Lecture 4 1/29/16

plot in R:

$$\text{plot}(x, y, \underbrace{\text{ylim} = c(0, 6000)}_{\text{rescales y-axis}})$$

Forecasting

Pure statistics "technical" no info other than the #s

Model based on process + abstract functional features

Model structure based on literature, fit parameters

Pure Model No data Information only

Naive persistence – raining today, rain tomorrow

Climate – what happens on Jan 30 will happen next year

Linear regression – problem: errors not indep!

Data = level + noise

↳ get level from moving average
- centered
- delayed – shifted

①

Exponentially-weighted Moving Avg.

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \longrightarrow \sum_{i=0}^{\infty} (1-a)a^i = 1$$

$$\mu_i = \sum_{i=0}^{\infty} x_i (1-a) a^i$$ ← weight data like this

pretty close w/ $\mu_i = \sum_{i=0}^{10} x_i (1-a) a^i$

$\lambda \equiv 1-a$ Smoothing term, $\lambda=0$ more smoothing

$$\mu_T = \lambda x_T + \lambda(1-\lambda) x_{T-1} + \lambda(1-\lambda)^2 x_{T-2} + \dots$$

$$\mu_T = \lambda x_T + (1-\lambda)\left\{ \lambda x_{T-1} + \lambda(1-\lambda) x_{T-1-1} + \lambda(1-\lambda)^2 x_{T-1-2} + \dots \right\}$$

$$\mu_t = \lambda x_T + (1-\lambda)\underbrace{\mu_{T-1}}_{\text{everything in past summarized}}$$

Holt-Winters Forecasting

Info we have

μ_{T-1} position yesterday

b_{T-1} Slope yesterday

$$\hat{x}_{T|T-1} = \mu_{T-1} + b_{T-1}$$

Update position:

$$\mu_T = \lambda_0 x_T + (1-\lambda_0)(\mu_{T-1} + b_{T-1})$$

Update slope:

$$b_T = \lambda_1 (\mu_T - \mu_{T-1}) + (1-\lambda_1) b_{T-1}$$

2 smoothing terms λ_0 & λ_1

Proper Forecasting - if know distribution should win

log-likelihood

proper linear forecasting

Prep of Galton-Watson Process

Last week:

Geometric - # of trials up to a ~~bf~~ success $\quad 1, 2, 3 \ldots \quad p(1-p)^i$

or $\quad$ PGF $\dfrac{up}{1-u(1-p)}$

This week: # of failures bf a success $\quad 0, 1, 2, 3$

(shifted 1 over from previous

both ways seen in literature

~~PGF $\frac{up}{1-u(1-p)}$~~

$$E[u^X] = \sum_{x=0}^{\infty} u^x p(1-p)^x$$

$$= \frac{p}{1-(1-p)u}$$

$\boxed{\text{R does it this way!}}$

Sum of 3 geometrics $\quad$ Negative Binomial!

↳ waiting time until 3 successes

$$W = X_0 + X_1 + X_2$$

$$E[u^W] = E[u^{X_0+X_1+X_2}] = E[u^{X_0} u^{X_1} u^{X_2}]$$

$$= E[u^{X_0}] \, E[u^{X_1}] \, E[u^{X_1}]$$

$$= \left(\frac{p}{1-(1-p)u}\right)^3 = g_W(u)$$