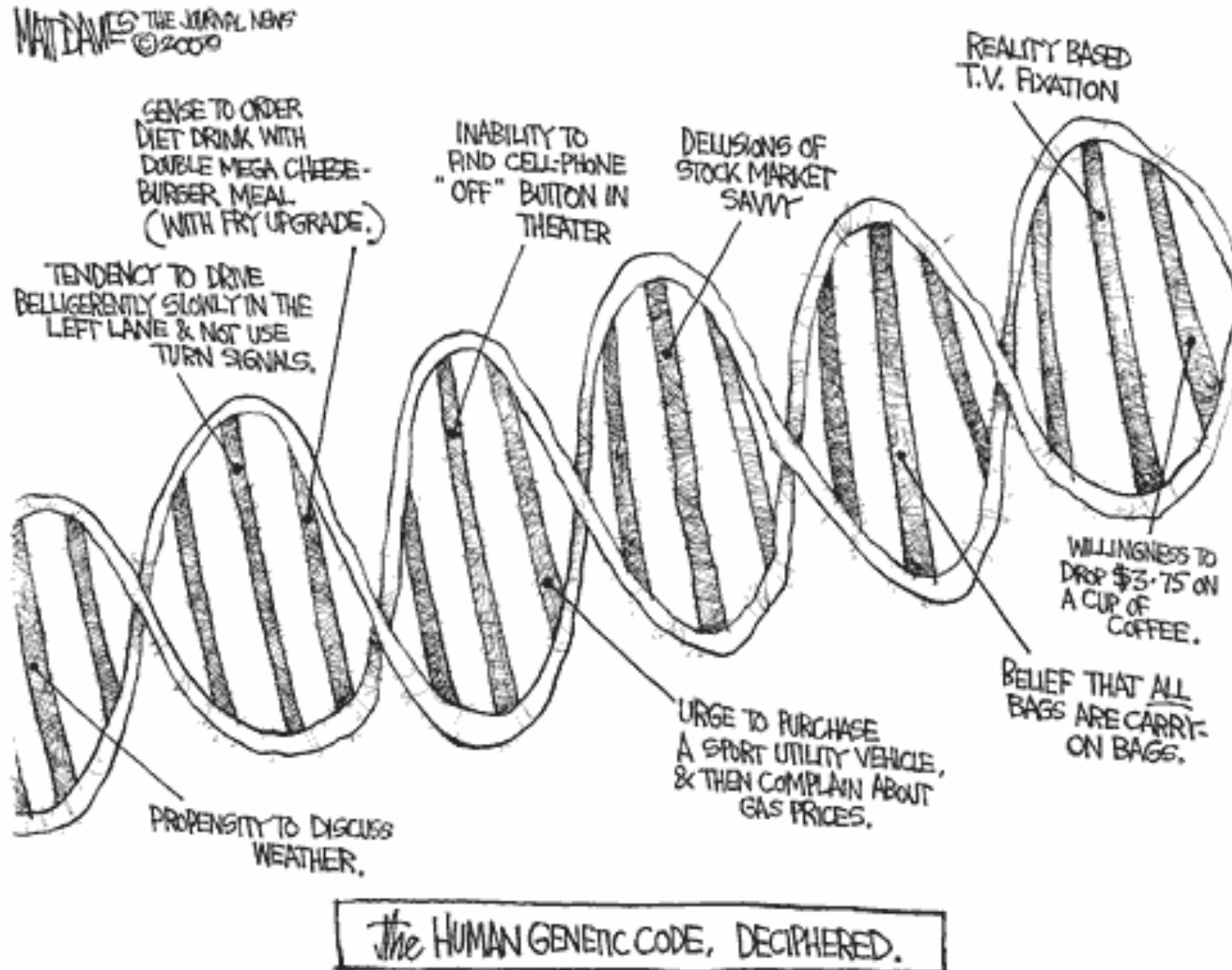


# Epidemiology 217

## Lecture #2: Population Genetics



# Outline

Several methods to answer the question: Is the trait genetic? [Austin Ch. 2]

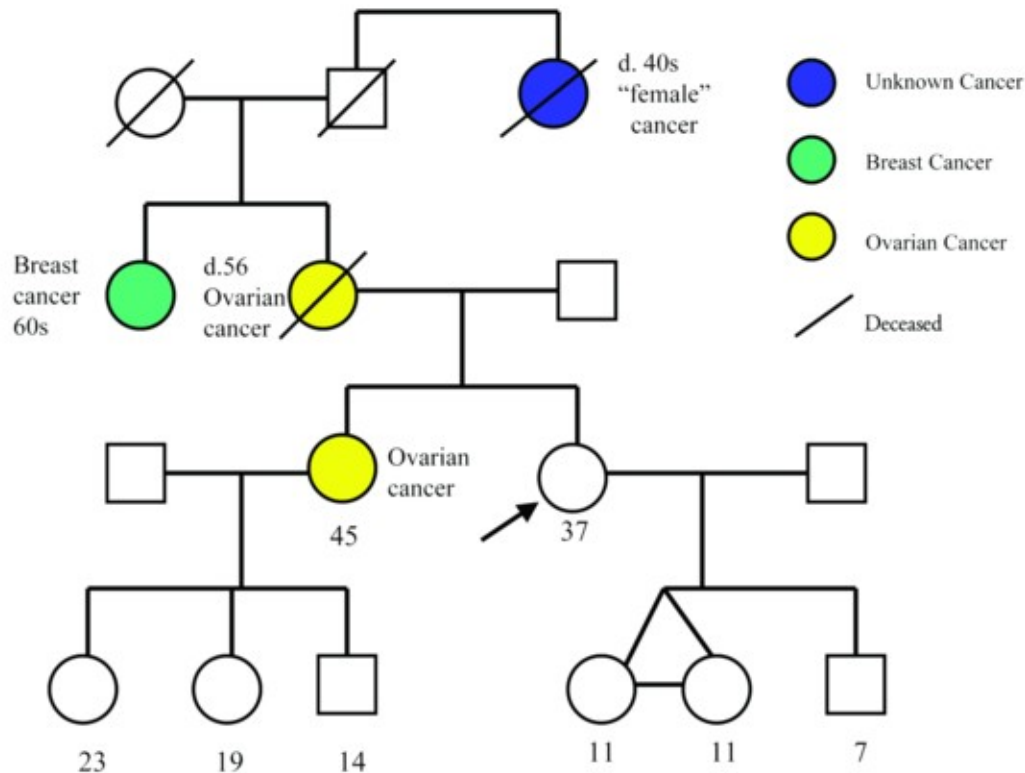
1. Familial Aggregation and recurrence risks
2. Heritability

Genetic concepts [Austin Ch. 3]

3. Allele Frequency Estimation
4. Hardy-Weinberg equilibrium (HWE)
5. Population Substructure/Stratification

# 1. Familial Aggregation

- Does the phenotype tend to run in families?
- Ascertain family members by “probands”, i.e., affected individuals



# Familial aggregation

## “Family case-control” approach

Family history of disease

Yes No

# Cases     a     b

# Controls   c     d

Odds Ratio = OR

$$= (a/b)/(c/d) = ad/bc$$

## “Family cohort” approach

Relative has disease?

Yes No

# Relatives of     a     b

Cases

# Relatives of     c     d

Controls

Relative Risk = RR

$$= (a/(a+b))/(c/(c+d))$$

- Be careful with “family history”
  - E.g., breast cancer, only females
  - E.g., late onset diseases, relative may not generally have disease
- Generally done by interview (cost reasons), though medical records would be more accurate

# Familial aggregation

|                                                                   | No of<br>Cases (%) | No of<br>Controls (%) | Adjusted<br>OR (95% CI) |
|-------------------------------------------------------------------|--------------------|-----------------------|-------------------------|
| Family history of pancreatic<br>cancer in 1st-degree relative     |                    |                       |                         |
| No                                                                | 1,107              | 1,152                 | 1.0 (ref)               |
| Yes                                                               | 76                 | 43                    | 1.76 (1.19, 2.61)       |
| Number of affected 1st-degree<br>Relatives with pancreatic cancer |                    |                       |                         |
| One                                                               | 70                 | 42                    | 1.70 (1.14, 2.53)       |
| Two or more                                                       | 6                  | 1                     | 4.26 (0.48, 37.7)       |

# Recurrence ('Familial') Risk Ratios

- Compares the probability a subject is affected given they have an affected family member to the population risk:

$$\lambda_R = K_R/K,$$

where  $K_R$  is the risk to relatives of type R  
 $K$  is the population risk

$\lambda_S$  = recurrence risk to siblings of probands  
versus the general population risk.

*(Recurrence Risks are not covered in Austin)*

# Estimating Recurrence Risk Ratios (RRR)

- With case-control data, calculate RRR as:  
Proportion of affected relatives of the cases  
(observed) /  
Proportion of affected relatives of controls  
(expected) (assumed to estimate  $K$ , the population prevalence of disease)

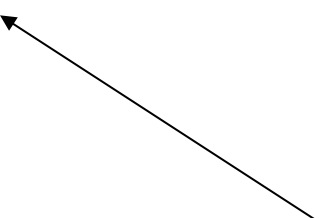
$$\text{Using } \lambda_R = P(Y_2 = 1 | Y_1 = 1) / K$$

- The higher the value of  $\lambda_R$  the stronger the genetic effect

# Examples of $\lambda_s$

|                        |        |
|------------------------|--------|
| • Alzheimer Disease    | 3-4    |
| • Rheumatoid Arthritis | 12     |
| • Schizophrenia        | 13     |
| • Type I Diabetes      | 15     |
| • Multiple Sclerosis   | 20-30  |
| • Neural Tube Defects  | 25-50  |
| • Autism               | 75-150 |

Defining the phenotype...!



# Limitations of Recurrence Risks

- Depend on mode of inheritance and disease frequency.
- Single gene diseases have high recurrence risks.
- More common complex diseases have lower values (e.g., CHD).
- Hard to distinguish genetic versus environmental effects.

## 2. Heritability Analysis

- Evaluates the genetic contribution to a trait in terms of variance explained.
- Phenotype = Mean + Genetics + Environment

$$P = \mu + G + E$$

- Overall variation in Phenotype P

$$\text{Var}(P) = \sigma^2_P = \sigma^2_G + \sigma^2_E$$

# Broad Sense Heritability

- Proportion of the overall phenotypic variance attributable to genetic influences

$$H^2 = \sigma^2_G / \sigma^2_P$$

# Narrow Sense Heritability

- Proportion of variance explained only by additive genetic effects.
- Most commonly calculated estimate since additive explains most variation.

$$(\text{Recall: } \sigma^2_P = \sigma^2_G + \sigma^2_E)$$

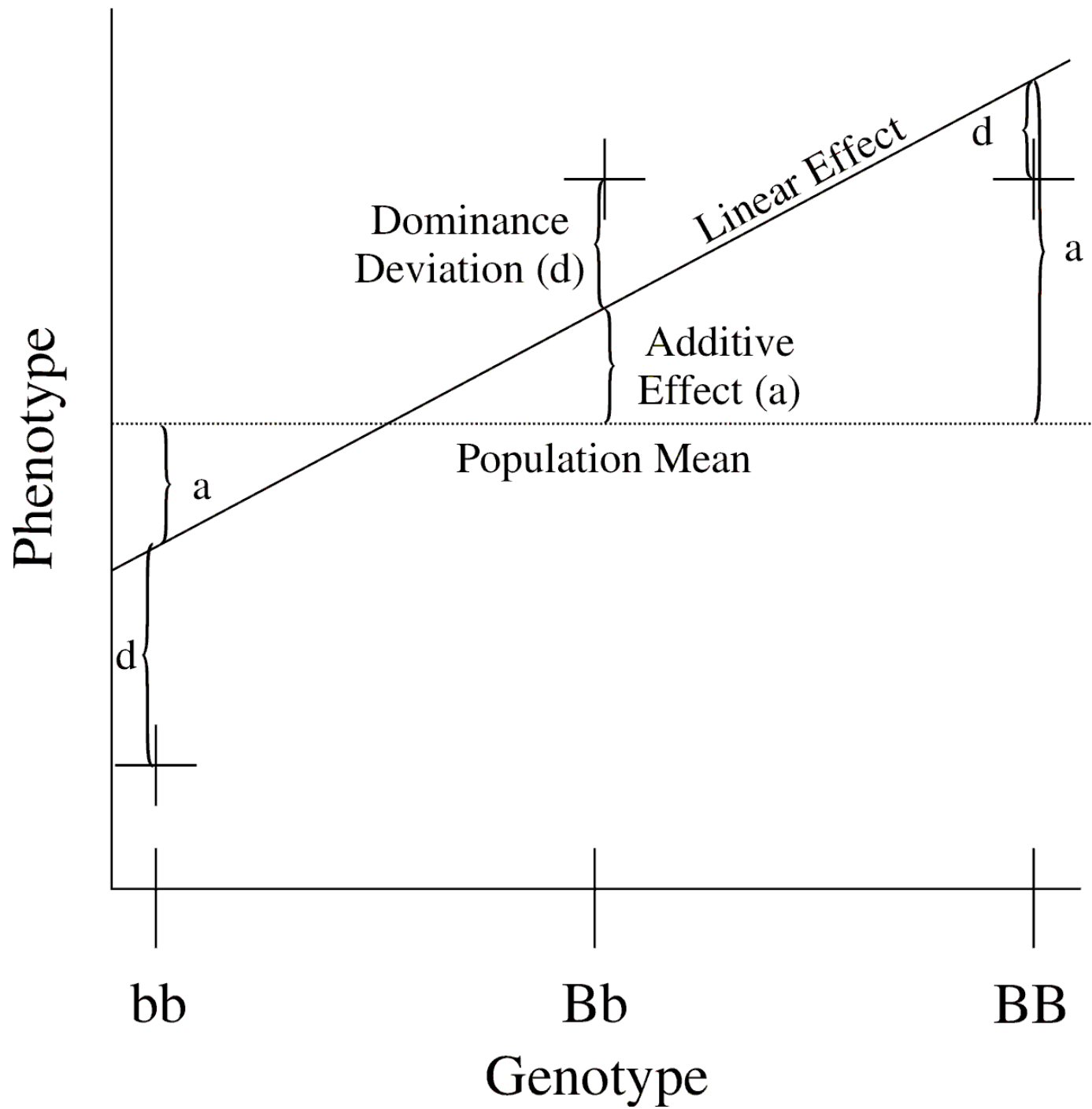
$$\sigma^2_G = \sigma^2_A + \sigma^2_D$$

A: Additive effect

D: Dominance effect

$$\text{“Broad sense”}: H^2 = \sigma^2_G / \sigma^2_P = (\sigma^2_A + \sigma^2_D) / \sigma^2_P$$

$$\text{“Narrow sense”}: h^2 = \sigma^2_A / \sigma^2_P$$



# Narrow Sense Heritability

- Can estimate from regression:
  - Additive: 0, 1, 2 for alleles
  - Dominance: 0, 1, 0 for departure
- *Or estimate from twin studies...*

# Liability model

Two copies of capital letter (risk allele)

- Two unlinked loci, with allele a and risk allele A; and allele b and risk allele B.
- Increase the risk by 1 unit on liability scale.

Table 2.3. Genotypes, liability scores, and genotype frequencies under a polygenic model with two loci.

| Genotype(s)                        | Liability score | Frequency |
|------------------------------------|-----------------|-----------|
| aabb                               | 0               | 1         |
| Aabb, aAbb, aaBb, aabB             | +1              | 4         |
| AAbb, AaBb, AabB, aABb, aAbB, aaBB | +2              | 6         |
| AABb, AaBB, AaBB, aABb             | +3              | 4         |
| AABB                               | +4              | 1         |

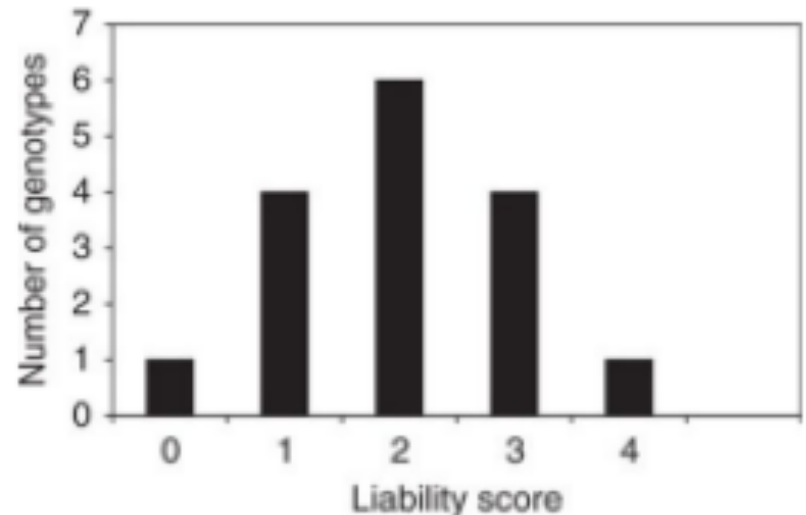


Fig. 2.1. Frequency distribution of genotypes by liability score under a polygenic model with two loci.

Table 2.4. Genotypes, liability scores, and genotype frequencies under a polygenic model with three loci.

| Genotype(s)                                    | Liability score | Frequency |
|------------------------------------------------|-----------------|-----------|
| aabbcc                                         | 0               | 1         |
| Aabbcc, aAbbcc, aaBbcc, aabBcc, aabbCc, aabbcC | +1              | 6         |
| AAbbcc, AaBbcc, AabBcc, ...                    | +2              | 15        |
| AAEbcc, AAbEcc, AaBEcc, ...                    | +3              | 20        |
| AAEBcc, AABbCc, AABbcC, ...                    | +4              | 15        |
| AABECc, AABECc, AABbCC, ...                    | +5              | 6         |
| AABECC                                         | +6              | 1         |

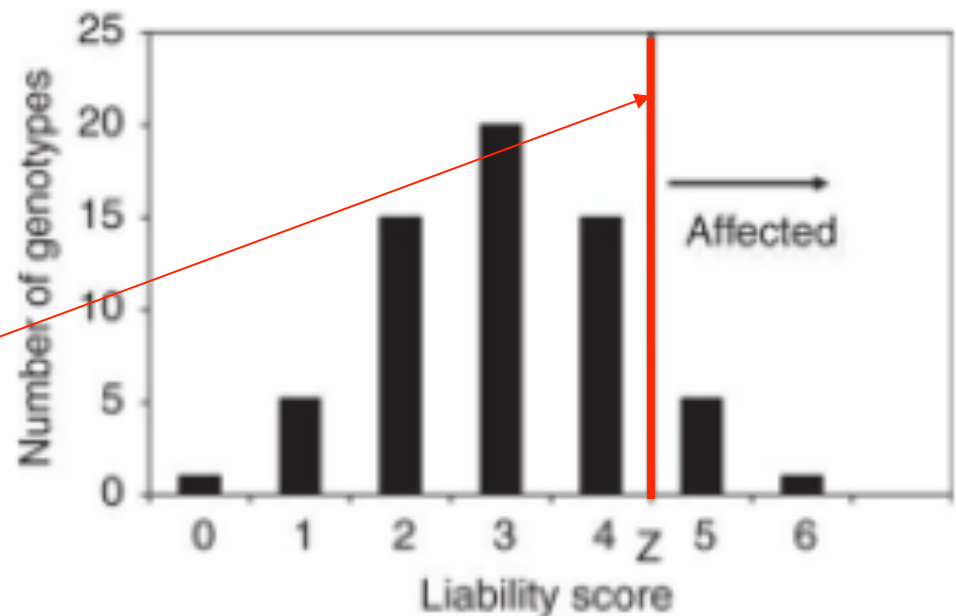
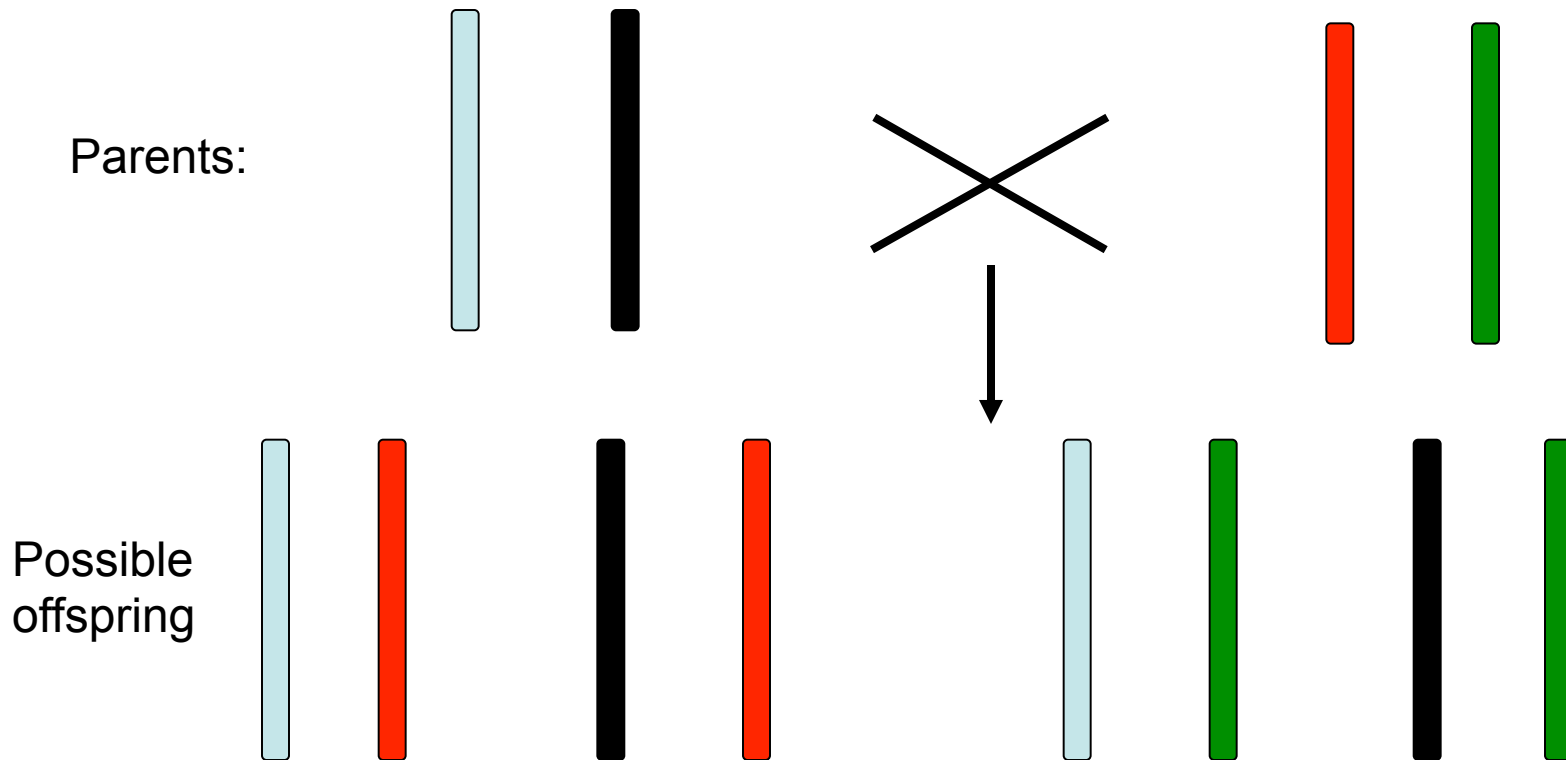


Fig. 2.2. Frequency distribution of genotypes by liability score under a polygenic model with three loci and threshold value Z for dichotomous traits.

- Three unlinked loci Aa, Bb, Cc (capital is risk allele)
- Increase the risk by 1 unit on liability scale
- If dichotomous, threshold determines disease status

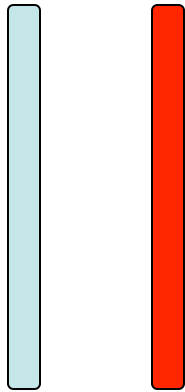
# Twin Studies

- Compare the phenotype correlation or disease concordance rates of MZ (identical) and DZ (fraternal) twins.



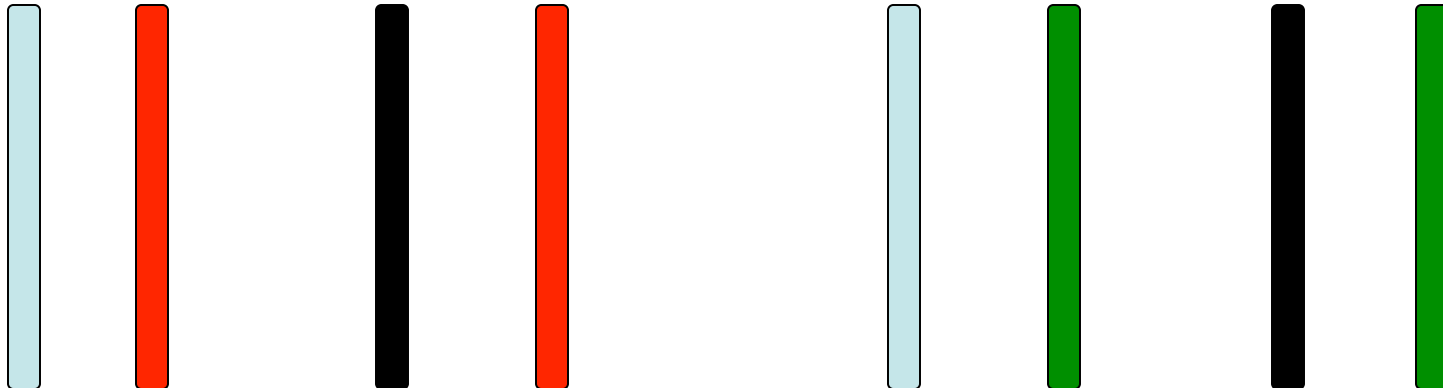
# MZ Twins (Identical)

If twin 1 is



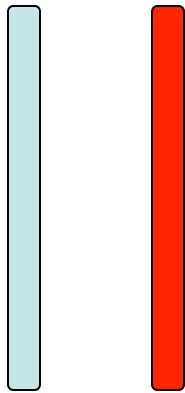
Both alleles are shared  
identical by descent (IBD)

Then twin 2 must be:



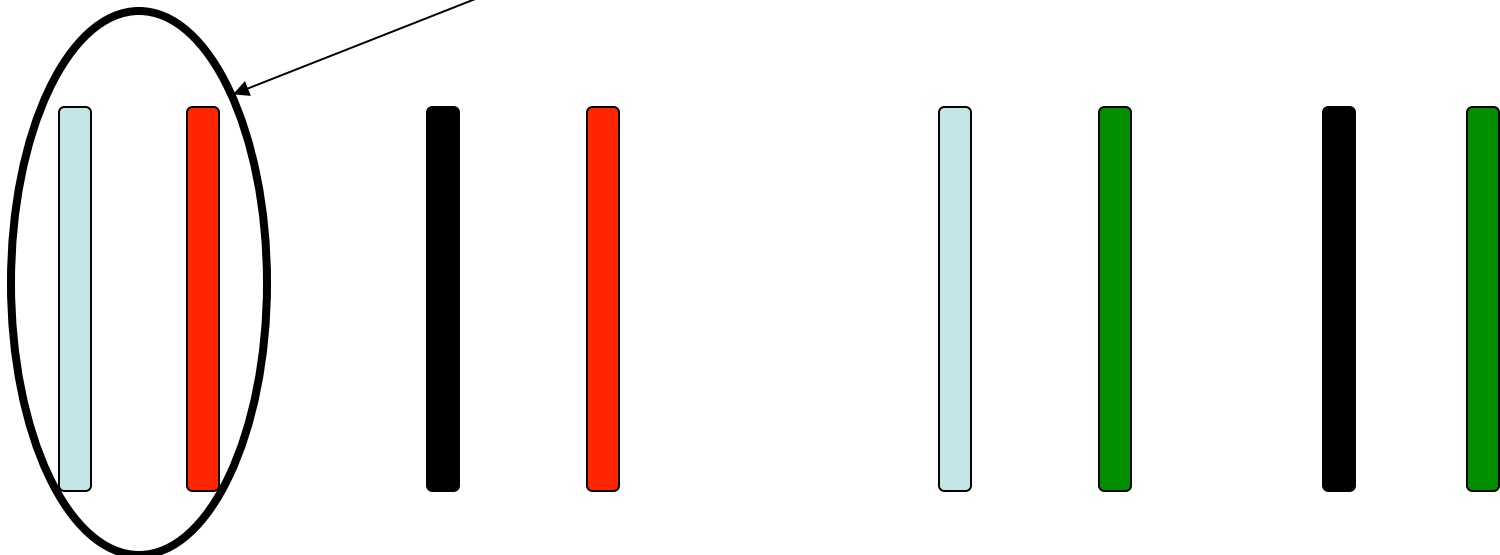
# MZ Twins (Identical)

If twin 1 is

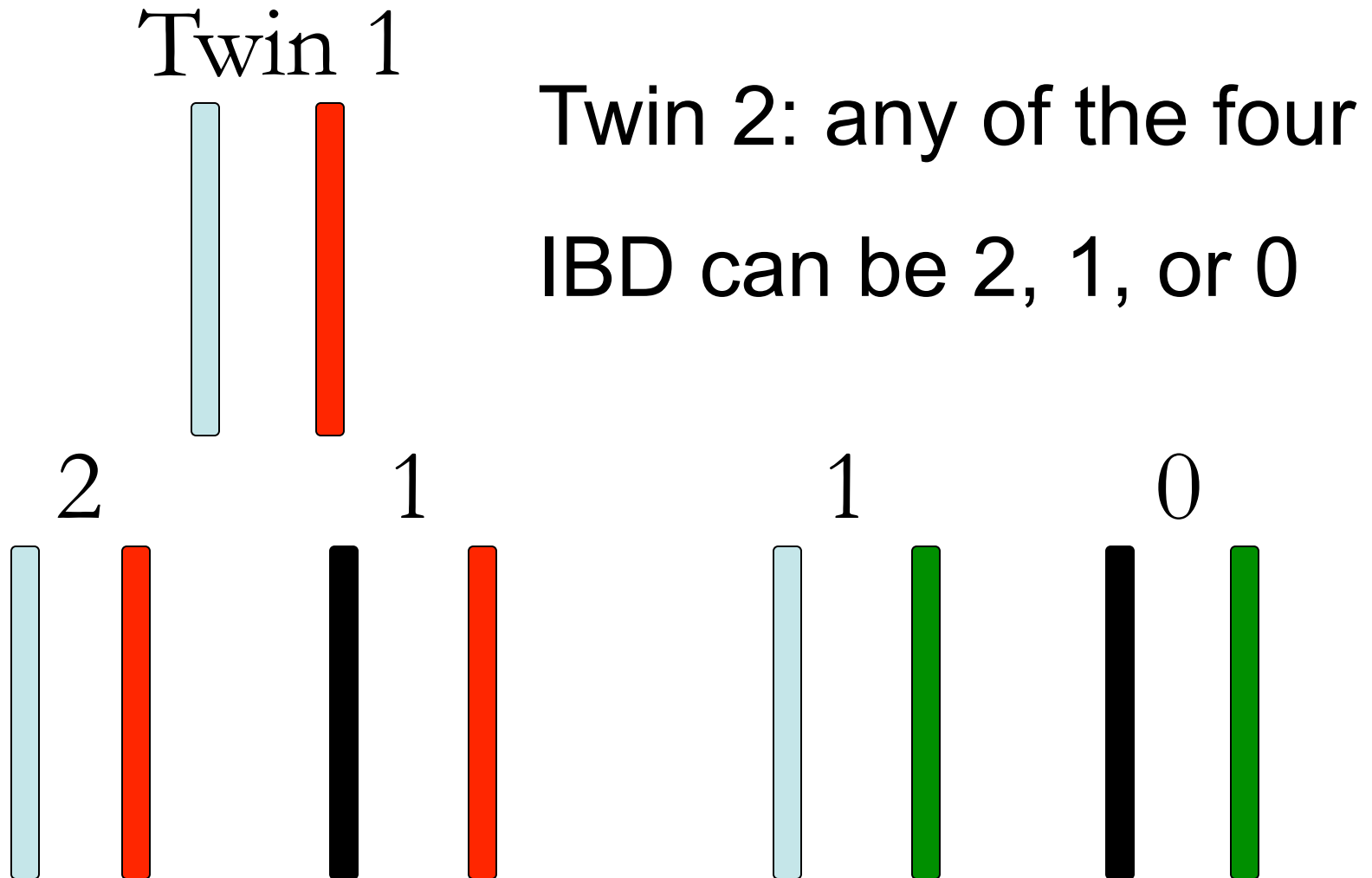


Both alleles are shared  
identical by descent (IBD)

Then twin 2 must be:

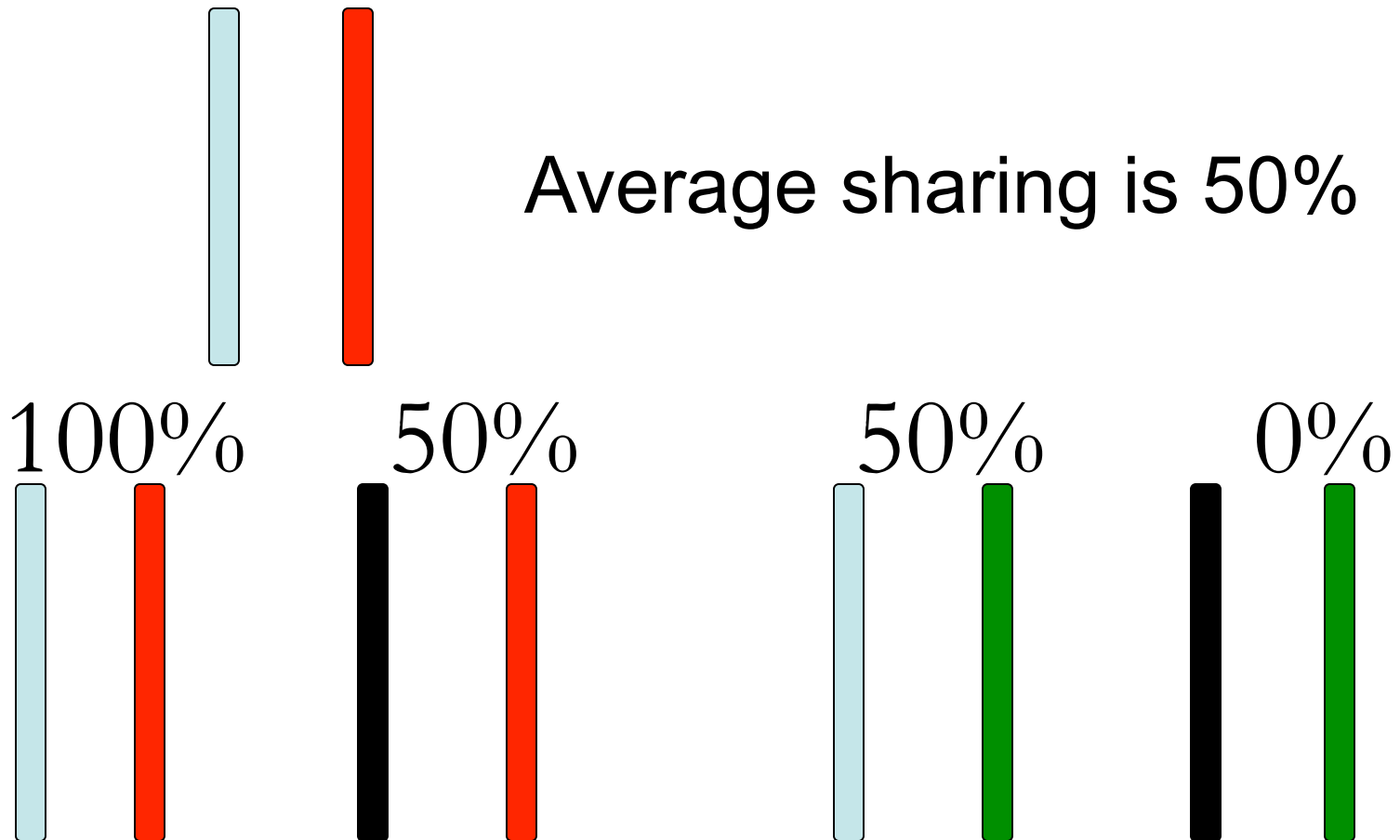


# DZ Twins (Fraternal)



# DZ Twins (Fraternal)

Twin 1



# IBD Sharing

| Relationship      | # of alleles shared IBD |       |       | Prop IBD |
|-------------------|-------------------------|-------|-------|----------|
|                   | 2                       | 1     | 0     |          |
|                   | Pr(2)                   | Pr(1) | Pr(0) |          |
| Self, MZ twins    | 1                       | 0     | 0     | 1        |
| Parent, Offspring | 0                       | 1     | 0     | 1/2      |
| Full siblings     | 1/4                     | 1/2   | 1/4   | 1/2      |
| Gr-child, Gr-prt  | 0                       | 1/4   | 3/4   | 1/4      |
| First cousins     | 0                       | 1/4   | 3/4   | 1/8      |

Proportion of alleles shared IBD

$$= \# \text{ alleles} \times \text{Pr}(\# \text{ alleles}) / 2$$

# Twin Studies

ACE Model:  $\sigma^2_P = \sigma^2_A + \sigma^2_C + \sigma^2_E$

A: Additive genetics

C: Common Environment

E: Unique Environment

Correlation in phenotype ( $P_1, P_2$ ) among twins:

- $\text{Corr}_{mz}(P_1, P_2) = r_{mz} = A + C$  [100% genes + Env]
- $\text{Corr}_{dz}(P_1, P_2) = r_{dz} = \frac{1}{2}A + C$  [50% genes + Env]

Heritability:  $h^2 = 2(r_{mz} - r_{dz})$

# Example of Twin Study: PCa

- Twin registry (Sweden, Denmark, and Finland)

| Twin | Concordant pairs (cp) | Discordant pairs (dp) | Concordance<br>$2cp / (2cp+dp)$ |
|------|-----------------------|-----------------------|---------------------------------|
| MZ   | 197                   | 807                   | 0.33 ( $r_{mz}$ )               |
| DZ   | 148                   | 1719                  | 0.15 ( $r_{dz}$ )               |

$$\begin{aligned}\text{heritability} &= 2(r_{mz} - r_{dz}) = 2(0.33 - 0.15) \\ &= 0.36\end{aligned}$$

Mucci et al. JAMA 2016;315:68  
(actual paper estimate uses Structural Equation Modeling, so differs)

# Heritability Extras

- Can also model interactions (see in text)
  - ACE model will overestimate the heritability in the presence of interactions
- Main point is that the ACE model is based on some assumptions that might not be true, and bias our heritability estimates.

# Heritability from GWAS

- Extensive recent work using linear mixed models to estimate heritability from GWAS array data.
- Called ‘chip heritability.’

# Outline

Several methods to answer the question: Is the trait genetic? [Austin Ch. 2]

1. Familial Aggregation and recurrence risks
2. Heritability

**Genetic concepts** [Austin Ch. 3]

3. Allele Frequency Estimation
4. Hardy-Weinberg equilibrium (HWE)
5. Population Substructure/Stratification

# 3. Allele Frequency Estimation

- Diploid, autosomal locus with 2 alleles: **A** and **a**
- Allele frequency is the fraction:

$$\frac{\text{No. of particular allele}}{\text{No. of all alleles in population}}$$

# Allele (Gamete) Frequency

- Let  $p = \text{Freq}(A)$  frequency of the dominant allele
- Let  $q = \text{Freq}(a)$  frequency of the recessive allele

Then,  $p + q = 1$

# Genotype Frequency

- $p^2$  = frequency of homozygous dominant genotype
- $q^2$  = frequency of homozygous recessive genotype
- $2pq$  = frequency of heterozygous genotype

Then,  $p^2 + 2pq + q^2 = 1$

# Estimating Allele Frequencies from Genotype Frequencies

Genotypes: AA   Aa   aa

Frequency:  $p^2$     $2pq$     $q^2$

Frequency of **A** allele =  $p^2 + \frac{1}{2} (2pq)$

Frequency of **a** allele =  $q^2 + \frac{1}{2} (2pq)$

# Ex. Calculation: Allele Frequencies

Assume N=200 in each of two

populations

- Pop 1: 90 AA 40 Aa 70 aa (N=200)
- Pop 2: 45 AA 130Aa 25 aa (N=200)

In Pop 1:

$$\blacksquare p = 90/200 + \frac{1}{2} (40/200) = 0.45 + 0.10 = 0.55$$

$$\blacksquare q = 70/200 + \frac{1}{2} (40/200) = 0.35 + 0.10 = 0.45$$

In Pop 2:

$$\blacksquare p = 45/200 + \frac{1}{2} (130/200) = 0.225 + 0.325 = 0.55$$

$$\blacksquare q = 25/200 + \frac{1}{2} (130/200) = 0.125 + 0.325 = 0.45$$

# Take home points

- $p + q = 1$  (sum of the allele frequencies = 1)
- $p^2 + 2pq + q^2 = 1$  (sum of the genotype frequencies = 1)
- Two populations with markedly different genotype frequencies can have the same allele frequencies

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# 4. Hardy-Weinberg Equilibrium

## **Hardy–Weinberg principle:**

Both allele and genotype frequencies in a population remain constant (i.e., in equilibrium) from generation to generation unless specific disturbing influences are introduced.

$$p^2 + 2pq + q^2 = 1$$

# Hardy-Weinberg Assumptions

Allele frequencies do not vary IF:

- Large population
- Random mating
- No in or out migration
- No isolated groups within the population
- No mutation
- No selection (no allele is advantageous)

# Example: Test of Hardy-Weinberg Equilibrium (HWE)

Assume we have the following observed genotype frequencies: 100 GG, 30 AG, 20AA

Frequency of the G allele:

$$p = 100/150 + 0.5(30/150) = 0.767$$

Frequency of the A allele

$$q = 20/150 + 0.5(30/150) = 0.233 = 1-p$$

# Example

Calculate expected genotype frequencies based on HW:  $p^2 + 2pq + q^2 = 1$

GG

$$p^2 = 0.767 * 0.767 = 0.588$$

$$n(p^2) = 150(0.588) = 88.2$$

AG

$$2pq = 2 * 0.767 * 0.233 = 0.357$$

$$n(2pq) = 150(0.357) = 53.6$$

AA

$$q^2 = 0.233 * 0.233 = 0.054$$

$$n(q^2) = 150(0.054) = 8.1$$

# Example

Compare expected genotype counts (E) to observed genotype counts (O)

|           | <u>E</u> | <u>O</u> | <u>(O-E)^2/E</u> |
|-----------|----------|----------|------------------|
| <b>GG</b> | 88.2     | 100      | 1.58             |
| <b>AG</b> | 53.6     | 30       | 10.39            |
| <b>AA</b> | 8.1      | 20       | 17.48            |
|           |          | -----    |                  |
|           |          |          | 29.45            |

Chi-square test

$= \sum_i (O_i - E_i)^2 / E_i = 29.4$  (chi square distribution with 1 degree of freedom)

$p = 6.6 \times 10^{-8} > \text{Out of H-W}$

HWE can be easily expanded to account for any number of alleles at a locus

- 3 allele case ( $p_1, p_2, p_3$ )

Allele frequencies:  $p_1 + p_2 + p_3 = 1$

Genotype frequencies:

$$p_1^2 + p_2^2 + p_3^2 + 2p_1p_2 + 2p_1p_3 + 2p_2p_3 = 1$$

- 4 allele case ( $p_1, p_2, p_3, p_4$ )

Allele frequencies:  $p_1 + p_2 + p_3 + p_4 = 1$

Genotype frequencies:

$$p_1^2 + p_2^2 + p_3^2 + p_4^2 + 2p_1p_2 + 2p_1p_3 + 2p_2p_3 + 2p_3p_4 = 1$$

# Application of HWE

For genetic association studies:

- Used as QC measure to assess the accuracy of the genotyping method
- Expect SNPs to be in HWE among control populations (ethnic-specific)
- Violations of HWE could indicate genotyping errors or bias in data

# HWE Game

1. Everyone receives ~5 pairs of cards
2. Two allele model: Red (R allele) & Black (B allele)
3. Random Mating: Exchange one card from each pair with another person (keep cards face down)
4. Determine genotype frequency: RR, RB, BB
5. Determine allele frequency: R, B

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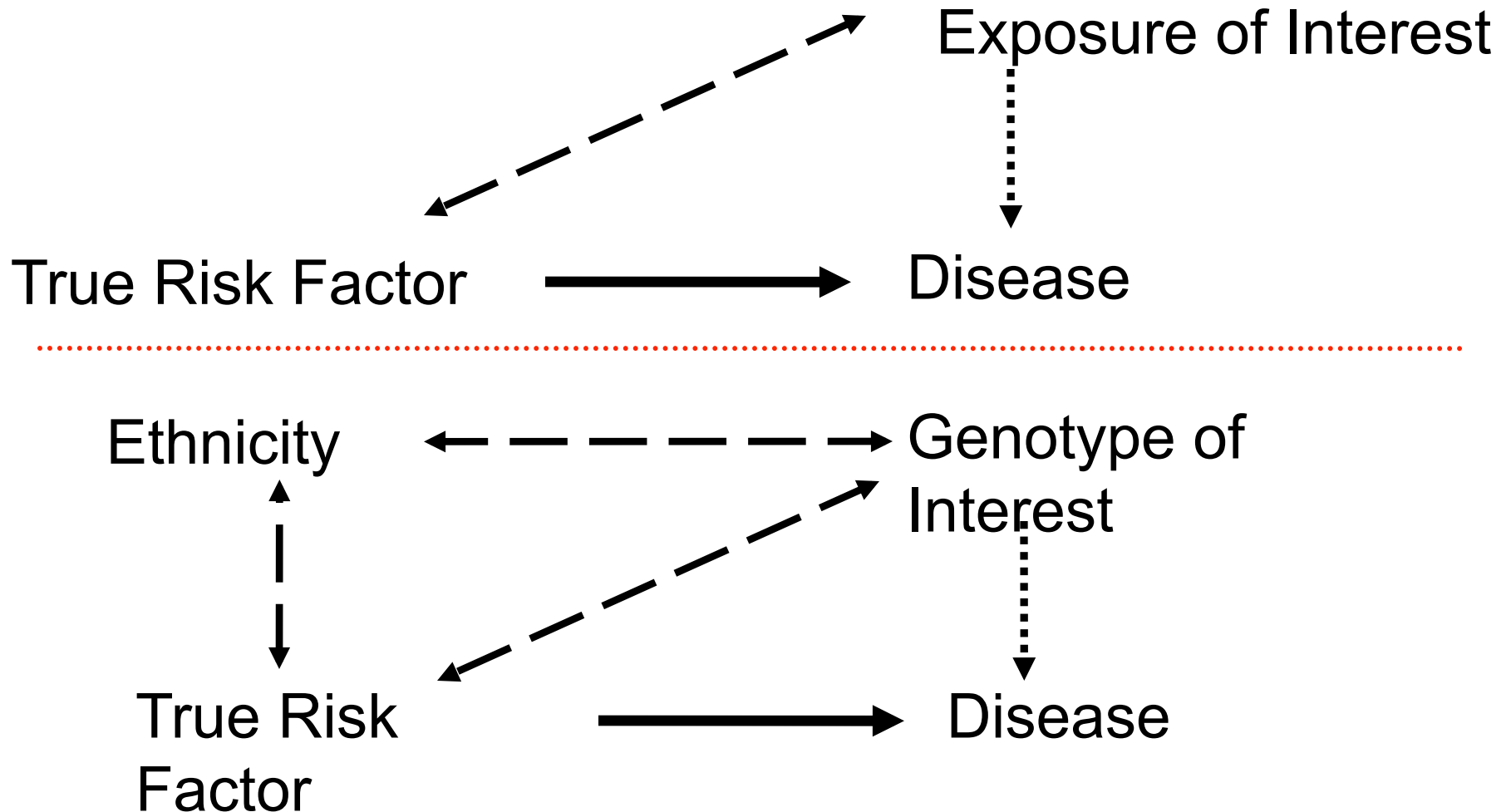
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# 5. Population Substructure/ Stratification

## **Population stratification:**

Confounding due to gene having marked variation in allele frequency across subgroups of a population and these subgroups differ in their baseline risk of disease.

# Population Stratification: Confounding



# Example

Study Population: 4,290 Pima and Papago Native Americans

Genetic Variant: Gm 3;5,13, 15 haplotype (Gm system of human immunoglobulin G)

Outcome: Type 2 diabetes

Question: Is the Gm 3; 5,13, 15 haplotype associated with Type 2 diabetes?

# Population Stratification: Gm3;5,13,14 in admixed sample of Native Americans of the Pima and Papago tribes

Full heritage American Indian population

|             | +   | -    |
|-------------|-----|------|
| Gm3;5,13,14 | ~1% | ~99% |

NIDDM prevalence ~40%

Caucasian population

|             | +    | -    |
|-------------|------|------|
| Gm3;5,13,14 | ~66% | ~34% |

NIDDM prevalence ~15%

Gm3,5,13,14  
haplotype

|   | Cases  | Controls |
|---|--------|----------|
| + | 7.80%  | 29.00%   |
| - | 92.20% | 71.00%   |

Unadjusted for ethnic background  
OR = 0.27 (95% 0.18-0.40)

Different genotype frequency,  
different phenotype frequency

# Population Stratification: Gm3;5,13,14 in admixed sample of Native Americans of the Pima and Papago tribes

| Index of Indian heritage | Gm3;5,13,14 haplotype | % Diabetes |
|--------------------------|-----------------------|------------|
| 0                        | 65.8%                 | 18.5%      |
| 4                        | 42.1%                 | 28.5%      |
| 8                        | 1.6%                  | 39.2%      |

| Gm3,5,13,14 haplotype | Cases  | Controls |
|-----------------------|--------|----------|
| +                     | 7.80%  | 29.00%   |
| -                     | 92.20% | 71.00%   |

Adjusted for ethnic background

OR = 0.83 (95% 0.58-1.18)

Previous result just picked out race/ethnicity!

# Ancestry Informative Markers

- Polymorphisms with known allele frequency differences across ancestral groups
- Useful in estimating ancestry in admixed individuals
- Example: Duffy locus (codes for blood group)
  - 100% sub-Saharan Africans vs. other groups
  - Protects against *P. vivax* (malaria)

# Example AIM: Duffy locus (rs2814778)

| ss#                             | Sample Ascertainment           |                       |                    |        | Genotype Detail |       |   |       |       | Alleles |       |
|---------------------------------|--------------------------------|-----------------------|--------------------|--------|-----------------|-------|---|-------|-------|---------|-------|
|                                 | Population                     | Individual Group      | Chrom. Sample Cnt. | Source | A/A             | A/G   | G | G/G   | HWP   | A       | G     |
| <a href="#">ss105436776</a> P1  |                                |                       | 204                | GF     | 0.706           | 0.118 |   | 0.176 | 0.001 | 0.765   | 0.235 |
|                                 | <a href="#">CAUC1</a>          |                       | 62                 | GF     | 1.000           |       |   |       |       | 1.000   |       |
|                                 | <a href="#">AFR1</a>           |                       | 48                 | GF     |                 | 0.292 |   | 0.708 | 0.752 | 0.146   | 0.854 |
|                                 | <a href="#">HISP1</a>          |                       | 46                 | GF     | 0.783           | 0.174 |   | 0.043 | 0.273 | 0.870   | 0.130 |
|                                 | <a href="#">PAC1</a>           |                       | 48                 | GF     | 0.958           | 0.042 |   |       | 1.000 | 0.979   | 0.021 |
|                                 | <a href="#">HAPMAP-ASW</a>     |                       | 98                 | IG     | 0.041           | 0.327 |   | 0.633 | 1.000 | 0.204   | 0.796 |
|                                 | <a href="#">HAPMAP-LWK</a>     |                       | 180                | IG     |                 | 0.011 |   | 0.989 | 1.000 | 0.006   | 0.994 |
|                                 | <a href="#">HAPMAP-MEX</a>     |                       | 100                | IG     | 0.920           | 0.080 |   |       | 1.000 | 0.960   | 0.040 |
|                                 | <a href="#">HAPMAP-MKK</a>     |                       | 286                | IG     | 0.007           | 0.126 |   | 0.867 | 0.752 | 0.070   | 0.930 |
|                                 | <a href="#">HAPMAP-TSI</a>     |                       | 176                | IG     | 0.977           | 0.023 |   |       | 1.000 | 0.989   | 0.011 |
|                                 | <a href="#">P3</a>             |                       | 532                | GF     | 0.737           | 0.019 |   | 0.244 | 0.001 | 0.746   | 0.254 |
|                                 | <a href="#">CAUC3</a>          |                       | 122                | GF     | 0.984           | 0.016 |   |       | 1.000 | 0.992   | 0.008 |
|                                 | <a href="#">AFR3</a>           |                       | 148                | GF     | 0.095           | 0.027 |   | 0.878 | 0.001 | 0.108   | 0.892 |
|                                 | <a href="#">HISP3</a>          |                       | 92                 | GF     | 0.957           | 0.043 |   |       | 1.000 | 0.978   | 0.022 |
|                                 | <a href="#">PAC3</a>           |                       | 170                | GF     | 1.000           |       |   |       |       | 1.000   |       |
|                                 | <a href="#">ENSEMBL_Watson</a> |                       | 2                  | IG     | 1.000           |       |   |       |       | 1.000   |       |
|                                 | <a href="#">ENSEMBL_Venter</a> |                       | 2                  | IG     | 1.000           |       |   |       |       | 1.000   |       |
| <a href="#">ss119047871</a> YRI |                                |                       | 2                  | IG     |                 |       |   | 1.000 |       | 1.000   |       |
| <a href="#">ss164197181</a> YRI |                                | Sub-Saharan African 2 |                    | IG     |                 |       |   | 1.000 |       | 1.000   |       |