Mathematical Modeling of Infectious Diseases, UCSF

Homework 4

Exercise 1. Let $\hat{e}_\tau$ be the one-step forecast error:

$$\hat{e}_\tau = x_\tau - \hat{x}_{\tau|\tau-1}.$$

Show the forecast can be conducted using these equations:

$$\mu_\tau = \mu_{\tau-1} + b_{\tau-1} + \lambda_0 \hat{e}_\tau$$

$$b_\tau = b_{\tau-1} + \lambda_0 \lambda_1 \hat{e}_\tau$$

∎

$$\mu_\tau = \lambda_0 x_\tau + (1-\lambda_0)\hat{x}_{\tau|\tau-1}.$$

$$\mu_\tau = \lambda_0 (x_\tau - \hat{x}_{\tau|\tau-1}) + \hat{x}_{\tau|\tau-1}.$$

$$\mu_\tau = \lambda_0 (\hat{e}_\tau) + \hat{x}_{\tau|\tau-1}.$$

$$\mu_\tau = \lambda_0 (\hat{e}_\tau) + \mu_{\tau-1} + b_{\tau-1}.$$

Here,

$$b_\tau = \lambda_1 (\mu_\tau - \mu_{\tau-1}) + (1-\lambda_1) b_{\tau-1}.$$

Substituting in from above:

$$b_\tau = \lambda_1 (\lambda_0 (\hat{e}_\tau) + b_{\tau-1}) + (1-\lambda_1) b_{\tau-1}.$$

$$b_\tau = \lambda_1 (\lambda_0 (\hat{e}_\tau) + b_{\tau-1}) + (1-\lambda_1) b_{\tau-1}.$$

$$b_\tau = \lambda_1 \lambda_0 (\hat{e}_\tau) + b_{\tau-1}.$$

Exercise 2. Show that if the data x_t lie along a straight line, the Holt-Winters forecasts will follow the line perfectly. ∎

Looking at the above equations, if the slope is constant, then $b_\tau = \lambda_1 \lambda_0 (\hat{e}_\tau) + b_{\tau-1} = b_{\tau-1}$. For this to be true, then $\hat{e}_\tau$ is equal to zero. Therefore, if x_τ lies along a straight line, then the one-step forecast error is zero and the forecasts fit a line perfectly.

Exercise 3. Use the California state gonorrhea counts to produce Holt-Winters forecasts for next year's case count. Please consider log transforming the data before analysis; you may wish to omit the first five years. Why do you think the peaks and troughs occurred? Do you think the Holt-Winters procedure is appropriate for the data? ∎

Answers will vary.

Exercise 4. What is the limit of μ_T for large T? ∎

Zero.

Exercise 5. Using R or some other language of your choice, plot simulated AR(1) time series for the normal model with unit variance. Choose $\alpha = 0.99, 0.8, 0.5, 0.2, 0.1, -0.1, -0.2, -0.5, -0.8, -0.99$. Run series longer than 64; run replications over and over again. (You only need to turn in one replication.) Please comment on the differences and similarities. ∎

Answers will vary.

Exercise 6. Find the variance of the negative binomial. ∎

$$\mathrm{var}Y = g''(1) + g'(1) - (g'(1))^2$$

To get var$Y$:

```
sage: u=var('u')
sage: p=var('p')
sage: k=var('k')
sage: gdoubleprime = diff(diff( p^k/(1-(1-p)*u)^k, u),u).subs(u=1).simplify()
sage: gprime = diff( p^k/(1-(1-p)*u)^k, u).subs(u=1).simplify()
sage: (gdoubleprime + gprime - gprime^2).simplify_full()
-(k*p - k)/p^2
```

$$\mathrm{var}Y = \frac{k(1-p)}{p^2}$$

Exercise 7. How long did this assignment take you? ∎