

Multiple Imputation

15 October 2013
Biostatistics 210

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UNOS Data

- Predictors of death in kids with kidney tx
- Some data on 9775 kids (465 deaths)
- A number of missing predictors
 - cold ischemia
 - # HLA loci
 - age of donor

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Missingness Mechanism

- MCAR: missing data is independent of value of data
- NMAR: missing data is dependent on value through variables we don't have
- MAR: missing data depends on value through data we have observed and we are going to use

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MAR Modeling

- MAR is wrt a series of variables
- Generally want to choose more rather than fewer: to make sure bias is eliminated
- Modeling requires insight into missing data
- Can't verify MAR assumption based on the data

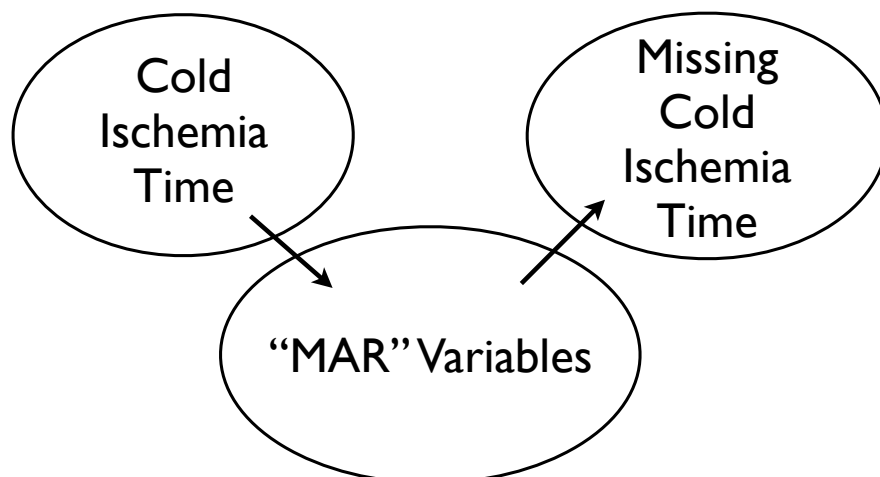
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Why Multiple Imputations?

- Don't know the value
- Single imputation: treats value as known
- Want to represent uncertainty
- Multiple draws from distribution

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MAR implies MI



Missingness only associated with missing values through "MAR" variables

Association of Cold Ischemia and "MAR" Variables same if Cold Ischemia missing or not

Multiple Imputation

1. Assume data MAR
2. Develop predictive model for missing data
3. Sample from predictive model
4. Make multiple complete datasets
5. Analyze datasets
6. Synthesize results

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MAR Assumption

- Assume that cold_ischemia is MAR
- That values MAR conditional on available data: txtype, HLA loci, age of recipient, age of donor, year of transplant, and survival
- Use a regression model as the basis for predictions

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mi package

- Powerful package for missing data
- Extensive capabilities
lots of choices, appear esoteric
- Can handle multiple missing predictors
- Presents results in easy to read format

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mi set

- Must declare data to be “mi set”
- `mi set mlong`
- Stores the imputations in long format
just governs how imputations are stored
- There is also `mi stset` & `mi xtset`

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mi register

- Must declare variable as needing or not needing imputation
- `mi register imputed cold_isc`
- `mi register regular death hlamat age_don age year`
- Says `cold_isc` will be imputed and others not

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mi impute

- This command actually does the imputations
- Can impute single or multiple variables
- Specifies:
 - predictors used for imputation
 - statistical model for imputation
 - number of imputations

imputing and analysis happen in separate steps

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Single Variable

- `mi impute regress cold_isc = death txttype i.hlamat age_don age year , add(20) rseed(123)`
- Use linear regression to impute `cold_isc`
- Predictors: `death`, `txttype`, etc
- `add(20)` = 20 imputations
- `rseed( )` = sets a random seed
means you can replicate imputations very very useful

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Predictive Model

```
. xi: reg cold_isc txtty death i.hla age_don age year
i.hlamat      _Ihlamat_0-6      (naturally coded; _Ihlamat_0 omitted)
```

cold_isc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
death	.243987	.3871391	0.63	0.529	-.5149169	1.002891
txttype	18.49092	.2235349	82.72	0.000	18.05273	18.92911
hlamat						
1	.6966437	.3575722	1.95	0.051	-.0043006	1.397588
2	.4241767	.3550638	1.19	0.232	-.2718505	1.120204
3	.7843253	.3517806	2.23	0.026	.0947341	1.473916
4	.9951523	.394421	2.52	0.012	.2219737	1.768331
5	1.636238	.5308462	3.08	0.002	.5956272	2.676849
6	1.722591	.560876	3.07	0.002	.6231127	2.822069
age_don	.0153432	.0067599	2.27	0.023	.0020918	.0285946
age	.0286415	.0161681	1.77	0.077	-.0030526	.0603356
year	-.2486191	.0229721	-10.82	0.000	-.2936511	-.2035872
_cons	495.8731	45.87233	10.81	0.000	405.9501	585.796

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Imputation Output

```
. mi impute regress cold_isc hlamat age_don age death year prevtx
txttype , add(50) rseed(123)
```

```
Univariate imputation                    Imputations =      50
Linear regression                        added =      50
Imputed: m=1 through m=50                updated =       0
```

Variable	Observations per m			Total
	Complete	Incomplete	Imputed	
cold_isc	7347	2020	2020	9367

(complete + incomplete = total; imputed is the minimum across m of the number of filled-in observations.)

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MI Dataset

```
tabulate _mi_m
```

_mi_m	Freq.	Percent	Cum.
0	9,367	18.82	18.82
1	2,020	4.06	22.88
2	2,020	4.06	26.94
3	2,020	4.06	31.00
4	2,020	4.06	35.06
5	2,020	4.06	39.12
6	2,020	4.06	43.18
7	2,020	4.06	47.23
8	2,020	4.06	51.29
9	2,020	4.06	55.35
10	2,020	4.06	59.41
11	2,020	4.06	63.47
12	2,020	4.06	67.53
13	2,020	4.06	71.59
14	2,020	4.06	75.65
15	2,020	4.06	79.71
16	2,020	4.06	83.76
17	2,020	4.06	87.82
18	2,020	4.06	91.88
19	2,020	4.06	95.94
20	2,020	4.06	100.00
Total	49,767	100.00	

← original data

← imputation "10"

9,347 record dataset
becomes 49,767
records

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Imputed Regression

the analysis step

- mi estimate, or : logistic death cold_isc
txty i.hlam age_don age year
- Fits a logistic regression with imputations
based on the last commands
- After “:” usual commands
- Need “or” to get odds ratios

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Results

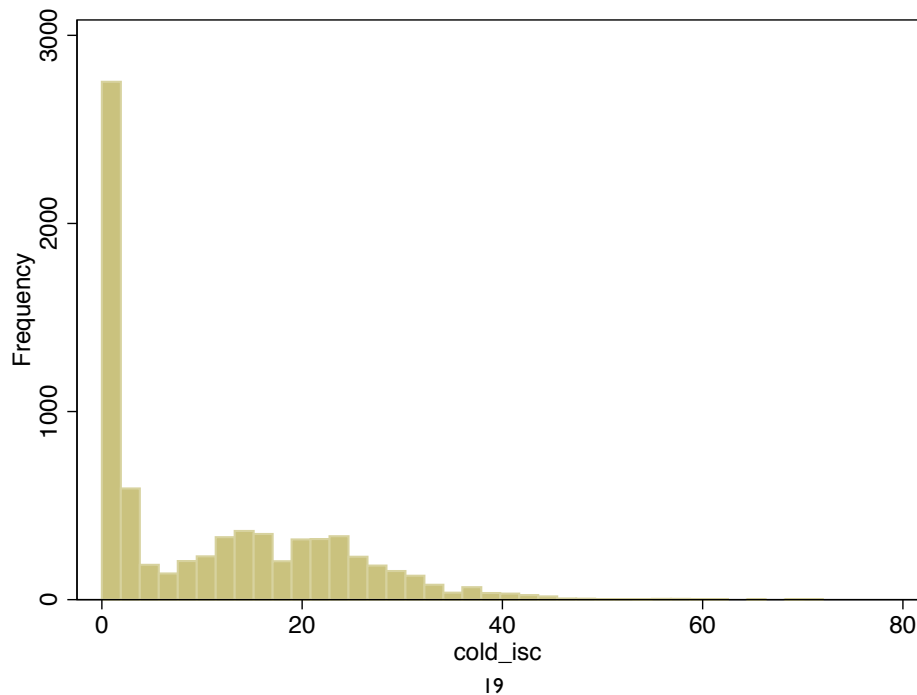
```
mi estimate, or: logistic death cold_isc age_don age year prevtx txtype i.hlamat
```

```
Multiple-imputation estimates      Imputations      =      20
Logistic regression              Number of obs    =     9367
```

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.002313	.0071502	0.32	0.746	.9883722	1.01645
age_don	.9979795	.0039896	-0.51	0.613	.9901907	1.00583
age	.9584372	.0090968	-4.47	0.000	.9407726	.9764335
year	.8578479	.0130779	-10.06	0.000	.8325948	.8838669
prevtx	1.108886	.1468931	0.78	0.435	.855321	1.437622
txtype	1.216516	.2422243	0.98	0.325	.8233093	1.797515
hlamat						
1	1.002488	.1990982	0.01	0.990	.679246	1.479555
2	.8181683	.1660688	-0.99	0.323	.549628	1.217914
3	.6849142	.1420574	-1.82	0.068	.4561307	1.02845
4	.6284968	.1535163	-1.90	0.057	.3893938	1.014419
5	.8206398	.2589599	-0.63	0.531	.4421287	1.523199
6	.7986734	.26867	-0.67	0.504	.4130751	1.544221
_cons	6.3e+131	1.9e+133	9.98	0.000	8.0e+105	4.9e+157

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Dist of Cold Ischemia Time



Predictive Means Matching

- Easy way to be faithful to quirky distns
- Imputes from observed distribution
- Choose match based on closest value of linear predictor
- Respect thresholds
- `mi impute pmm cold_isc hlamat age_don
age death year prevtx txttype , add(20)
rseed(567)`

Transformed Results

```

Multiple-imputation estimates
Logistic regression

DF adjustment:   Large sample

Model F test:    Equal FMI
Within VCE type: OIM

Imputations      =      20
Number of obs    =     9367
Average RVI      =     0.0091
Largest FMI      =     0.1067
DF:              min =    4339.77
                  avg =   1.59e+10
                  max =   1.24e+11
F( 12, 6.0e+06)=    13.07
Prob > F         =     0.0000

```

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.002321	.0070415	0.33	0.741	.9886026	1.01623
age_don	.9979781	.0039895	-0.51	0.613	.9901895	1.005828
age	.9584332	.0090967	-4.47	0.000	.9407689	.9764293
year	.8577968	.0130501	-10.08	0.000	.8325966	.8837597
prevtx	1.108817	.1468895	0.78	0.436	.8552597	1.437547
txtype	1.216574	.2400089	0.99	0.320	.8263761	1.791016
hlamat						
1	1.002359	.1990838	0.01	0.991	.6791438	1.479398
2	.8180324	.1660531	-0.99	0.322	.5495211	1.217746
3	.6847243	.1420257	-1.83	0.068	.4559942	1.028187
4	.6284921	.1535049	-1.90	0.057	.3894033	1.014379
5	.8208394	.2589845	-0.63	0.531	.4422767	1.523429
6	.798598	.2686421	-0.67	0.504	.4130386	1.544066
_cons	7.1e+131	2.1e+133	10.00	0.000	1.0e+106	4.9e+157

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Models in MI Impute

mi impute model

- Continuous variable
 - regress -- linear regression
 - pmm -- non-parametric regression
 - truncreg -- regression with a boundary (e.g., cold ischemia can't be less than 0)
- Categorical
 - logit -- logistic regression
 - ologit -- ordered logit (order category)
 - mlogit -- unordered logit (unordered)

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Data Management

- Imputing creates phantom observations
- Data is only good for imputations
- Useful command -- “mi extract” creates a dataset based on one “imputation”
- **mi extract 0, clear**
back to regular incomplete data
- **mi extract 10, clear**
complete dataset using imputation #10
allows you to check imputation set

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Imputation 10

```
. logistic death cold_isc age_don age year prevtx txttype i.hlamat
```

Logistic regression

Number of obs	=	9367
LR chi2(12)	=	168.51
Prob > chi2	=	0.0000
Pseudo R2	=	0.0491

Log likelihood = -1633.2127

death	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
cold_isc	1.000447	.0065814	0.07	0.946	.9876305	1.01343
age_don	.997997	.0039921	-0.50	0.616	.9902032	1.005852
age	.9584903	.0090981	-4.47	0.000	.9408232	.9764891
year	.8573443	.0130516	-10.11	0.000	.8321415	.8833103
prevtx	1.109332	.1469424	0.78	0.433	.855679	1.438175
txttype	1.261713	.2416046	1.21	0.225	.8668913	1.836355
hlamat						
1	1.004259	.1993966	0.02	0.983	.6805166	1.482015
2	.8191515	.1662461	-0.98	0.326	.5503179	1.219312
3	.6863007	.1423103	-1.82	0.069	.4570993	1.03043
4	.630004	.1538591	-1.89	0.059	.3903584	1.016771
5	.8238943	.259907	-0.61	0.539	.4439664	1.528949
6	.8018133	.2697156	-0.66	0.511	.4147099	1.550251
_cons	2.0e+132	6.1e+133	10.03	0.000	2.8e+106	1.4e+158

but this dropped all other imputations

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mi xeq

simpler approach

- mi xeq l0: summ cold_isc
summarize cold_isc from imputation l0

m=10 data:
-> summ cold_isc

Variable	Obs	Mean	Std. Dev.	Min	Max
cold_isc	9367	10.04335	11.53007	-20.96038	72

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mi xeq

examining results

. mi xeq l0: logistic death cold_isc age_don age year prevtx txttype i.hlamat

m=10 data:
-> logistic death cold_isc age_don age year prevtx txttype i.hlamat

Logistic regression	Number of obs	=	9367
	LR chi2(12)	=	168.51
	Prob > chi2	=	0.0000
Log likelihood = -1633.2127	Pseudo R2	=	0.0491

death	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
cold_isc	1.000447	.0065814	0.07	0.946	.9876305 1.01343
age_don	.997997	.0039921	-0.50	0.616	.9902032 1.005852
age	.9584903	.0090981	-4.47	0.000	.9408232 .9764891
year	.8573443	.0130516	-10.11	0.000	.8321415 .8833103
prevtx	1.109332	.1469424	0.78	0.433	.855679 1.438175
txttype	1.261713	.2416046	1.21	0.225	.8668913 1.836355
hlamat					
1	1.004259	.1993966	0.02	0.983	.6805166 1.482015
2	.8191515	.1662461	-0.98	0.326	.5503179 1.219312
3	.6863007	.1423103	-1.82	0.069	.4570993 1.03043
4	.630004	.1538591	-1.89	0.059	.3903584 1.016771
5	.8238943	.259907	-0.61	0.539	.4439664 1.528949
6	.8018133	.2697156	-0.66	0.511	.4147099 1.550251
_cons	2.0e+132	6.1e+133	10.03	0.000	2.8e+106 1.4e+158

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How Many Imputations

- Surprisingly few
- 50% missingness 5 draws is highly efficient
- Stata on UNOS data
 - 20 imputations in 6 sec.
 - 50 imputations in 16 sec.
 - 100 imputation in 31 sec.
 - 500 imputations in 360 sec.

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50 Imputations

```
. mi estimate, or: logistic death cold isc age_don age year prevtx txttype i.hlamat
```

Multiple-imputation estimates	Imputations	=	50
Logistic regression	Number of obs	=	9367
	Average RVI	=	0.0126
DF adjustment: Large sample	DF: min	=	2492.02
	avg	=	2.42e+10
	max	=	2.69e+11
Model F test: Equal FMI	F(12, 3.2e+06)	=	13.04
Within VCE type: OIM	Prob > F	=	0.0000

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold isc	1.001093	.0071376	0.15	0.878	.9871925	1.015189
age_don	.9979925	.003991	-0.50	0.615	.9902007	1.005846
age	.9584724	.0090979	-4.47	0.000	.9408057	.9764708
year	.8575193	.0130689	-10.09	0.000	.8322835	.8835204
prevtx	1.109199	.14693	0.78	0.434	.8555691	1.438017
txttype	1.245769	.2475528	1.11	0.269	.8438537	1.839111
hlamat						
1	1.003516	.1992915	0.02	0.986	.6799574	1.481042
2	.8187808	.1661875	-0.99	0.325	.5500469	1.218809
3	.6857152	.1422276	-1.82	0.069	.4566588	1.029664
4	.6294577	.153747	-1.90	0.058	.389994	1.015957
5	.822776	.2596321	-0.62	0.536	.4432816	1.527156
6	.8006779	.2693564	-0.66	0.509	.4140995	1.548143

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500 Imputations

Multiple-imputation estimates
Logistic regression

DF adjustment: Large sample

Model F test: Equal FMI
Within VCE type: OIM

Imputations = 500
Number of obs = 9367
Average RVI = 0.0130
DF: min = 23812.19
avg = 1.54e+11
max = 1.54e+12
F(12, 3.1e+07)= 13.02
Prob > F = 0.0000

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.002006	.0070717	0.28	0.776	.9882409	1.015964
age_don	.9979834	.0039899	-0.50	0.614	.9901938	1.005834
age	.9584474	.009097	-4.47	0.000	.9407824	.9764441
year	.8577653	.0130703	-10.07	0.000	.8325267	.8837691
prevtx	1.109026	.1469091	0.78	0.435	.8554327	1.437797
txtype	1.223848	.2423487	1.02	0.308	.8301736	1.804205
hlamat						
1	1.002682	.1991348	0.01	0.989	.6793796	1.479836
2	.8183211	.1660985	-0.99	0.323	.5497324	1.218137
3	.6850761	.1420932	-1.82	0.068	.4562356	1.028699
4	.6287796	.1535788	-1.90	0.057	.3895769	1.014854
5	.8212404	.259145	-0.62	0.533	.4424569	1.524297
6	.7992063	.2688505	-0.67	0.505	.4133495	1.545256

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Complete Case

. logistic death cold_isc age_don age year prevtx txtype i.hlamat

Logistic regression

Log likelihood = -1355.1485

Number of obs = 7347
LR chi2(12) = 109.63
Prob > chi2 = 0.0000
Pseudo R2 = 0.0389

death	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
cold_isc	1.001708	.0070643	0.24	0.809	.9879571	1.015649
age_don	.9976003	.0042907	-0.56	0.576	.989226	1.006045
age	.9693436	.0102045	-2.96	0.003	.9495481	.9895518
year	.8672756	.0150721	-8.19	0.000	.8382322	.8973253
prevtx	1.025197	.1484016	0.17	0.864	.7719559	1.361515
txtype	1.308204	.2723144	1.29	0.197	.8699419	1.967256
hlamat						
1	1.116539	.253119	0.49	0.627	.71599	1.741169
2	.9675642	.2217915	-0.14	0.886	.617393	1.516344
3	.8108275	.1908674	-0.89	0.373	.5111603	1.286174
4	.8051673	.2183204	-0.80	0.424	.4732423	1.369899
5	.9409304	.3416137	-0.17	0.867	.461871	1.916877
6	.892732	.331529	-0.31	0.760	.4311368	1.848533

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Relative Efficiency

of MI compared to complete case

Variable	RE
cold_isc	1.00
txtype	1.27
age_don	1.22
age	1.21
year	1.33

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Comparison

- Big gains unlikely for missing variables
- Instead seen in other variables
- Deleting them bias and loss of efficiency
- Bias: certain covs patterns left out
recent years, living tx, younger recipients
- Efficiency: just lose numbers
- Imputed value not big diff here: not pred.

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Did it work?

Compared to complete/available case analysis?

- No way to really know
- Must believe the MAR assumption
- No difference? Could be small % missing values
- Imputations close to overall mean for variables

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Multiple Missing Variables

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Missing Data

Variable	Obs	Mean	Std. Dev.	Min	Max
hlamat	9541	2.57363	1.378919	0	6
age_don	9662	31.29952	13.38866	0	73
age	9766	11.64653	5.291385	0	18
cold_isc	7525	10.85967	11.51735	0	72
death	9775	.0475703	.2128662	0	1
prevtx	9702	1.1443	.3514119	1	2
txtype	9775	.4733504	.4993148	0	1

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Multiple Missing Variable

- Can be handled by multiple imputation
- Means developing a model for several variable simultaneous
- Two main ways to do this
 - assume variables are normal
so called multivariate normal
 - “chained” regression equal
so called iterative chained equations

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Chained Imputation

- Regression model for each missing variable
- “ICE”: iterative chained equations
- Combined for multivariate imputations
- Builds a multivariate distribution
- Separate models: gives big flexibility
- Only implemented in Stata 12

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Possible Imputation Models

- Continuous: regress, pmm
- Binary: logit
- Ordinal: ologit
- Nominal: mlogit
- Discrete: poisson, nbreg
- Continuous+threshold: truncreg

separate models for each missing variable

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Chained Syntax

- `mi register imputed hlamat age_don age cold_isc prevtx death txttype`
register everything
- `mi impute chained (pmm) cold_isc (ologit) hlamat (regress) age_don (regress) age (logit) prevtx = death txttype, add(50) rseed(897)`
- *(method) variable = nonmissing*
- Very flexible syntax

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mi impute reply

```
. mi impute chained (pmm) cold_isc (ologit) hlamat (regress) age_don (regress) age (logit)
prevtx = death txttype, add(20) rseed(897) dots
```

Conditional models:

```
    age: regress age i.prevtx age_don i.hlamat cold_isc death txttype
    prevtx: logit prevtx age age_don i.hlamat cold_isc death txttype
    age_don: regress age_don age i.prevtx i.hlamat cold_isc death txttype
    hlamat: ologit hlamat age i.prevtx age_don cold_isc death txttype
    cold_isc: pmm cold_isc age i.prevtx age_don i.hlamat death txttype
```

Performing chained iterations:

```
    imputing m=1 through m=20 .....10.....20 done
```

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mi impute reply

```
Multivariate imputation          Imputations =      20
Chained equations                 added =      20
Imputed: m=1 through m=20        updated =       0

Initialization: monotone          Iterations =     200
                                burn-in =      10
```

```
cold_isc: predictive mean matching
hlamat: ordered logistic regression
age_don: linear regression
age: linear regression
prevtx: logistic regression
```

Variable	Observations per m			Total
	Complete	Incomplete	Imputed	
cold_isc	7525	2250	2250	9775
hlamat	9541	234	234	9775
age_don	9662	113	113	9775
age	9766	9	9	9775
prevtx	9702	73	73	9775

(complete + incomplete = total; imputed is the minimum across m of the number of filled-in observations.)

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Chained Results

```
. mi estimate, or: logistic death cold_isc age_don age prevtx txttype i.hlamat
```

```
Multiple-imputation estimates          Imputations =      20
Logistic regression                   Number of obs =     9775
                                      Average RVI   =     0.0524
                                      Largest FMI    =     0.2545
DF adjustment: Large sample           DF: min      =     305.01
                                      avg          =    345849.80
                                      max          =    2868239.03
Model F test: Equal FMI               F( 11,68922.5) =      8.87
Within VCE type: OIM                  Prob > F      =     0.0000
```

death	Odds Ratio	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.009929	.0069456	1.44	0.152	.9963536	1.023689
age_don	.9904639	.0038147	-2.49	0.013	.9830154	.9979688
age	.9483929	.0084812	-5.93	0.000	.9319149	.9651624
prevtx	1.382299	.1809094	2.47	0.013	1.069402	1.786747
txttype	1.23375	.2337621	1.11	0.268	.8506975	1.789283
hlamat						
1	1.04634	.1895773	0.25	0.803	.7335592	1.492486
2	.8583751	.1603099	-0.82	0.414	.595238	1.237837
3	.8070057	.1548374	-1.12	0.264	.5540192	1.175515
4	.6971202	.1609292	-1.56	0.118	.4433924	1.096042
5	.8441431	.2567099	-0.56	0.577	.4651095	1.532064
6	.8949769	.2920161	-0.34	0.734	.4721403	1.696495
_cons	.1035634	.0236399	-9.93	0.000	.0662056	.1620011

Multiple Imputation

- One method for handling missing data
- Can be various others: have a similar spirit *averaging over distn for missing values*
- Imputation is flexible and can be used in many problems
- Useful with missing predictor(s)
- Conceptually straightforward
- Lots of issues in practice

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Issues in Modeling

- Inclusivity over parsimony
more predictor: MAR more plausible
- Will remove biases -- primary objective
- Two things to include: outcome, predictor in your analytic model
- Functional form tends not to be critical

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Missing Values

- Important to explore and understand
- Informs model
- MAR assumption permits valid analysis
missing depend on values thru observed data
- Missing data methods needed for validity and/or efficiency

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Making Up Data?

- No.
- Making a model for missingness
- Making guess at missing data
- Fully representing uncertainty
 - this is the key to it's validity

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Further Topics

- MAR assumption animates two other approaches
 - maximum likelihood approaches
 - inverse weighting
- Will discuss next week