

Survival Analysis III

John Kornak
April 18, 2017
john.kornak@ucsf.edu

Reading VGSM 6.3 - 6.5

- Homework #1 due Today in class
- Homework Q/A after class today 12-1
- Lab 3 on Thursday 10.30 – MH-1400
- Homework #2 due next Tuesday (4/25) in class

So far...

- Survival data and censoring
- Reviewed Kaplan-Meier survival curves and Logrank test
- Hazard function and hazard ratio (HR)
- Proportional hazards model
- Cox Model (no baseline hazard model)
- Binary, categorical and continuous predictors
- Wald and likelihood ratio tests
- Zero/infinite HR
- Confounding, mediation, adjusting for other variables
- Interactions and lincom statements (danger of extrapolation)
- Adjusted survival curves (for Cox model)

In this lecture (extensions to the Cox model)

- Time-dependent covariates
- Diagnostics (model checking) - proportional hazards?
- Non-proportional Hazards: Stratification
- Non-proportional Hazards: generate time-dependent covariates trick
- Diagnostics (model checking) – linearity of predictors?
- Other methods: Clustered data, Left-Truncation, Interval-censoring

3

Time Dependent Covariates

*A time-dependent covariate in a Cox model
is a predictor whose values may vary with time*

*... and is recorded at multiple times during
the study*

4

Example

- Risk factors for pregnancy in a cohort of HIV infected women in Uganda
- Is the development of pregnancy affected by CD4 cell counts?
- We could consider only baseline CD4 count as a predictor (i.e. CD4 value at study onset)
- *But, CD4 cell count measured throughout the study!*
- Multiple measures of CD4 during study could provide additional prognostic information

5

Example

E.g., Patient #24901:

CD4 at baseline: 143
CD4 at day 123: 202
CD4 at day 216: 344
CD4 at day 284: 373
Pregnant on day 380

6

Data

multiple records per subject

	idno	t_from	t_to	cd4	prg
218.	24901	0	123	143	0
219.	24901	123	216	202	0
220.	24901	216	284	344	0
221.	24901	284	380	373	1
229.	25601	0	117	112	0
230.	25601	117	216	304	0
231.	25601	216	293	319	0
232.	25601	293	379	297	0
233.	25601	379	468	302	0
234.	25601	468	560	264	0
235.	25601	560	574	277	0
236.	25601	574	651	277	0
237.	25601	651	738	268	0

- idno: subject id #
- t_from: start of interval
- t_to: end of interval
- cd4: cd4 cell count during interval
- prg: pregnancy (1 = yes, 0 = no)

Stata syntax to define dataset:

stset t_to, failure(prg) **id(idno)**

7

TDC Cox model

```
gen cd4_50 = cd4/50
stcox cd4_50
```

```
No. of subjects = 702      Number of obs = 4935
No. of failures = 85
Time at risk = 448321
Log likelihood = -485.32684      LR chi2(1) = 129.91
                                Prob > chi2 = 0.0000
```

t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
cd4_50	.5456291	.0344751	-9.59	0.000	.4820756 .6175612

Interpretation: a 50 cell increase in CD4 cell count (at any time point) is associated with a 45% reduction in the rate of pregnancy, 95% CI (-52% to -38%), $p < 0.001$

8

Another TDC example...

- Does lung transplant extend life of patients with Cystic Fibrosis?
- Outcome: Time from listing to death or censoring
- Predictor: Received lung transplant (yes or no)

Any concerns?

9

Another TDC example...

- Does lung transplant extend life of patients with Cystic Fibrosis?
- Outcome: Time from listing to death or censoring
- Predictor: Received lung transplant (yes or no)

Bias: waiting list mortality!

Short-term survivors unlikely to get a transplant!

10

Solution

Treat transplant as a time-dependent covariate

$$tx(t) = \begin{cases} 0 & \text{before transplantation} \\ 1 & \text{after transplantation} \end{cases}$$

$$h(t|tx) = \begin{cases} h_0(t) & \text{before transplantation} \\ \exp(\beta) h_0(t) & \text{after transplantation} \end{cases}$$

group membership changes over time

11

Summary

TDC Cox Model

- TD Covariates useful when values of predictors change
- Key is to set up dataset properly
- Straightforward fitting the Cox model
- We will look at another use of TDC to accommodate non-proportional hazards later...

12

Diagnostics for model checking: *testing the proportional hazards assumption*

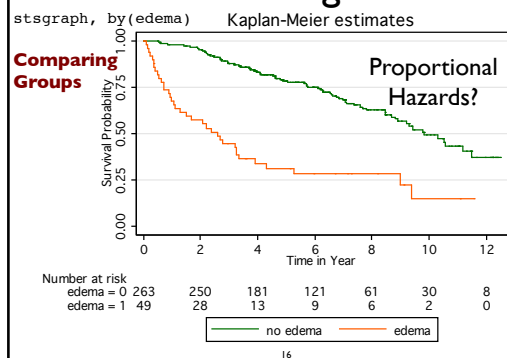
“Essentially, all models are wrong, but some are useful.”

George Box - statistician (1987)

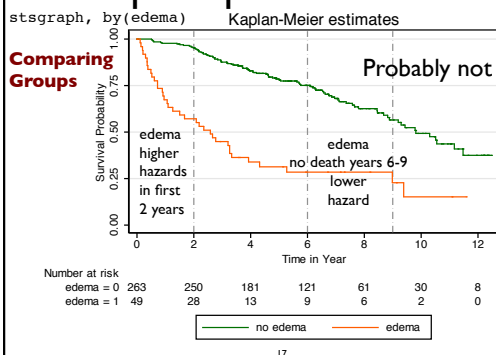
“Essentially, all models are wrong, but some are useful.
However, the approximate nature of the model must always be borne in mind.”

George Box and Norman Draper - statisticians (1987)

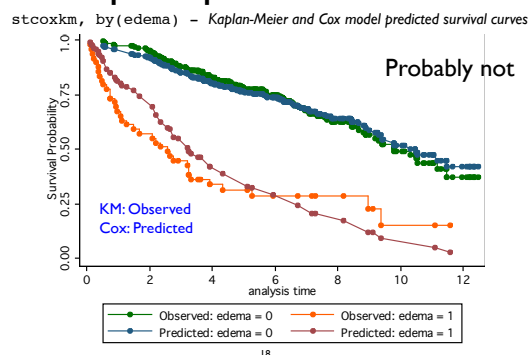
Model Checking PBC Data



Group Proportional Hazards?



Group Proportional Hazards?



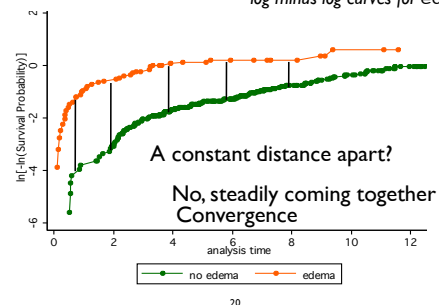
Log minus log plots for categorical predictors

Under the Cox model:

- $S(t|x) = S_0(t) \exp(\beta_1 x_1 + \dots + \beta_p x_p)$
- $S_1(t) = S_0(t) \exp(\beta)$
- $\log(-\log(S_1(t))) = \beta + \log(-\log(S_0(t)))$
- Estimate survival curves and transform them by: (1) take logs, (2) multiply by -1, then (3) take logs again
- Therefore, under Proportional Hazards assumption the curves $\log(-\log(S_1(t)))$ and $\log(-\log(S_0(t)))$ should be a constant distance apart

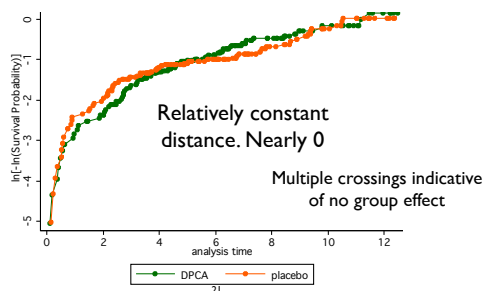
Log minus log plot: edema

stphplot, by(edema) nonegative nolntime -
log minus log curves for edema



Log minus log plot: rx

stphplot, by(rx) nonegative nolntime -
log minus log curves for rx



Summary: log minus log curves

- Easily calculated (pro)
- Naturally subjective (con)
 - Not so easy to interpret
 - Look for pronounced convergence/divergence, or marked crossing
- Only works for categorical variables (con)
- Multiple crossing is evidence of a lack of overall effect (i.e., difference=0, HR=1)

Smoothed Hazard Ratio (for continuous and categorical predictors)

- Possible to use “residuals” to estimate shape of hazard ratio over time
- $HR(t)$: hazard ratio at time t
 - If $HR(t)$ is reasonably constant: prop. hazards
 - If not, gives description of shape of HR
- The method estimates $\log(HR(t)) = \beta(t)$

How does it work?

- Fit Cox model with relevant predictors
- Obtain “scaled Schoenfeld residuals”
complex formula to generate residuals for each predictor & time point
- LOWESS: smooth residuals vs. time
- Plot the smooth curve estimates of $\beta(t)$
- Note that estimated curves change with bandwidth selection

Stata

```

gen age10 = age/10

stcox edema age10, scaledsch(r_edema r_age)
=> saves residuals r_edema for edema, r_age for age10

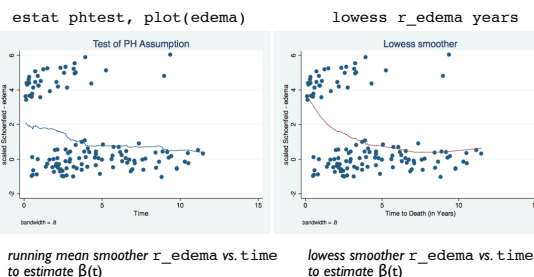
```

	No. of subjects =	312	Number of obs =	312
	Log likelihood =	-614.3788	Prob > chi2 =	0.0000

	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
edema	3.471158	.7099928	6.08	0.000	2.324711 5.182981
age10	1.355471	.1166134	3.54	0.000	1.145143 1.604429

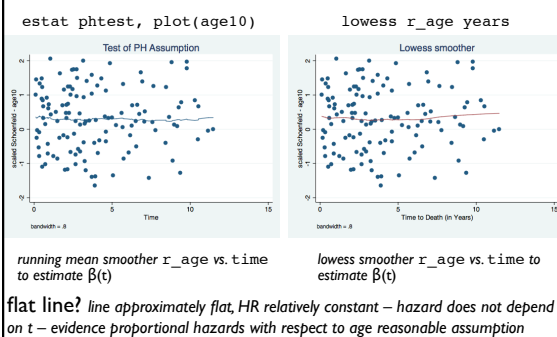
25

$\beta(t)$ for edema



flat line? line is not flat, HR is not constant – hazard depends on t – evidence against proportional hazards with respect to edema

$\beta(t)$ for age10



Smoothing Hazard Ratio

- Present the smoothed curves as a summary
- Augment it with the table to explain the HR
- Get those values by typing

```

lowess r_edema years, gen(smloghr) nogr
gen smhr=exp(smloghr)
sort years
list years sm* if status==1

```

Lowess $\beta(t)$ values for edema

Years	Log HR	HR
1	2.5	12.2
2	1.6	5.1
4	0.86	2.4
6	0.58	1.8

28

Test of Proportional Hazards

- Null hypothesis: Hazards are proportional
i.e., $\beta(t)$ is constant over time
i.e., no association between residuals & time
- Alternative: Hazards are not proportional
i.e., $\beta(t)$ changes with time
i.e., association between residuals & time

Idea is to look at correlation between residuals and time?

29

Schoenfeld Test

- A test for non-proportional hazards:
correlation between residual and time
stcox edema age10
estat phtest, detail
- Small p-value means
proportional hazards is rejected –
the proportional hazards assumption can be shown false

30

Schoenfeld Test

```
stcox edema age10
estat phtest, detail
```

Test of proportional-hazards assumption

Time: Time		rho	chi2	df	Prob>chi2
edema		-0.33749	13.09	1	0.0003
age10		0.01747	0.03	1	0.8540
global test			13.52	2	0.0012

rho is correlation between residuals and time

We see that edema is statistically significant
= *non-proportional hazards*

Scaled Schoenfeld Residuals (plot & test)

- Technical/subjective, so hard to explain (con)
- Poor for multilevel categorical variables would need a plot for each level of category (con)
- Handles continuous variables well (pro)
- Can display effects on HR over time

32

Graphs vs. Tests

- Graphs and tests are complementary
- Need to look at whether the graph shows evidence of important violation
- Test helps as an objective assessment of graph
- However, tests have low power when n is small (and "too much" power when n is large)
- Graphs can show problem with test
e.g., single outlier can affect test

33

Handling Non-Proportionality

- Stratification (categorical predictors)
- Time Dependent Covariates (continuous predictors)

34

Handling non-proportionality I

Stratification

for categorical predictors

Stratified Cox Model

PBC data

- We have seen that baseline edema does not obey proportional hazards, but age does... so model,
- $h(t|edema=1, age) = h_{01}(t) \exp(\beta \times age)$
 $h(t|edema=0, age) = h_{00}(t) \exp(\beta \times age)$
- Two separate baseline reference groups – one for each edema group
- Proportional within edema but not across:
relative effect of a 1-unit change in age on hazard is the same for edema = 1 or edema = 0;
implicitly assumes no interaction between edema and age

35

Stratification Approach

- Fit a Cox model with terms for proportional variable and stratify by non-proportional variable

```
stcox age10, strata(edema)
      (proportional)      (non-proportional)
```

- Can use adjusted survival curves to present the effect of edema

37

Stratified Cox Model

- Easily implemented in Stata:
 - Proportional hazards model for age
 - Stratified by edema

```
. stcox age10, strata(edema)

Stratified Cox regr. -- Breslow method for ties
No. of subjects =      312      Number of obs =      312
No. of failures =      125
Time at risk =    1713.853528
Log likelihood =   -546.68714      LR chi2(1) =    11.60
                                Prob > chi2 =    0.0007
```

_t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
age10	1.342085	.1162448	3.40	0.001	1.132539 1.590402

Stratified by edema

No p-value, HR etc. for edema

38

Interpretation

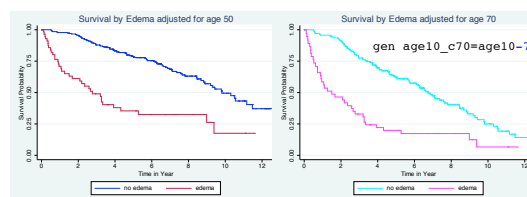
For each 10 years increase in age, there is a 34% increase in the hazard of death after adjusting for edema via stratification, 95% CI (13% increase to 59% increase), $p=0.001$

Could also mention you used a stratified model in the method section.

39

Examining Effect of Edema

- gen age10_c50=age10-5
should center first, graph sets adjusted variables to 0
- sts graph, by(edema) adjustfor(age10_c50)



Notice that the effect of edema is fading in time

40

Stratification Pros/Cons

- Fairly simple and non-technical approach
- What if the non-proportional variable is continuous?
- What if more than one non-proportional variable?

41

Summary

Stratified Cox Model

- Stratification requires multiple baseline hazards
- Stratification of a continuous variable (e.g., bilirubin) requires cutting it into categories
- Need to be at least 3-5 events per stratum
- Can use stratification as a way to adjust for non-proportional variable or to *avoid* proportional hazard assumption
- Gives no summary of the effect of stratum, but *adjusted survival curves can show strata effects*

42

Handling non-proportionality 2

The time-dependent covariates “trick”

for categorical and continuous predictors

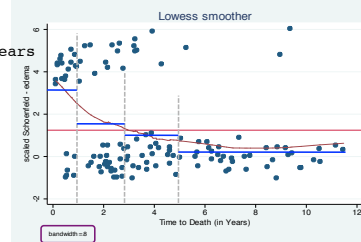
Time-Dep Cov Approach

Divide time into ****discrete**** periods: e.g., Year 0-1, 1-3, 3-5, 5+

Edema residuals

lowest r_edema years

Edema hazard approximately proportional within segments but not between segments



The trick here is that the time interval itself becomes the time varying covariate!

Time-Dep Cov Approach

- Divide time into a series of periods (e.g., Year 0-1, 1-3, 3-5, 5+)
- Estimate HR for edema for each period
- Achieved by creating a series of TD covariates: edema01, edema13, edema35, edema5p that separately give the effect of edema in each period

```
stset years, failure(status) id(number) // generates _t0, _t, _d
stsplot grp, at(1 3 5) // split time variable (years) at these times into groups s years (grp)
// that is, generate multiple rows for each subject; one for each timepoint up to and including
// the time of censoring or time of death

recode status .=0 // recodes all newly generated rows to "censored" status

gen edema01=edema* (grp==0) // This set of commands generates 4 separate
gen edema13=edema* (grp==1) // edema variables specific to each time interval;
gen edema35=edema* (grp==3) // that is, edemaXX only equals 1 if the patient has edema
gen edema5p=edema* (grp==5) // AND the dataset row corresponds to period xx
```

45

TD Cov Set-Up

```
. list number _t0 _t status edema grp edema01 edema13 edema35 edema5p in 1/12, sepby(number)
// lists values of the variables: number _t0 _t status edema grp edema01 edema13 edema 35 edema5p;
// 'in 1/12' restricts to first 12 rows; 'sepby(number)' draws line between each subject:
```

	number	_t0	_t	status	edema	grp	edema01	edema13	edema35	edema5p
1.	1	0	1	Censored	1	0	1	0	0	0
2.	1	1	1.0951403	Dead	1	1	0	1	0	0
3.	2	0	1	Censored	0	0	0	0	0	0
4.	2	1	3	Censored	0	1	0	0	0	0
5.	2	3	5	Censored	0	3	0	0	0	0
6.	2	5	12.320329	Censored	0	5	0	0	0	0
7.	3	0	1	Censored	1	0	1	0	0	0
8.	3	1	2.770705	Dead	1	1	0	1	0	0
9.	4	0	1	Censored	1	0	1	0	0	0
10.	4	1	3	Censored	1	1	0	1	0	0
11.	4	3	5	Censored	1	3	0	0	1	0
12.	4	5	5.2703629	Dead	1	5	0	0	0	1

A separate edema variable is set up for each time period so that we can have hazard ratio estimates for edema specific to each time interval!

46

```
gen age10=age/10
stcox edema?? age10
```

Output

```
No. of subjects = 312           Number of obs = 1001
No. of failures = 125
Time at risk = 1713.853528
Log likelihood = -605.36554      LR chi2(5) = 69.23
                                Prob > chi2 = 0.0000
```

	_t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
edema01		14.45344	6.974774	5.53	0.000	5.613169 37.21639
edema13		3.423855	1.241054	3.40	0.001	1.682588 6.967111
edema35		3.187902	1.495416	2.47	0.013	1.2712 7.994587
edema5p		.8742166	.526164	-0.22	0.823	.2687244 2.844009
age10		1.33777	.1153185	3.38	0.001	1.129812 1.584006

HR declines with time,
does not statistically significantly differ from 1 after year 5

47

Interpretation

“Adjusted for age, during the first year of follow-up, subjects with edema at baseline have about 14-fold (5.6-37) higher hazard of death. During years 1-3 and 3-5, it is 3.4-fold (1.7, 7.0) and 3.2-fold (1.3, 8.0) higher, respectively compared to those with no edema. After year 5, the relative hazard is 0.87 (0.3, 2.8), not statistically significantly different from 1.0 (although the confidence interval is wide enough to include important differences).”

48

Output

```
gen age10=age/10
stcox edema?? age10
```

No. of subjects = 312
 No. of failures = 125
 Time at risk = 1713.853528
 Log likelihood = -605.36554

Similar?

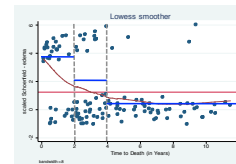
Number of obs = 1001
 LR chi2(5) = 69.23
 Prob > chi2 = 0.0000

_t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
edema01	14.45344	6.974774	5.53	0.000	5.613169 37.21639
edema13	3.423855	1.241054	3.40	0.001	1.682588 6.967111
edema35	3.187902	1.495416	2.47	0.013	1.2712 7.994587
edema5p	.8742166	.526164	-0.22	0.823	.2687244 2.844009
age10	1.33777	.1153185	3.38	0.001	1.129812 1.584006

49

Output

- What if divide time into Year 0-2, 2-4, 4+?



No. of subjects = 312
 Log likelihood = -604.93786

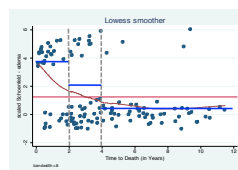
Number of obs = 784
 Prob > chi2 = 0.0000

_t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
edema02	10.54145	3.869923	6.42	0.000	5.133481 21.64657
edema24	3.2131	1.138287	3.29	0.001	1.604626 6.433907
edema4p	.9417363	.4928127	-0.11	0.909	.3376709 2.626425
age10	1.342123	.1157712	3.41	0.001	1.13336 1.58934

50

Output

- What if divide time into Year 0-2, 2-4, 4+?



No. of subjects = 312
 Log likelihood = -604.93786

Number of obs = 784
 Prob > chi2 = 0.0000

_t	Haz. Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
edema02	10.54145	3.869923	6.42	0.000	5.133481 21.64657
edema24	3.2131	1.138287	3.29	0.001	1.604626 6.433907
edema4p	.9417363	.4928127	-0.11	0.909	.3376709 2.626425
age10	1.342123	.1157712	3.41	0.001	1.13336 1.58934

51

What about Follow Up Edema Values?

- Recall, edema only codes for baseline edema
- Subjects with edema die off fairly fast
- PBC is a progressive disease
- Subjects are developing edema over time
- If used the yearly information on edema as a TD covariate, the effect may not fade with time

52

TDC for non-proportional hazards pros and cons

- Con: Bit of programming to set up
- Con: Somewhat artificial on choice of cut-points (though remember "all models are wrong but some are useful")
- Pro: Works for continuous and categorical predictors
- Pro: Estimates time-varying HRs and 95% CIs
- Pro: Clinicians love cutpoints e.g. *might say "edema doesn't matter after 4-5 yrs"*

53

Linearity in Predictors?

continuous predictors

- We considered the proportional hazard assumption
- But what about the linearity of predictors assumption?
- $\log(h(t|x)/h_0(t)) = \beta_1 x_1 + \dots + \beta_p x_p$

54

Linearity in Predictors?

continuous predictors

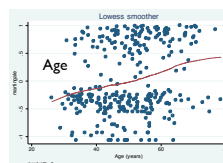
- Possible to use “martingale residuals” to discover a suitable functional form for a predictor (possibly adjusted for other predictors)
- Fit Cox model with no predictor, and give a name “mg_resi” to the martingale residuals
 - `stcox, mgale(mg_resi) estimate`
- LOWESS: smooth mg_resi vs. predictor
- Does it appear to be approx. a straight line?

55

Model Checking: PBC

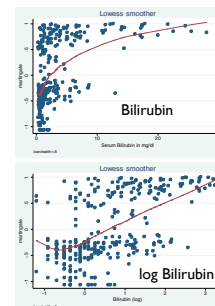
Linearity in predictors?

```
. use pbc.dta
. stset years, failure(status)
. stcox, mgale(mg_resi) estimate
. lowess mg_resi age
. lowess mg_resi bili
. lowess mg_resi logbili
```



Similar plots can be made while adjusting for other covariates by `stcox covariates, mgale(mg_rescov)` and then, e.g., `lowess mg_rescov age`

56



Other Survival Topics

- **Continuously varying Time Dependent Covariates: continuous interactions with time**
 - Alternative time dependent covariates approach: *continuous interaction with time* - allows effect of predictor to vary continuously with time
 - `stcox` with `tvc()` option and maybe `texp(fn(_t))`, or `stsplit, at(failures)` -- [see Sections 6.4.2.6 and 6.9 of VGSM plus time-varying subsection of `help stcox` and `help tvc_note`: especially see hyperlink to Cox regression with continuous time-varying covariates]
 - The continuous interaction with time model is more realistic than splitting at specific times (pro)
 - Typically requires transformation so that hazard ratio is a linear (or other “smooth”) function of the interaction with time (con)

Other Survival Topics

- **Clustered data**
 - Dr. McCulloch’s lectures have a strong focus on the analysis of clustered data and you will see many examples (but not survival)
 - Clustered survival data consist of e.g., multiple subjects clustered by center, or multiple events (multiple failure times) on a subject (c.f. Dr. McCulloch’s lectures to come)
 - use `shared(cluster_id)`, `cluster(id)` or `vce(cluster id)` in Stata with `stcox`
 - The key is to appropriately set up the data for the type of clustering – ordered vs. non-ordered events, single or multiple event types etc.
 - See <http://www.stata.com/support/faqs/statistics/multiple-failure-time-data>

Other Survival Topics

- **Interval Censoring**
 - Conventional survival data is right-censored only
 - Often only know event happens in an interval (e.g., between scheduled hospital visits).
 - Regular intervals can use pooled logistic regression (e.g., all patients visit at time 0, 12 months, 24 months and 36 months)
 - Non-regular intervals need non-parametric Kaplan-Meier-Turnbull or parametric modeling (i.e. with a model for baseline hazard or survival distribution)
 - Use `intcens` in STATA from SSC for parametric models
 - type `ssc install intcens` in STATA then look at: `help intcens`

Other Survival Topics

- **Left-Truncation**
 - PBC used time from enrolment for time 0, but makes more sense to use diagnosis time
 - Why LT a problem? subjects with early death less likely to be enrolled – if not accounted for will underestimate early deaths
 - `stset years_since_diag, failure(status)`
`enter(disease_dur)`
`disease_dur = truncation times (time from Dx to enrollment)`
 - Survival function drops faster early on after accounting for left-truncation
 - See section 6.6.6 in VGSM

Summary

- Time dependent covariates
- Testing proportional hazards: graphs and test
- Non-proportional hazards solutions
 - 1) Stratified Cox
 - 2) Time dependent covariate trick (discrete or continuous)

Other extensions: clustering, left-truncation, interval censoring – competing risks (later...)

Don't forget...

- HW 1 discussion will follow now
- Next lecture: "Common Biostatistical Problems" 4/25
- Upload HW 2 by start of 4/25 lecture.
- HW 2 discussion will follow lecture on 4/25
- **The homework of 4/25 will be due by the lecture on 4/27 -- only a two day window!!!!**