) = $[(4.00 \times (51 + 51)) + [4.00 \times (288 + 18)] + [4.00 \times (153 + 153)] \div [(51 + 51 + 288 + 18 + 153 + 153 + 96 + 6)] = 4.00$.

the exposed, and (3) the average ratio change in risk for all exposed individuals (ie, the average risk ratio) (Table 1).

The risk ratios in Table 4 are collapsible. Any weighted average of the constant risk ratio of 3.00 should be 3.00; in Table 4, all 5 collapsed tables do indeed yield risk ra-

tios of 3.00. Collapsibility means that if we adjust the risk ratio of 3.00 for all persons for a variable that is not a confounder, sex in this example, the adjusted risk ratio will still be 3.00. This is true in Table 4.

The risk ratios for males and females are also collapsible in Table 5. Sex is not a confounder in Table 5, and a Mantel-Haenszel-weighted average of the risk ratios for males (1.28) and females (1.74) is equal to the risk ratio of 1.44 from the collapsed table of counts for all persons. If we adjust the risk ratio of 1.44 for sex, the result is still 1.44.

Individual risks when not exposed would not be known in real data. But the 2×2 table of counts for all persons could be obtained in a study, and from those counts, the ratio change in average risk (1.44) in Table 5 can be estimated. Calculations in Table 5 show that the ratio change in average risk is indeed 1.44. However, unlike the example in Table 4, the risk ratio of 1.44 in Table 5 is not the average ratio change in risk (the average risk ratio) for all individuals, which is 1.84 (calculations in Table 5). The average risk ratio could be estimated in a study only if the risk ratio was the same for all exposed persons¹; this unlikely scenario was shown in Table 4.

Odds ratios are not collapsible. The odds ratio is constant (4.00) for all exposed persons in Table 5. However, when we collapse across the categories of risk, the odds ratios from each of the 3 collapsed tables are not equal to a weighted average of 4.00; all are closer to 1. Furthermore, the odds ratio of 2.58 for all persons is not a weighted average of the odds ratios of 2.65 for men and 2.91 for women, as 2.58 is closer to 1 than either stratum-specific estimate. Adjusting the odds ratio of 2.58 for sex, using Mantel-Haenszel methods, produces an odds ratio of 2.79, though sex is not a confounder. If we could adjust for more variables, such as age, the adjusted odds ratio would tend to move away from 2.58 and closer to 4.00.¹

The odds ratio of 2.58 in Table 5 is an unbiased estimate of the ratio change in odds due to exposure for the exposed group. However, it is not the ratio change in the average odds due to exposure among the exposed, which is 4.00; nor is it the average ratio change in odds for all exposed individuals (the average odds ratio), which is also 4.00. The odds ratio will estimate the average change in odds (the average odds ratio) among exposed individuals only when all individual odds ratios are equal and all individual outcome risks without exposure are equal¹; this implausible scenario is shown in Table 5, where collapsed counts for low- (or high-) risk subjects only produce a 2×2 table with an odds ratios of 4.00.

Similarly, the odds ratio of 4.64 for all persons in Table 4 is an unbiased estimate of the ratio change in odds due to exposure for the exposed group, but 4.64 is not the ratio change in average odds due to exposure, which is 8.23 (bottom of Table 4); and 4.64 is not the average ratio change in odds (the average odds ratio), which is 6.18. If we use Mantel-Haenszel methods to adjust the odds ratio of 4.64 for sex, a variable that is not a confounder, the adjusted odds ratio is 5.00.

In summary, the risk ratio has a useful interpretation as the ratio change in average risk due to exposure among the exposed.¹ The odds ratio lacks any interpretation as an average. The odds ratio estimated from observed data

will be closer to 1 than the ratio for the change in average odds.^{1,17,21} If we had data from a large trial in which randomization balanced all measured variables (thereby removing confounding by these factors), estimating the ratio change in average odds due to treatment would require adjustment for variables related to outcome risk; if variation in outcome risk under no exposure persisted within the adjustment covariate patterns, the adjusted odds ratio would still be closer to 1 than the desired estimate.

The odds ratio does not estimate the average change in odds (the average odds ratio) among exposed individuals either, except under implausible restrictions.¹ This means that estimating a constant (homogenous) odds ratio that applies to all exposed individuals, as proposed in the argument that a constant odds ratio is more plausible, will usually be impossible, even if a constant odds ratio actually exists.

Useful discussions of this topic, with examples, can be found in publications by Greenland¹ and Newman.¹¹ Greenland suggests that odds ratios should not be used for inference unless they approximate risk ratios. Newman acknowledges the collapsibility problem and argues that the exposure-outcome association cannot be summarized by a single odds ratio.¹¹ He suggests reporting 2 odds ratios. The first odds ratio is for the effect of exposure on the entire exposed group; this corresponds to the ratio change in incidence odds (Table 1), which was 4.64 in Table 4 and 2.58 in Table 5. The second odds ratio is a summary across whatever stratum-specific odds ratios are available; this corresponds to a Mantel-Haenszel-adjusted odds ratio of 5.0 for Table 4 and 2.79 for Table 5. However, all of these odds ratios lack any interpretation as an average.

SUMMARY OF AGREEMENTS AND ARGUMENTS

Odds ratios approximate risk ratios when outcomes are rare in all noteworthy strata used for an analysis. When outcomes are rare, all 4 arguments can be ignored. This is most useful in case-control studies, in which odds ratios can be interpreted as risk ratios; whether the estimates are called odds or risk ratios is a matter of style.

When event outcomes are common, odds ratios will not approximate risk ratios. Odds ratios are conveniently symmetrical regarding the outcome, and a constant odds ratio may be more plausible than a constant risk ratio, but estimating a constant odds ratio will usually be impossible. Even estimating the ratio change in average odds may involve insurmountable practical difficulties.

Because risk ratios are not symmetrical, analysts using risk ratios may wish to present 2 risk ratios when the outcome is common: one for outcome Y and another for outcome not Y. Because odds ratios are not collapsible, those who report odds ratios could say explicitly that the estimated odds ratio will be closer to 1 than the ratio change in average odds. Also, because odds ratios are sometimes misinterpreted as risk ratios,²²⁻²⁵ studies that report odds ratios when outcomes are common could state that the estimates do not approximate risk ratios.

I have tried to give sufficient information to allow readers to choose between odds ratios or risk ratios. For myself, I prefer risk ratios because they can be interpreted as a ratio change in average risk.

HOW TO ESTIMATE RISK RATIOS

Methods for estimating crude and adjusted risk ratios are not widely described in textbooks, so I will briefly list some with citations. Reviews are available.^{26,27}

When outcomes are rare, odds ratios will approximate risk ratios, so Mantel-Haenszel methods for odds ratios or logistic regression can be used to estimate risk ratios.^{11,28} When outcomes are rare or common, risk ratios can be estimated using several methods:

1. Mantel-Haenszel methods for risk ratios.^{11,28}
2. Regression with marginal standardization.^{26,29-33} After fitting a regression model that can estimate the risk for a binary outcome, one can estimate, for each subject, the outcome risk if exposed ($X=1$) and if not exposed ($X=0$), adjusted for each subject's values of other variables in the regression model. The average adjusted risk if exposed divided by the average if not exposed is the standardized risk ratio; it is standardized to the distribution of the variables in the study sample. This method can be used with logistic, probit, log-log, or complementary log-log regression models.^{34,35} Confidence intervals can be estimated with delta³⁶ or bootstrap³⁷ methods.
3. Regression methods that directly estimate risk ratios.^{26,38-44} Exponentiated coefficients from a generalized linear model with a log link and binomial outcome distribution are risk ratios.
4. Poisson regression.^{26,42-47} If Poisson regression is applied to data with binary outcomes, the exponentiated coefficients are risk ratios or prevalence ratios. The Poisson confidence intervals will be too wide, but approximately correct intervals can be estimated using a robust (sandwich, survey, or generalized estimating equation) estimator of variance.
5. For matched cohort data, such as studies of twins, risk ratios can be estimated with Mantel-Haenszel methods, conditional Poisson regression, or stratified proportional hazards models.⁴⁸

A crude odds ratio can be converted to a crude risk ratio: $\text{risk ratio} = \text{odds ratio} / [(1 - p_0) + (p_0 \times \text{odds ratio})]$, in which p_0 is the outcome prevalence (risk) among the unexposed. Some have applied this formula to an adjusted odds ratio to obtain an adjusted risk ratio.⁴⁹ This method can produce biased risk ratios and incorrect confidence intervals.^{26,32,41,50-52}

COMMENT

To assist a personal or public health decision or make a treatment choice, the usefulness of both risk and odds ratios is often enhanced if we are also given information about the frequency of the outcome disease and the prevalence of the exposure risk factor. For example, a ratio estimate of treatment effect from a clinical trial should be accompanied by information about the cumulative incidence of the outcome in each trial arm.

I have focused only on odds and risk ratios. However, for some studies with binary outcomes, other measures of association may be preferred, including the rate ratio, hazard ratio, risk difference, and rate difference.

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What silent wonder is waked in the boy by
blowing bubbles from soap and water with
a pipe.

—Ralph Waldo Emerson