Biostat 200 – Lab 6
Lecture Review

October 24, 2013

Overview

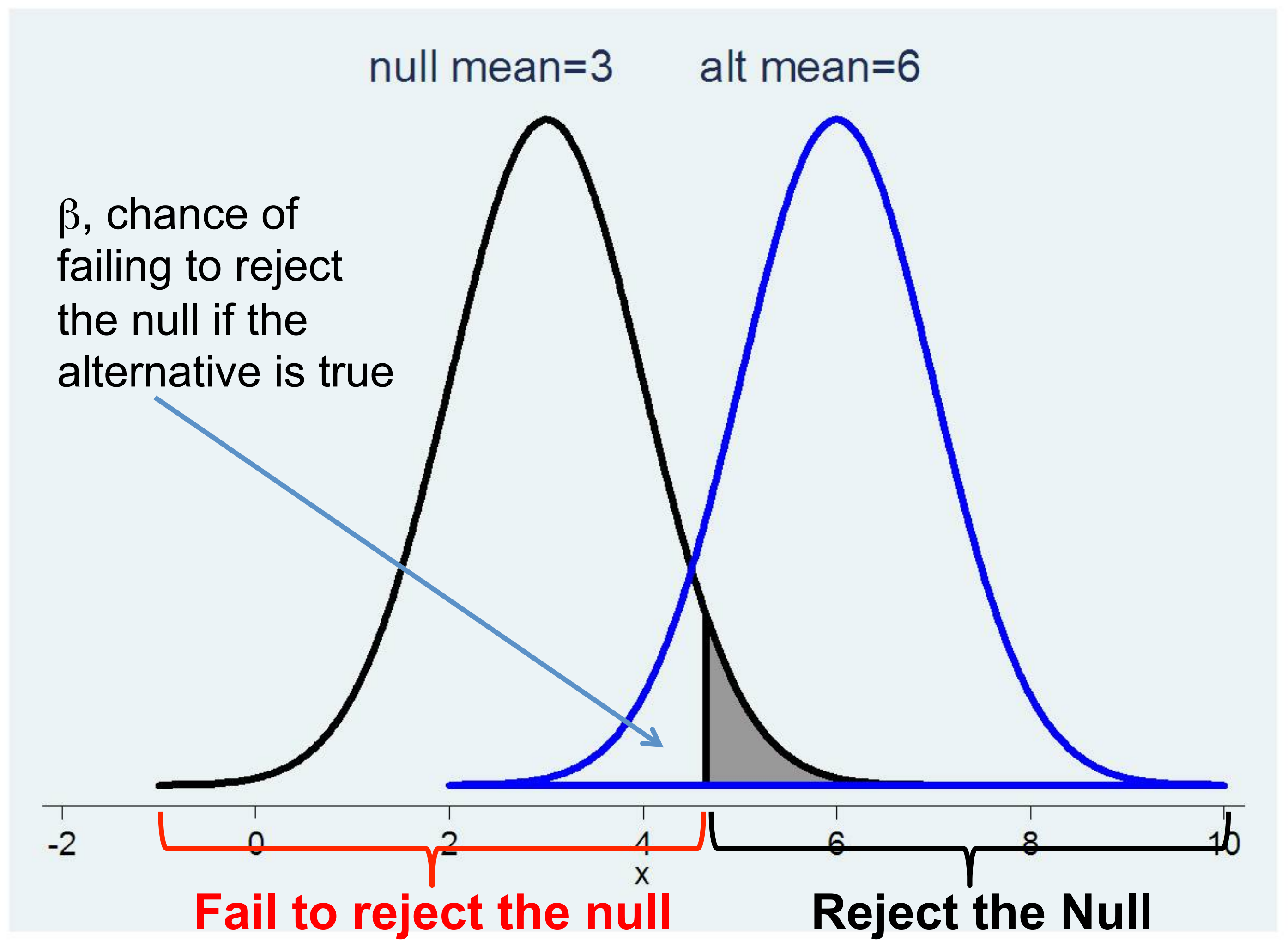
- Brief review of error and sample size
  - Type I and Type II error
  - Power and sample size
- Comparison of two means
  - Paired means
  - Independent means
- Comparison of two proportions

Types of error

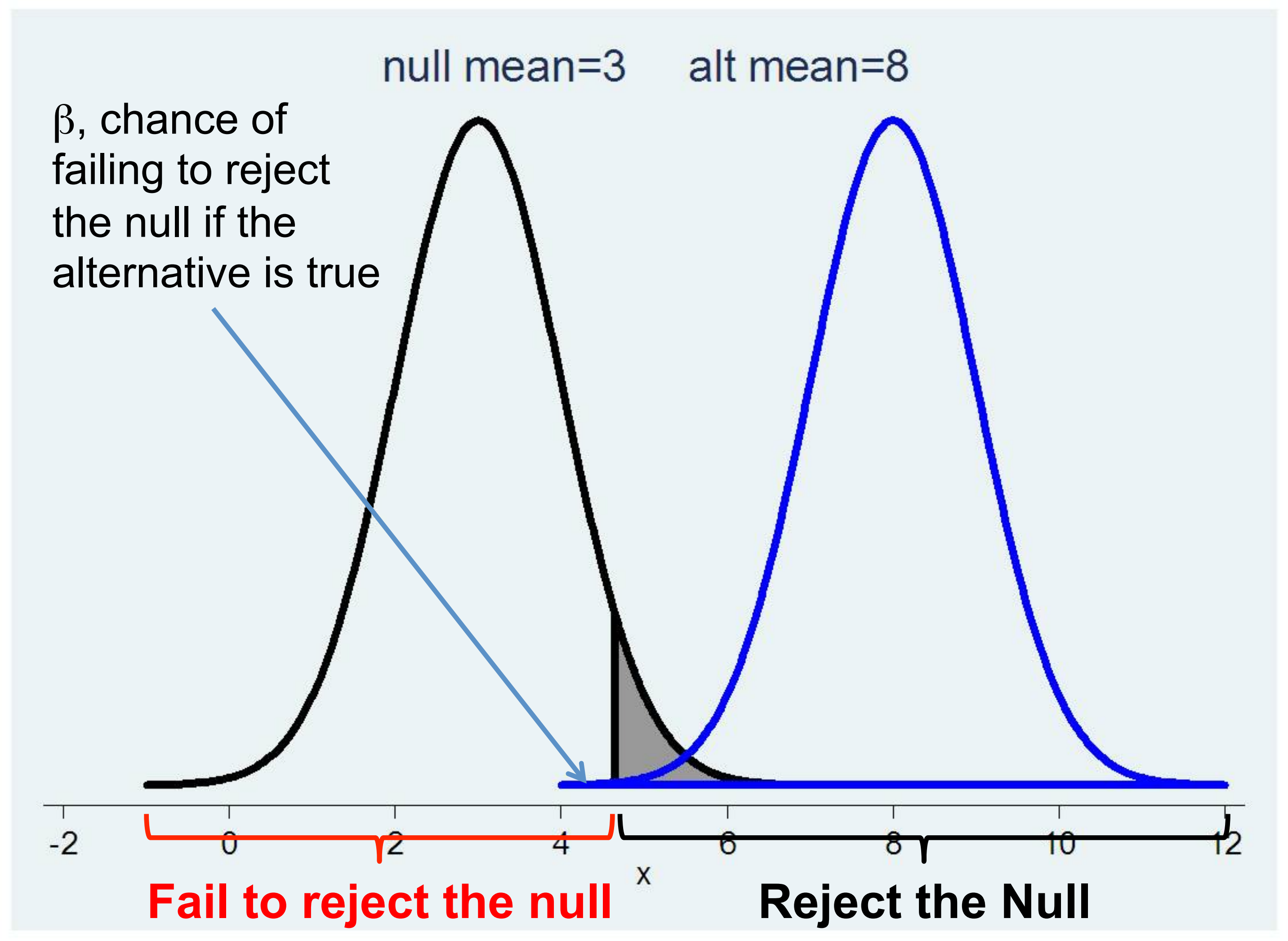
- Type I: incorrectly reject the null
  - α: probability of Type I error = P(reject H_0 | H_0 is true)
- Type II: incorrectly fail to reject the null
  - β: probability of Type II error = P(do not reject H_0 | H_0 is false)
- Both are conditional upon whether H_0 is true or false
- To minimize these errors:
  - set α
  - design study to minimize the chance of a Type II error

	True state	
Decision	**H_0 is true**	**H_0 is false**
Do not reject H_0 ($p \geq 0.05$)	Correct	Type II error=β
Reject H_0 ($p < 0.05$)	Type I error=α	Correct

Chance of a Type II error



If the alternative is very different from the null, the chance of a Type II error is low



Power

- $1-\beta$
- Lower power for alternative values that are closer to the null value
  - β higher, so $(1-\beta)$ is lower
- Higher power for more extreme alternative values
  - β lower, so $(1-\beta)$ is higher
- Used to calculate sample size

Sample size (n)

- Standard to fix α and β and determine n
  - α=0.05 and β=0.20 (80% power)
- Fixed n
  - Calculate minimum detectable difference
  - How big/far away the alternative has to be from the null to reject the null with 80% probability, if the alternative is true

Hypothesis testing review

- Step 1: State the hypothesis
- Step 2: Calculate the test statistic
- Step 3: Compare the test statistic to the distribution to determine the probability of obtaining a value as large or larger than the observed value (p-value)
- Step 4: Use the p-value to reject or fail to reject the null

Comparison of two means

Paired samples

Paired t-test

- For paired samples with numerical variables
  - Before and after on same person
  - Matched samples
- Calculate the difference for each person or pair
- Use one-sample methods to test hypotheses about value of the difference

Paired t-test – Step 1 (hypothesis)

Two sided

- H_0: $\mu1-\mu2=\delta$
- H_A: $\mu1-\mu2\neq\delta$

- Often $\delta=0$, no difference
- H_0: $\mu1-\mu2=0 \rightarrow H_0$: $\mu1=\mu2$
- H_A: $\mu1-\mu2 \neq 0 \rightarrow H_A$: $\mu1\neq\mu2$

One sided

- H_0: $\mu1-\mu2\geq\delta$ <u>or</u> H_0: $\mu1-\mu2\leq\delta$
- H_A: $\mu1-\mu2<\delta$ <u>or</u> H_A: $\mu1-\mu2>\delta$

- Often $\delta=0$, no difference
- H_0: $\mu1 \geq \mu2$ or H_0: $\mu1 \leq \mu2$
- H_A: $\mu1 < \mu2$ or H_A: $\mu1 > \mu2$

Paired t-test – Step 2 (test statistic)

- If δ=0, the formula for t_{stat} is

$$t_{stat} = \frac{\bar{d}}{s_d / \sqrt{n}} \text{ where}$$

$$s_d = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$

- d-bar is the average difference between the pairs

Paired t-test – Steps 3 & 4 (calculate p-value and reject null?)

- Calculate the p-value
  - Use ttail in Stata (*2 for two-sided hypothesis)
- p-value: probability of observing a difference as large or larger, given the null hypothesis is true
- Reject or fail to reject the null
  - $p < \alpha$: reject null
  - $p \geq \alpha$: fail to reject the null

Paired t-test – Using Stata

- `ttest var_difference==0`
  - Use after you've created a variable for the difference of post-pre, var2-var1, etc
- `ttest var1==var2`
  - Use without calculating the difference
- Example: BREATH study, PEth at baseline and 3 months

Comparison of two means

Independent samples

t-test for independent samples

- To compare means from two independent samples
  - Treatment vs placebo
  - Exposed vs unexposed

t-test – Step 1 (hypothesis)

- Two sided hypothesis
  - H_0: $\mu_1 = \mu_2$
  - H_A: $\mu_1 \neq \mu_2$
- One sided hypothesis
  - H_0: $\mu_1 \geq \mu_2$
  - H_A: $\mu_1 < \mu_2$
- One sided hypothesis
  - H_0: $\mu_1 \leq \mu_2$
  - H_A: $\mu_1 > \mu_2$

t-test – Step 2 (test statistic)

- Calculate t_{stat} (σ is unknown)

$$t_{stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(1/n_1 + 1/n_2\right)}} \text{ where}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

- The formula for the pooled SD is a weighted average of the individual sample SDs
- The degrees of freedom for the test are n_1+n_2-2

t-test – Steps 3 & 4 (calculate p-value and reject null?)

- (Same as paired t-test…)
- Calculate the p-value
- Reject or fail to reject the null
  - $p < \alpha$: reject null
  - $p \geq \alpha$: fail to reject the null
- Example: blood loss pre and post anti-shock garment intervention

t-test – Using Stata

- `ttesti n1 mean1 sd1 n2 mean2 sd2`
  - Use immediate command to enter the values
  - Remember that for a one-sample t-test, use `ttesti n mean sd hypothesizedmean`
- `ttest var1==var2, unpaired`
  - For variables in different columns
- `ttest var, by(groupvar)`
  - For data all in one variable, grouped by another variable

95% confidence intervals for the difference between 2 means

$$\text{Lower bound}: (\bar{x}_1 - \bar{x}_2) - t_{df,.025}\sqrt{s_p^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}$$

$$\text{Upper bound}: (\bar{x}_1 - \bar{x}_2) + t_{df,.025}\sqrt{s_p^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}$$

- Assumes equal variance in two populations
- If the confidence interval for the difference does not include 0, then you can reject the null hypothesis of no difference

t-test for unequal variances

$$t_{stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{(s_1^2 / n_1) + (s_2^2 / n_2)}}$$

with degrees of freedom ν (pronounced "nu")

$$\nu = \frac{\left[(s_1^2 / n_1) + (s_2^2 / n_2)\right]^2}{\left[(s_1^2 / n_1)^2 / (n_1 - 1) + (s_2^2 / n_2)^2 / (n_2 - 1)\right]}$$

- `ttesti n1 mean1 sd1 n2 mean2 sd2, unequal`
- `ttest var1==var2, unpaired unequal`
- `ttest var, by(groupvar) unequal`

95% CI for the difference between 2 means with unequal variances

$$\text{Lower bound}: (\bar{x}_1 - \bar{x}_2) - t_{\nu,.025}\sqrt{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]}$$

$$\text{Upper bound}: (\bar{x}_1 - \bar{x}_2) + t_{\nu,.025}\sqrt{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right]}$$

- Example: mean log PEth by sex

Comparison of two proportions

2 proportions – Step 1 (hypothesis)

- Null hypothesis for two proportions, p_1 and p_2
  - H_0: $p_1 = p_2$
  - H_A: $p_1 \neq p_2$
- If n_1 and n_2 are sufficiently large, the difference between the two proportions follows a <u>normal</u> distribution.

2 proportions – Step 2 (test statistic)

- Calculate the z-statistic

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1 - \hat{p})[1/n_1 + 1/n_2]}}$$

$$\text{where} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

2 proportions – Steps 3 & 4 (calculate p-value and reject null?)

- Calculate the p-value
  - Use normal in Stata (*2 for two-sided hypothesis)
- Reject or fail to reject the null
  - $p < \alpha$: reject null
  - $p \geq \alpha$: fail to reject the null

2 proportions – Using Stata

- `prtesti n1 p1 n2 p2`
  - Use immediate command to enter the values
- `prtest var, by(groupvar)`
  - For data all in one variable, grouped by another variable
- Example: rate of starting ART by sex

Statistical hypothesis tests

Data and comparison type	Alternative hypotheses	Parametric test Stata command
Numerical; One mean	H_a: $\mu \neq \mu_a$ (two-sided) H_a: $\mu > \mu_a$ or $\mu < \mu_a$ (one-sided)	Z or t-test • ttest var1=hypoth val.*
Numerical; Two means, paired data	H_a: $\mu_1 \neq \mu_2$ (two-sided) H_a: $\mu_1 > \mu_2$ or $\mu < \mu_a$ (one-sided)	Paired t-test • ttest var1=var2*
Numerical; Two means, independent data	H_a: $\mu_1 \neq \mu_2$ (two-sided) H_a: $\mu_1 > \mu_2$ or $\mu < \mu_a$ (one-sided)	T-test (equal or unequal variance) • ttest var1, by(byvar) • ttest var1, by(byvar) unequal
Numerical, Two or more means, independent data		
Dichotomous; One proportion	H_a: $p \neq p_a$ (two-sided) H_a: $p > p_a$ or $p < p_a$ (one-sided)	Proportion test • prtest var1=hypoth value* • bitest var1=hypoth value
Dichotomous; two proportions	H_a: $p_1 \neq p_2$ (two-sided) H_a: $p_1 > p_2$ (one-sided)	Proportion test (z-test) • prtest var1, by(byvar)
Categorical by categorical (nxk)		