Multiple Imputation

VGSM, Chapter 11

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Last Week

- Missing data grouped by relationship between missingness and it's value
- MCAR/MAR and NMAR
- MAR is the assumption where most action is
- Motivates multiple imputation
- Model-based guesses at missing data

UNOS Dataset

- Data on 9775 pediatric kidney transplants from 1990-2002
- Followed for transplant outcomes
  465 deaths
- Predictors: age, age of donor, year,
  HLA loci, txtype
- Cold ischemia time: missing in 23%

Multiple Imputation

1. Assume data MAR
2. Develop predictive model for missing data
3. Sample from predictive model
4. Make multiple complete datasets
5. Analyze datasets
6. Synthesize results

Selecting MAR Variables

- Bias is the biggest threat
  - More predictors make MAR plausible
  - Include the outcome
- Unimportant covariates: little cost
  - probably not much gained
- Any predictor in model -- include impute
  - possible some that aren't

Modeling Choice

- Specify a parametric model for imputations
- cold_ischemia: continuous variable
  but also has many "0" s
- Many missing values like to be "0"
- Difficult distribution to model
- Try Normal distribution

Predictive Model

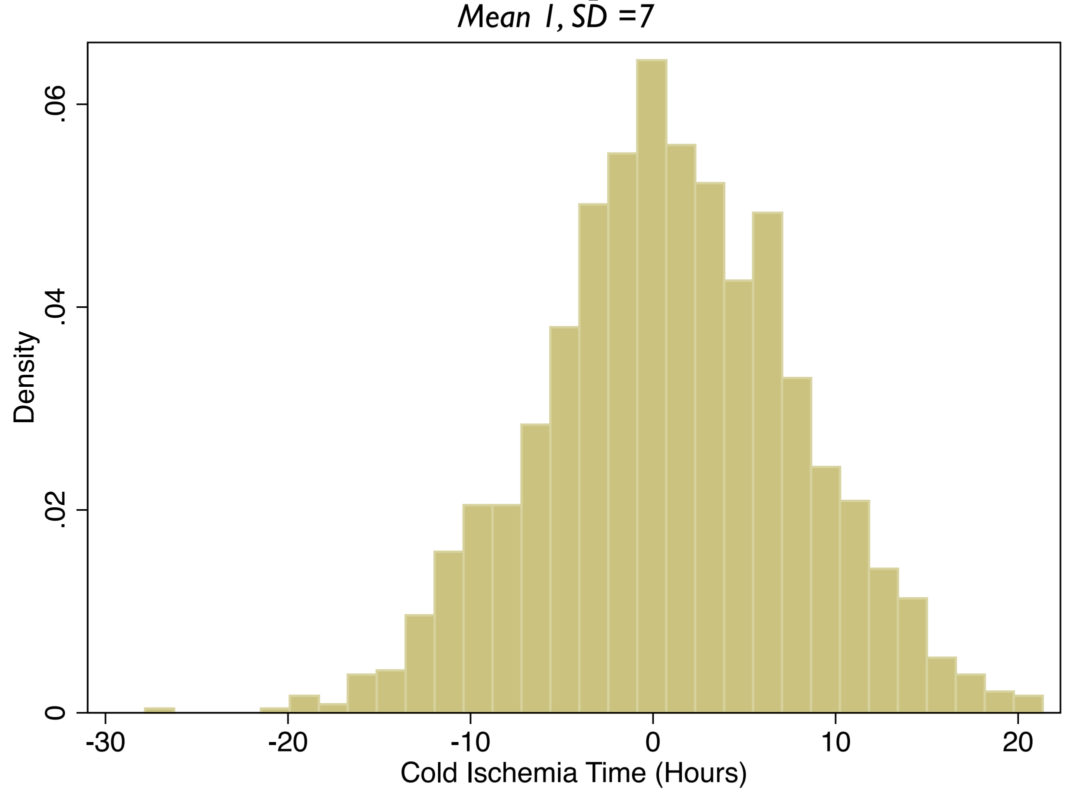
```
. . xi: reg cold_isc txty death i.hla age_don age year
i.hlamat          _Ihlamat_0-6        (naturally coded; _Ihlamat_0 omitted)

      Source |       SS       df       MS              Number of obs =    7347
-------------+------------------------------           F( 11,  7335) = 1126.79
       Model |  611905.778    11  55627.798            Prob > F      =  0.0000
    Residual |  362116.493  7335  49.3683017           R-squared     =  0.6282
-------------+------------------------------           Adj R-squared =  0.6277
       Total |  974022.271  7346  132.592196           Root MSE      =  7.0263

------------------------------------------------------------------------------
    cold_isc |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      txtype |   18.49092   .2235349    82.72   0.000     18.05273    18.92911
       death |    .243987   .3871391     0.63   0.529    -.5149169    1.002891
  _Ihlamat_1 |   .6966437   .3575722     1.95   0.051    -.0043006    1.397588
  _Ihlamat_2 |   .4241767   .3550638     1.19   0.232    -.2718505    1.120204
  _Ihlamat_3 |   .7843253   .3517806     2.23   0.026     .0947341    1.473916
  _Ihlamat_4 |   .9951523    .394421     2.52   0.012     .2219737    1.768331
  _Ihlamat_5 |   1.636238   .5308462     3.08   0.002     .5956272    2.676849
  _Ihlamat_6 |   1.722591    .560876     3.07   0.002     .6231127    2.822069
     age_don |   .0153432   .0067599     2.27   0.023     .0020918    .0285946
         age |   .0286415   .0161681     1.77   0.077    -.0030526    .0603356
        year |  -.2486191   .0229721   -10.82   0.000    -.2936511   -.2035872
       _cons |   495.8731   45.87233    10.81   0.000     405.9501     585.796
------------------------------------------------------------------------------
```

Normal Imputations

Mean 1, SD =7



Misspecified Distribution

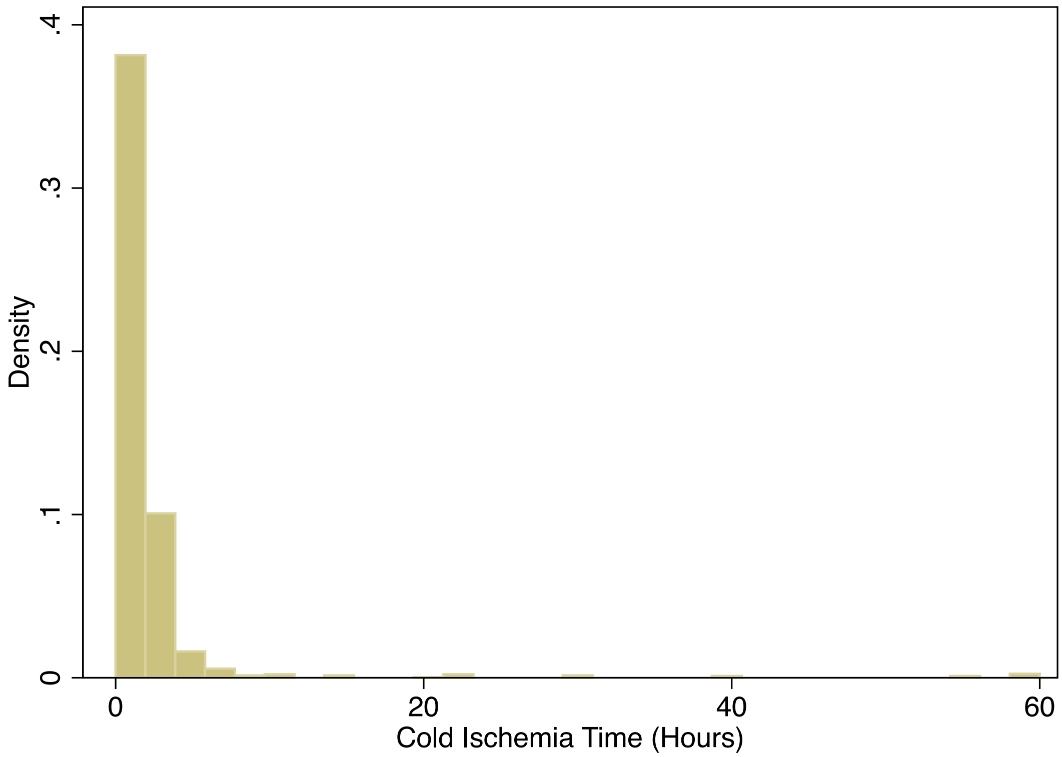
- Minimal effect
- Most important to get moments correct
  means, SD, covariance of imputed data
- Unappealing to get implausible values
- Option to force plausible value

Predictive Means Matching

- Linear regression fit to missing data
  $E(Y_{mis}) = \beta_0 + \beta_1 x_1 + \ldots \beta_p x_p$
- Sample from "posterior" of β | Data
- With each beta calculate predictions
  for all observations
- Impute Y_{mis} from observed values k nearest neighbors
- Forces plausible value
  must specify *k* — *Stata default, k=1*

PMM Imputations

Mean 2, SD =6



Missing Data

Variable	Obs	Mean	Std. Dev.	Min	Max
hlamat	9541	2.57363	1.378919	0	6
age_don	9662	31.29952	13.38866	0	73
age	9766	11.64653	5.291385	0	18
cold_isc	7525	10.85967	11.51735	0	72
death	9775	.0475703	.2128662	0	1
prevtx	9702	1.1443	.3514119	1	2
txtype	9775	.4733504	.4993148	0	1

Multiple Missing Variable

- Can be handled by multiple imputation
- Means developing a model for several variable simultaneous
- Two main ways to do this
  - assume variables are normal
    so called multivariate normal
  - "chained" regression equal
    so called iterative chained equations

Multivariate Normal Imputation

- Treats all variables as normal
- Imputes using linear regression
  imputes well for linear
- Can be coaxed to do binary and ordinal
- Can't impute for unordered categorical

Chained Imputation

- Regression model for each missing variable
- "ICE": iterative chained equations
- Combined for multivariate imputations
- Builds a multivariate distribution
- Separate models: gives big flexibility

Possible Imputation Models

- Continuous: regress, pmm
- Binary: logit
- Ordinal: ologit
- Nominal: mlogit
- Discrete: poisson, nbreg
- Continuous+threshold: truncreg

separate models for each missing variable

3. Make multiple complete datasets

Imputing in Stata

- Extensive suite of capabilities
- Data management
- Creating imputations
- Analyzing and synthesizing

mi package

- Powerful package for missing data
- Extensive capabilities
  lots of choices, appear esoteric
- Can handle multiple missing predictors
- Presents results in easy to read format

mi set

- Must declare data to be "mi set"
- `mi set mlong`
- Stores the imputations in long format
  just governs how imputations are stored
- There is also `mi stset` & `mi xtset`

mi register

- Must declare variable as needing or not needing imputation
- `mi register imputed cold_isc`
- `mi register regular death hlamat age_don age year`
- Says cold_isc will be imputed and others not

mi impute

- This command actually does the imputations
- Can impute single or multiple variables
- Specifies:
    - predictors used for imputation
    - statistical model for imputation
    - number of imputations

imputing and analysis happen in separate steps

Imputing Cold Ischemia

- mi impute regress cold_isc = death txtype i.hlamat age_don age year , add(20) rseed(123)
- Use linear regression to impute cold_isc
- Predictors: death, txtype, etc
- add(20) = 20 imputations
- rseed( ) = sets a random seed
  means you can replicate imputations
  very very useful

Imputation Output

```
. mi impute regress cold_isc hlamat age_don age death year prevtx
txtype , add(50) rseed(123)

Univariate imputation                          Imputations =       50
Linear regression                                    added =       50
Imputed: m=1 through m=50                          updated =        0

------------------------------------------------------------------
                   |               Observations per m
                   |----------------------------------------------
          Variable |   Complete   Incomplete   Imputed |     Total
-------------------+-----------------------------------+----------
          cold_isc |       7347         2020      2020 |      9367
------------------------------------------------------------------
(complete + incomplete = total; imputed is the minimum across m
 of the number of filled-in observations.)
```

Predictive Mean Matching

```
. mi impute pmm cold_isc hlamat age_don age death year prevtx
txtype , add(50) rseed(123)

Univariate imputation                           Imputations =       50
Linear regression                                     added =       50
Imputed: m=1 through m=50                           updated =        0

------------------------------------------------------------------
                   |               Observations per m
                   |----------------------------------------------
          Variable |   Complete   Incomplete   Imputed |     Total
-------------------+-----------------------------------+----------
          cold_isc |       7347         2020      2020 |      9367
------------------------------------------------------------------
(complete + incomplete = total; imputed is the minimum across m
 of the number of filled-in observations.)
```

MI Dataset

```
tabulate _mi_m

      _mi_m |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      9,367       18.82       18.82
          1 |      2,020        4.06       22.88
          2 |      2,020        4.06       26.94
          3 |      2,020        4.06       31.00
          4 |      2,020        4.06       35.06
          5 |      2,020        4.06       39.12
          6 |      2,020        4.06       43.18
          7 |      2,020        4.06       47.23
          8 |      2,020        4.06       51.29
          9 |      2,020        4.06       55.35
         10 |      2,020        4.06       59.41
         11 |      2,020        4.06       63.47
         12 |      2,020        4.06       67.53
         13 |      2,020        4.06       71.59
         14 |      2,020        4.06       75.65
         15 |      2,020        4.06       79.71
         16 |      2,020        4.06       83.76
         17 |      2,020        4.06       87.82
         18 |      2,020        4.06       91.88
         19 |      2,020        4.06       95.94
         20 |      2,020        4.06      100.00
------------+-----------------------------------
      Total |     49,767      100.00
```

original data

imputation “10”

9,347 record dataset becomes 49,767 records

Chained Syntax

- mi register imputed hlamat age_don age cold_isc prevtx death txtype
  register everything
- mi impute chained (pmm) cold_isc (ologit) hlamat (regress) age_don (regress) age (logit) prevtx = death txtype, add(50) rseed(897)
- *(method) variable = nonmissing*
- Very flexible syntax

Chained Output

```
. mi impute chained (pmm) cold_isc (ologit) hlamat (regress) age_don (regress) age (logit)
prevtx = death txtype, add(20) rseed(897) dots

Conditional models:
               age: regress age i.prevtx age_don i.hlamat cold_isc death txtype
            prevtx: logit prevtx age age_don i.hlamat cold_isc death txtype
           age_don: regress age_don age i.prevtx i.hlamat cold_isc death txtype
            hlamat: ologit hlamat age i.prevtx age_don cold_isc death txtype
          cold_isc: pmm cold_isc age i.prevtx age_don i.hlamat death txtype

Performing chained iterations:
  imputing m=1 through m=20 .........10.........20 done
```

mi impute reply

```
Multivariate imputation                      Imputations =       20
Chained equations                                  added =       20
Imputed: m=1 through m=20                        updated =        0

Initialization: monotone                      Iterations =      200
                                                 burn-in =       10

          cold_isc: predictive mean matching
            hlamat: ordered logistic regression
           age_don: linear regression
               age: linear regression
            prevtx: logistic regression

------------------------------------------------------------------
                   |               Observations per m
                   |----------------------------------------------
          Variable |   Complete   Incomplete   Imputed |     Total
-------------------+-----------------------------------+----------
          cold_isc |       7525         2250      2250 |      9775
            hlamat |       9541          234       234 |      9775
           age_don |       9662          113       113 |      9775
               age |       9766            9         9 |      9775
            prevtx |       9702           73        73 |      9775
------------------------------------------------------------------
(complete + incomplete = total; imputed is the minimum across m
 of the number of filled-in observations.)
```


Multiple Imputation

1. Assume data MAR
2. Develop predictive model for missing data
3. Sample from predictive model
4. Make multiple complete datasets
5. Analyze datasets
6. Synthesize results

Analyze and Synthesize

Source of Uncertainty

- Predictive distn of cold_isc
  - $cold\_isc_i = \alpha+\beta_1*x_1+\beta_2*x_2+..\ \beta_p*x_p+\varepsilon_i$
- Sources of uncertainty
  1. prediction: ε_i
  2. parameters: $\beta_1, \beta_2 ..., \beta_p$
  3. form of model: *predictors, transformation*
- Imputation deals with 1 & 2

Result of 1st Imputation

```
Logistic regression                               Number of obs   =       9367
                                                  LR chi2(12)     =     168.53
                                                  Prob > chi2     =     0.0000
Log likelihood = -1633.2037                       Pseudo R2       =     0.0491

------------------------------------------------------------------------------
       death | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.000987   .0065701     0.15   0.881     .9881921    1.013947
     age_don |   .9979897   .0039913    -0.50   0.615     .9901974    1.005843
         age |   .9584817   .0090968    -4.47   0.000     .9408171    .9764779
        year |   .8574911   .0130516   -10.10   0.000     .8322883    .8834571
      prevtx |   1.109361     .14694     0.78   0.433     .8557115    1.438197
      txtype |   1.248548   .2388952     1.16   0.246     .8580995    1.816656
             |
      hlamat |
          1  |   1.004009    .199318     0.02   0.984     .6803856    1.481562
          2  |    .818982   .1662035    -0.98   0.325     .5502147    1.219036
          3  |   .6860487   .1422434    -1.82   0.069     .4569506    1.030008
          4  |   .6296867   .1537688    -1.89   0.058     .3901773    1.016218
          5  |   .8229399   .2596461    -0.62   0.537     .4434097    1.527323
          6  |   .8010403   .2694053    -0.66   0.509      .414361    1.548566
```

Imputations

Imputation #	OR txtype	SE(OR)
1	1.25	0.24
2	1.13	0.22
3	1.21	0.23
4	1.29	0.25
5	1.24	0.24
mean	1.22	0.24
SD	0.06	

Combining Imputations

K = Number of Imputations = 5

MI Estimate = $(OR_1 + OR_2 + ... + OR_K)/K$
= 1.22

MI Variance = $(1 + 1/K)*\{ SE(OR_k) \}^2 +$
$\{ mean(\{SE_k(OR)\}^2) \}$

= $(1+1/5)*\{ 0.06 \}^2 +$
$(0.24)^2$
= .062

MI SE = sqrt(MI Variance) = 0.25

Imputed Regression

the analysis step

- mi estimate, or : logistic death cold_isc txty i.hlam age_don age year
- Fits a logistic regression with imputations based on the last commands
- After ":" usual commands
- Need "or" to get odds ratios

Results

```
 mi estimate, or: logistic death cold_isc age_don age year prevtx txtype i.hlamat

Multiple-imputation estimates                   Imputations     =          5
Logistic regression                             Number of obs   =       9367

------------------------------------------------------------------------------
       death |     exp(b)   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.002313   .0071502     0.32   0.746     .9883722     1.01645
     age_don |   .9979795   .0039896    -0.51   0.613     .9901907     1.00583
         age |   .9584372   .0090968    -4.47   0.000     .9407726    .9764335
        year |   .8578479   .0130779   -10.06   0.000     .8325948    .8838669
      prevtx |   1.108886   .1468931     0.78   0.435      .855321    1.437622
      txtype |   1.216516   .2422243     0.98   0.325     .8233093    1.797515
             |
      hlamat |
          1  |   1.002488   .1990982     0.01   0.990      .679246    1.479555
          2  |   .8181683   .1660688    -0.99   0.323      .549628    1.217914
          3  |   .6849142   .1420574    -1.82   0.068     .4561307     1.02845
          4  |   .6284968   .1535163    -1.90   0.057     .3893938    1.014419
          5  |   .8206398   .2589599    -0.63   0.531     .4421287    1.523199
          6  |   .7986734     .26867    -0.67   0.504     .4130751    1.544221
             |
       _cons |   6.3e+131   1.9e+133     9.98   0.000     8.0e+105    4.9e+157
------------------------------------------------------------------------------
```


PMM Results

```
 mi estimate, or: logistic death cold_isc age_don age year prevtx txtype i.hlamat

Multiple-imputation estimates                   Imputations     =          5
Logistic regression                             Number of obs   =       9367

------------------------------------------------------------------------------
       death |     exp(b)   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.002321   .0070415     0.33   0.741     .9886026     1.01623
     age_don |   .9979781   .0039895    -0.51   0.613     .9901895    1.005828
         age |   .9584332   .0090967    -4.47   0.000     .9407689    .9764293
        year |   .8577968   .0130501   -10.08   0.000     .8325966    .8837597
      prevtx |   1.108817   .1468895     0.78   0.436     .8552597    1.437547
      txtype |   1.216574   .2400089     0.99   0.320     .8263761    1.791016
             |
      hlamat |
          1  |   1.002359   .1990838     0.01   0.991     .6791438    1.479398
          2  |   .8180324   .1660531    -0.99   0.322     .5495211    1.217746
          3  |   .6847243   .1420257    -1.83   0.068     .4559942    1.028187
          4  |   .6284921   .1535049    -1.90   0.057     .3894033    1.014379
          5  |   .8208394   .2589845    -0.63   0.531     .4422767    1.523429
          6  |    .798598   .2686421    -0.67   0.504     .4130386    1.544066
             |
       _cons |   7.1e+131   2.1e+133    10.00   0.000     1.0e+106    4.9e+157
------------------------------------------------------------------------------
```

Data Management

- Imputing creates phantom observations
- Data is only good for imputations
- Useful command -- "mi extract" creates a dataset based on one "imputation"
- mi extract 0, clear
  back to regular incomplete data
- mi extract 10, clear
  complete dataset using imputation #10
  allows you to check imputation set

Imputation 10

```
. logistic death cold_isc age_don age year prevtx txtype i.hlamat

Logistic regression                               Number of obs   =       9367
                                                  LR chi2(12)     =     168.51
                                                  Prob > chi2     =     0.0000
Log likelihood = -1633.2127                       Pseudo R2       =     0.0491

------------------------------------------------------------------------------
       death | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.000447   .0065814     0.07   0.946     .9876305     1.01343
     age_don |    .997997   .0039921    -0.50   0.616     .9902032    1.005852
         age |   .9584903   .0090981    -4.47   0.000     .9408232    .9764891
        year |   .8573443   .0130516   -10.11   0.000     .8321415    .8833103
      prevtx |   1.109332   .1469424     0.78   0.433      .855679    1.438175
      txtype |   1.261713   .2416046     1.21   0.225     .8668913    1.836355
             |
      hlamat |
          1  |   1.004259   .1993966     0.02   0.983     .6805166    1.482015
          2  |   .8191515   .1662461    -0.98   0.326     .5503179    1.219312
          3  |   .6863007   .1423103    -1.82   0.069     .4570993     1.03043
          4  |    .630004   .1538591    -1.89   0.059     .3903584    1.016771
          5  |   .8238943    .259907    -0.61   0.539     .4439664    1.528949
          6  |   .8018133   .2697156    -0.66   0.511     .4147099    1.550251
             |
       _cons |   2.0e+132   6.1e+133    10.03   0.000     2.8e+106    1.4e+158
------------------------------------------------------------------------------
```

but this dropped all other imputations

mi xeq

simpler approach

- mi xeq 10: summ cold_isc
  summarize cold_isc from imputation 10

```
m=10 data:
-> summ cold_isc

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
    cold_isc |      9367    10.04335    11.53007  -20.96038         72
```


mi xeq

examining results

```
. mi xeq 10: logistic death cold_isc age_don age year prevtx txtype i.hlamat

m=10 data:
-> logistic death cold_isc age_don age year prevtx txtype i.hlamat

Logistic regression                               Number of obs   =       9367
                                                  LR chi2(12)     =     168.51
                                                  Prob > chi2     =     0.0000
Log likelihood = -1633.2127                       Pseudo R2       =     0.0491

------------------------------------------------------------------------------
       death | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.000447   .0065814     0.07   0.946     .9876305     1.01343
     age_don |    .997997   .0039921    -0.50   0.616     .9902032    1.005852
         age |   .9584903   .0090981    -4.47   0.000     .9408232    .9764891
        year |   .8573443   .0130516   -10.11   0.000     .8321415    .8833103
      prevtx |   1.109332   .1469424     0.78   0.433      .855679    1.438175
      txtype |   1.261713   .2416046     1.21   0.225     .8668913    1.836355
             |
      hlamat |
          1  |   1.004259   .1993966     0.02   0.983     .6805166    1.482015
          2  |   .8191515   .1662461    -0.98   0.326     .5503179    1.219312
          3  |   .6863007   .1423103    -1.82   0.069     .4570993     1.03043
          4  |    .630004   .1538591    -1.89   0.059     .3903584    1.016771
          5  |   .8238943    .259907    -0.61   0.539     .4439664    1.528949
          6  |   .8018133   .2697156    -0.66   0.511     .4147099    1.550251
             |
       _cons |   2.0e+132   6.1e+133    10.03   0.000     2.8e+106    1.4e+158
------------------------------------------------------------------------------
```

mi extract

- Useful command
- Analysis not supported in mi estimate:
- Use "mi extract, `k'" looped for k
- Save results
- Analyze using published formulae
  not optimal, but possible

How Many Imputations

- Feasibly many:
  "50% missingness 5 draws is highly efficient"
- True but gives poor standard errors
  better guideline 20-40
- Stata on UNOS data
  - 20 imputations in 6 sec.
  - 50 imputations in 16 sec.
  - 500 imputations in 360 sec.

50 Imputations

```
. mi estimate, or: logistic death cold_isc age_don age year prevtx txtype i.hlamat

Multiple-imputation estimates                     Imputations     =         50
Logistic regression                               Number of obs   =       9367
                                                  Average RVI     =     0.0126
DF adjustment:   Large sample                     DF:      min    =    2492.02
                                                           avg    =   2.42e+10
                                                           max    =   2.69e+11
Model F test:       Equal FMI                     F(  12, 3.2e+06)=      13.04
Within VCE type:          OIM                     Prob > F        =     0.0000

------------------------------------------------------------------------------
       death |     exp(b)   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.001093   .0071376     0.15   0.878     .9871925    1.015189
     age_don |   .9979925    .003991    -0.50   0.615     .9902007    1.005846
         age |   .9584724   .0090979    -4.47   0.000     .9408057    .9764708
        year |   .8575193   .0130689   -10.09   0.000     .8322835    .8835204
      prevtx |   1.109199     .14693     0.78   0.434     .8555691    1.438017
      txtype |   1.245769   .2475528     1.11   0.269     .8438537    1.839111
             |
      hlamat |
          1  |   1.003516   .1992915     0.02   0.986     .6799574    1.481042
          2  |   .8187808   .1661875    -0.99   0.325     .5500469    1.218809
          3  |   .6857152   .1422276    -1.82   0.069     .4566588    1.029664
          4  |   .6294577    .153747    -1.90   0.058      .389994    1.015957
          5  |    .822776   .2596321    -0.62   0.536     .4432816    1.527156
          6  |   .8006779   .2693564    -0.66   0.509     .4140995    1.548143
------------------------------------------------------------------------------
```


500 Imputations

```
Multiple-imputation estimates                     Imputations     =        500
Logistic regression                               Number of obs   =       9367
                                                  Average RVI     =     0.0130
DF adjustment:   Large sample                     DF:      min    =   23812.19
                                                           avg    =   1.54e+11
                                                           max    =   1.54e+12
Model F test:       Equal FMI                     F(  12, 3.1e+07)=      13.02
Within VCE type:          OIM                     Prob > F        =     0.0000

------------------------------------------------------------------------------
       death |     exp(b)   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cold_isc |   1.002006   .0070717     0.28   0.776     .9882409    1.015964
     age_don |   .9979834   .0039899    -0.50   0.614     .9901938    1.005834
         age |   .9584474    .009097    -4.47   0.000     .9407824    .9764441
        year |   .8577653   .0130703   -10.07   0.000     .8325267    .8837691
      prevtx |   1.109026   .1469091     0.78   0.435     .8554327    1.437797
      txtype |   1.223848   .2423487     1.02   0.308     .8301736    1.804205
             |
      hlamat |
          1  |   1.002682   .1991348     0.01   0.989     .6793796    1.479836
          2  |   .8183211   .1660985    -0.99   0.323     .5497324    1.218137
          3  |   .6850761   .1420932    -1.82   0.068     .4562356    1.028699
          4  |   .6287796   .1535788    -1.90   0.057     .3895769    1.014854
          5  |   .8212404    .259145    -0.62   0.533     .4424569    1.524297
          6  |   .7992063   .2688505    -0.67   0.505     .4133495    1.545256
------------------------------------------------------------------------------
```

Approach to Imputation Model

- Inclusivity over parsimony
  more predictor: MAR more plausible
- Will remove biases -- primary objective
- Two things to include: outcome, predictor in your analytic model
- Functional form tends not to be critical

Making Up Data?

- No.
- Making a model for missingness
- Making guess at missing data
- Fully representing uncertainty
  - this is the key to it's validity

Maximum Likelihood (ML)

Based on implication #1

- Maximum likelihood valid for MAR data
  as long as CMAR variables incorporated
- Works especially well for missing outcome
  especially in longitudinal analysis
  where attrition depends on previous values
- ML often fit with the EM algorithm
  EM also iterative fills in missing data
- Imputation & analysis models consistent
  MI and maximum like very similar

Disadvantages of ML

- Cumbersome for missing predictors
  don't usual model their distribution
- Awkward if CMAR variables are not a part of desired
- Lacks flexibility of MI

Inverse Weighting

Based on implication #1

- Model pr(non missingness|C_{MAR})
  weight by chance of being observed
- Two step approach
  not implemented in software
  easy to mess up (my experience)
- Clear connection to complete data analysis
- Inefficient

3 Methods

- Same MAR assumption
- Inverse weight:
  model missingness, data model flexible
- MI/ML:
  model data, missingness model unspecified