Homework 6

Exercise 1. Just in words, without writing any formulas or doing any math, explain how the mean can increase geometrically, even though sometimes the process goes extinct. ∎

If we imagine an ensemble of epidemics, the epidemics that do not go extinct are increasing faster than geometrically so that the mean epidemic size for the ensemble is geometric.

Exercise 2. Assume $Y_0 = 1$, $X_0 = N - 1$. Compute the expected number of secondary cases for a single index case in an otherwise susceptible population. ∎

$$r(N-1)$$

Exercise 3. Suppose you wanted to allow superspreaders in the Reed-Frost model. How might you vary the model to allow this? ∎

We might imagine that R, a random variable representing the per contact transmission probability for an infected individual, is normally distributed with mean r and variance σ^2. We get the per contact transmission probability for each individual by sampling from R.

Exercise 4. Repeat the above exercise for the same p; can you find an epidemic where it went extinct? Try it for smaller p, say for $p = 0.01$; try it for $p = 1/64$. Try different population sizes. What is the longest epidemic you see? What is the largest epidemic? How often do you see it drop near zero and then have a large epidemic? Try it dozens of times (but only turn in 3). You can use the `multi.rf` function. ∎

Answers will vary.

Exercise 5. Let $N = X + Y + Z$. Assume the Kermack-McKendrick model. Prove $\frac{dN}{dt} = 0$; interpret. ∎

$$N = X + Y + Z$$

$$\dot{N} = \dot{X} + \dot{Y} + \dot{Z}$$

$$\dot{N} = -\beta XY + \beta XY - \rho Y + \rho Y = 0$$

The total population remains constant over time.

Exercise 6. Play with the Kermack-McKendrick epidemic; vary β, vary ρ, vary N. To vary N, just change the sum of the number of people you use in `init`. You can do all your explorations as one-liners:

```
kmplot(dosim(pars=c(beta=0.001,rho=0.1),init=c(1999,1,0),endtime=40))
```

where you just change β, ρ, etc. Do a bunch of them; make sure you look at one with $R_0 < 1$ and one with R_0 around 2 or so. Only turn in 3 graphs. ∎

Answers will vary.

Exercise 7. How long did this assignment take you? ∎

Answers will vary.