Model Validation

Dave Glidden

17 df predictor model

coefficients

```
. xi: logistic dead i.agecat male hrsdm hrsca hrslung hrschf bmi smoke bath money walkjog push, coef
i.agecat          _Iagecat_0-6        (naturally coded; _Iagecat_0 omitted)

Logistic regression                               Number of obs   =      11652
                                                  LR chi2(17)     =    2080.12
                                                  Prob > chi2     =     0.0000
Log likelihood = -3132.1774                       Pseudo R2       =     0.2493

------------------------------------------------------------------------------
        dead |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  _Iagecat_1 |   .6345948   .1454731     4.36   0.000     .3494727    .9197169
  _Iagecat_2 |   1.039148   .1413782     7.35   0.000     .7620513    1.316244
  _Iagecat_3 |   1.314136   .1370846     9.59   0.000     1.045455    1.582817
  _Iagecat_4 |   1.689447   .1371779    12.32   0.000     1.420583    1.958311
  _Iagecat_5 |    2.11972   .1429917    14.82   0.000     1.839461    2.399979
  _Iagecat_6 |   2.789075   .1451113    19.22   0.000     2.504662    3.073487
        male |    .712165   .0704824    10.10   0.000     .5740221    .8503079
       hrsdm |   .5781861   .0848568     6.81   0.000     .4118699    .7445023
       hrsca |   .7208027   .0851805     8.46   0.000      .553852    .8877534
     hrslung |   .8225759   .1244037     6.61   0.000     .5787491    1.066403
      hrschf |    .851144    .145293     5.86   0.000     .5663751    1.135913
      bmicat |   .5017212   .0698006     7.19   0.000     .3649147    .6385278
       smoke |   .7204122   .0927079     7.77   0.000     .5387081    .9021163
        bath |   .6721684   .1055242     6.37   0.000     .4653448    .8789919
       money |    .643781   .0961402     6.70   0.000     .4553498    .8322122
     walkjog |   .7358055   .0786537     9.36   0.000     .5816471    .8899639
        push |   .4231833   .0791656     5.35   0.000     .2680215     .578345
       _cons |  -4.903092   .1321857   -37.09   0.000    -5.162171   -4.644013
------------------------------------------------------------------------------
```

Calculating Points

Push: HR = 1.53
coef = 0.42 (smallest coef)
points = +1 0.42/0.42

Walk/Jog HR = 2.1
coef = 0.736
points = +2 0.736/0.42 ≈ 2

Variable	Lee Points
Age 60-64	+1
Age 65-69	+2
Age 70-74	+3
Age 75-79	+4
Age 80-84	+5
Age > 85	+7
Male	+2
Diabetes	+1
Cancer	+2
Lung Disease	+2
Heart Failure	+2
BMI < 25	+1
Smoking	+2
Bathing	+2
Money	+2
Walking	+2
Pushing	+1

Last Step

- The score be used directly
- $p_{death} \approx \exp(-4.9+0.42*score)$
- Can also be used predict/stratify
- Would need a cutpoint
  Lee uses quantiles 0-5, 6-9, 10-13, ≥14

What groups to form gets to fundamental issues of why you are making a predictive model

End Result

- Score from 0-23
- Lower score -> lower risk of death
- 90% of people have score of 10 or less
- Is it a good summary of risk?
- Can it predict who will die?
- What does a "7" on the score mean?

Model Validation

Model Validation

Altman and Royston

- Statistical and clinical considerations
  - best model for predictors?
  - accurate enough for purposes?
- Interplay between statistic and context

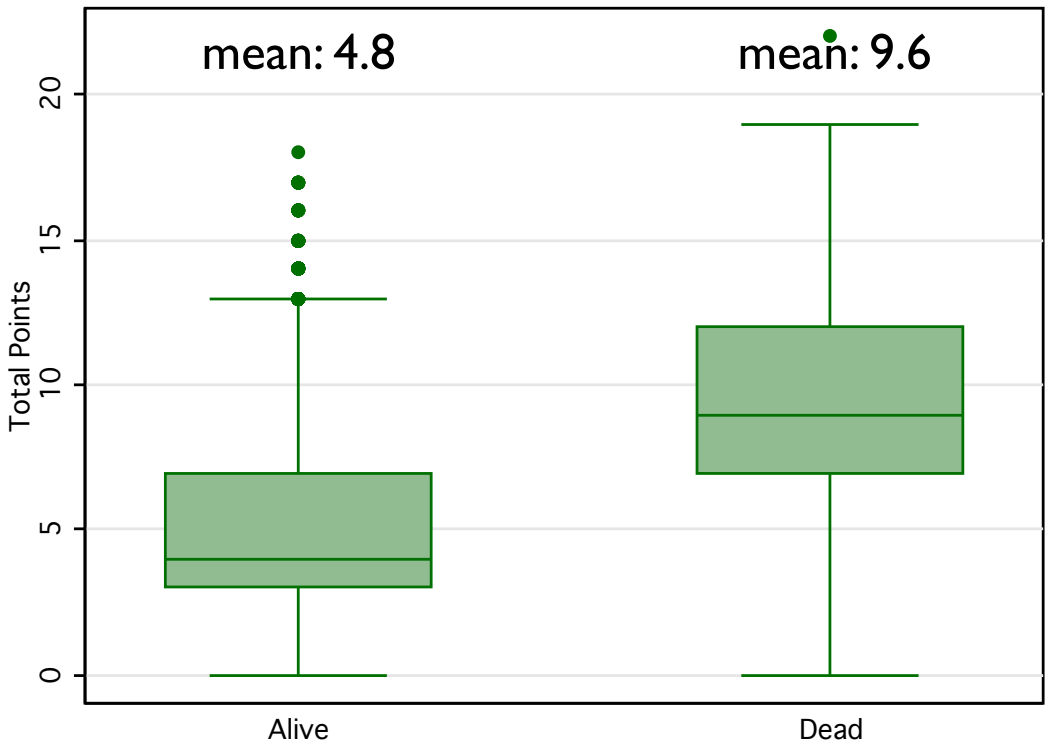
Statistical Fit

- Discrimination
  - Is it a strong model?
  - Can we effectively predict?
- Calibration
  - Are predictions accurate?

Discrimination

- Difference between groups
- Various measures
  - mean difference in score
  - c statistic

Score by Group

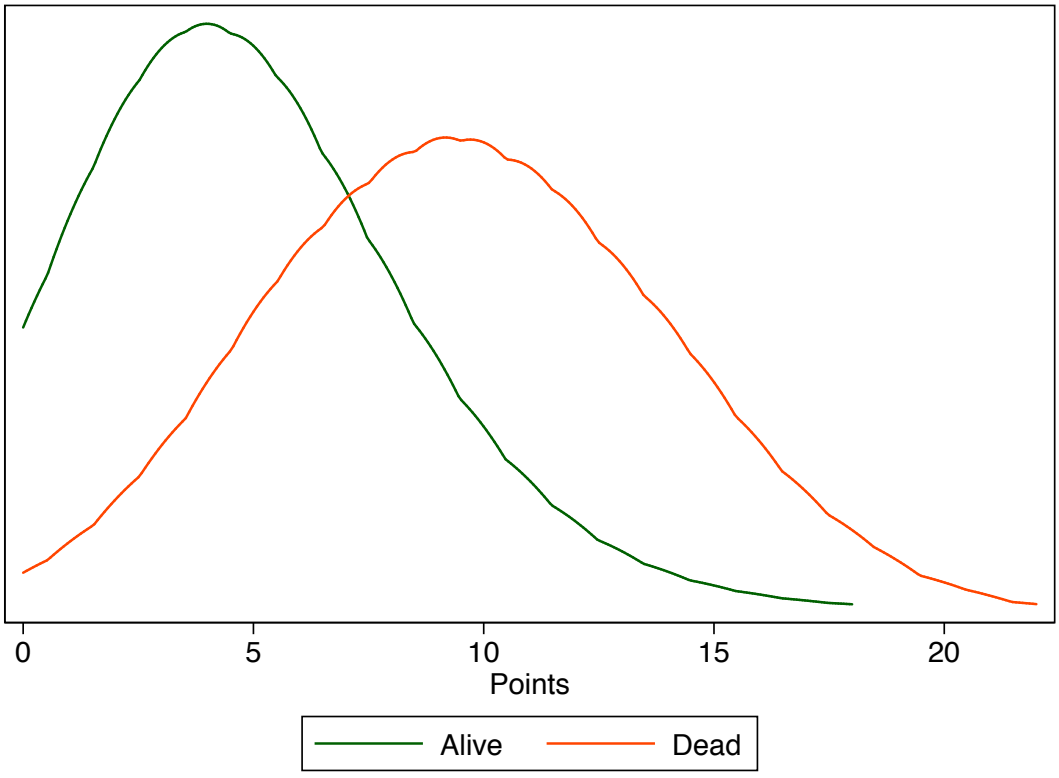


Area Under ROC

- Also known as C-statistic
- Summary measure of AUC curve *between 0 and 1*
- Probability that dead person has more points than a living person
- Measures purity of groups
- Clearly related to discrimination

C-statistic (test):
0.8424 (linear predictor), 0.8405 (points)

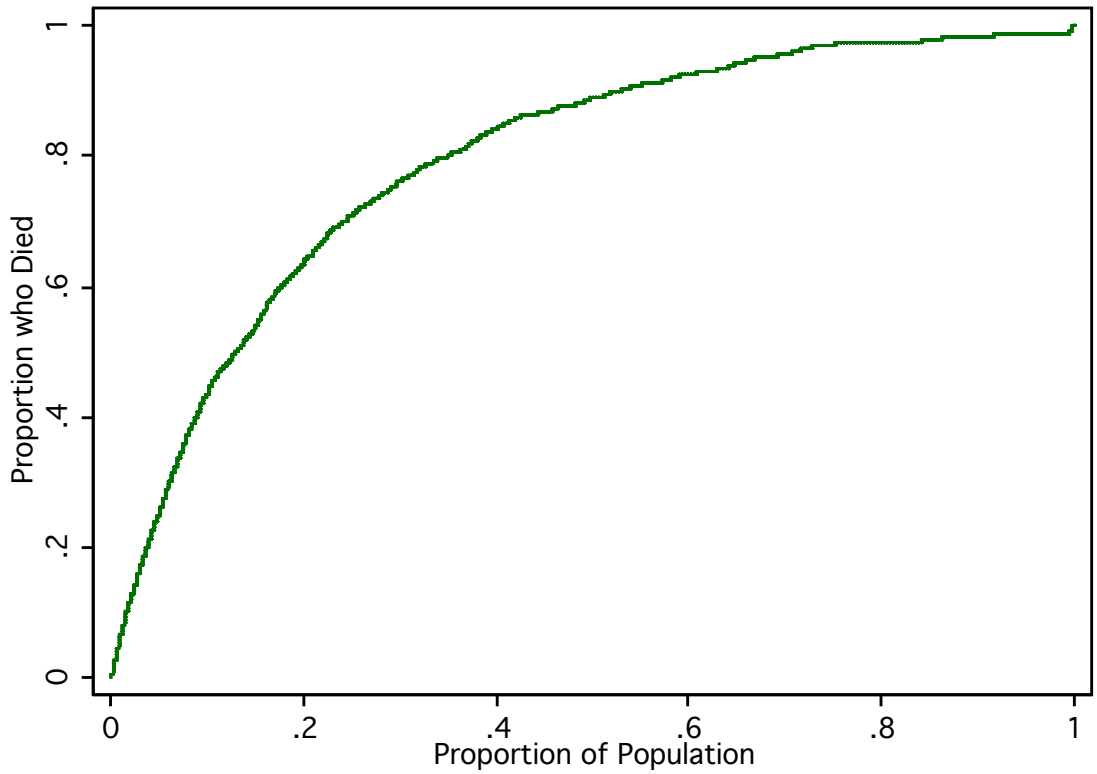
Histogram by Outcome



Lorenz Curve

- Cumulative proportion of outcome by cumulative positive classifications
- Works in prevalence information
- Gives a sense of screening performance
- Also used for income distributions
  10% of pop^{n} has 40% of mortality risk

Lorenz




```
table points2, c(n prob mean prob)
```

Risk for Each Levels of Points

points2	N(prob)	Pr Death (Model)
0	840	.0073689
1	1,228	.0126862
2	2,255	.0177149
3	2,445	.0277034
4	2,431	.0408263
5	2,172	.0587947
6	1,920	.0850401
7	1,523	.1187199
8	1,256	.1655665
9	952	.2236966
10	715	.2950674
11	552	.3817409
12	436	.4617796
13	333	.5622715
14	184	.6385003
15	194	.7240268
16	55	.7923723
17	86	.8453672
18	31	.8894324
19	14	.9191387
20	1	.9487265
22	1	.9775991
23	1	.981464

Validation Set

- Total dataset 19,710 people available
- 11,701: east, west, central region US
  learning set
- 8009: south
  test set
- Deliberately non-comparable
- Rationale: *assess portability*

Lee Model
validation set

```
. xi: logistic dead i.agecat male hrsdm hrsca hrslung hrschf bmi smoke bath money walkjog push
i.agecat          _Iagecat_0-6        (naturally coded; _Iagecat_0 omitted)

Logistic regression                               Number of obs   =       7973
                                                  LR chi2(17)     =    1413.63
                                                  Prob > chi2     =     0.0000
Log likelihood = -2423.8528                       Pseudo R2       =     0.2258

------------------------------------------------------------------------------
        dead | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  _Iagecat_1 |   1.537822   .2259016     2.93   0.003       1.1531    2.050903
  _Iagecat_2 |   2.248315   .3291624     5.53   0.000     1.687476    2.995549
  _Iagecat_3 |   2.586506    .381196     6.45   0.000     1.937602     3.45273
  _Iagecat_4 |   4.109389   .5813912     9.99   0.000     3.114228    5.422558
  _Iagecat_5 |   7.608406    1.14937    13.43   0.000     5.658564    10.23013
  _Iagecat_6 |   14.62968   2.233807    17.57   0.000     10.84588    19.73353
        male |   2.166964   .1712289     9.79   0.000     1.856058    2.529949
       hrsdm |   1.761092   .1639891     6.08   0.000     1.467303    2.113704
       hrsca |   1.664218   .1689592     5.02   0.000      1.36393    2.030617
     hrslung |   1.587757   .2179521     3.37   0.001     1.213219     2.07792
      hrschf |   2.136519   .3437415     4.72   0.000     1.558685    2.928569
      bmicat |   1.736036   .1361793     7.03   0.000     1.488635    2.024553
       smoke |   1.594206   .1606953     4.63   0.000     1.308409    1.942429
        bath |   1.474593   .1711166     3.35   0.001     1.174616    1.851179
       money |   1.420027   .1534369     3.25   0.001     1.149006    1.754974
     walkjog |   1.995648   .1817115     7.59   0.000      1.66947    2.385553
        push |    1.49066   .1350732     4.41   0.000     1.248098    1.780363
------------------------------------------------------------------------------
```

Lee Model

training set

```
. xi: logistic dead i.agecat male hrsdm hrsca hrslung hrschf bmi smoke bath money walkjog push
i.agecat          _Iagecat_0-6        (naturally coded; _Iagecat_0 omitted)

Logistic regression                               Number of obs   =      11652
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------------------------------------------------------------------------------
        dead | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  _Iagecat_1 |   1.886258   .2743998     4.36   0.000     1.418319     2.50858
  _Iagecat_2 |   2.826806   .3996489     7.35   0.000     2.142667    3.729387
  _Iagecat_3 |   3.721534   .5101651     9.59   0.000     2.844693    4.868651
  _Iagecat_4 |   5.416485   .7430223    12.32   0.000     4.139534    7.087346
  _Iagecat_5 |   8.328805    1.19095    14.82   0.000     6.293148    11.02294
  _Iagecat_6 |   16.26596   2.360374    19.22   0.000     12.23942    21.61716
        male |     2.0384   .1436712    10.10   0.000     1.775394    2.340367
       hrsdm |   1.782802   .1512828     6.81   0.000     1.509638    2.105393
       hrsca |   2.056083   .1751382     8.46   0.000     1.739942    2.429665
     hrslung |   2.276356   .2831871     6.61   0.000     1.783806    2.904911
      hrschf |   2.342325   .3403233     5.86   0.000     1.761869    3.114015
      bmicat |   1.651562   .1152799     7.19   0.000     1.440391    1.893691
       smoke |    2.05528   .1905406     7.77   0.000     1.713791    2.464814
        bath |   1.958479   .2066669     6.37   0.000     1.592563    2.408471
       money |   1.903665   .1830186     6.70   0.000     1.576725    2.298398
     walkjog |   2.087163    .164163     9.36   0.000     1.788983    2.435042
        push |   1.526814   .1208712     5.35   0.000     1.307375    1.783085
------------------------------------------------------------------------------
```
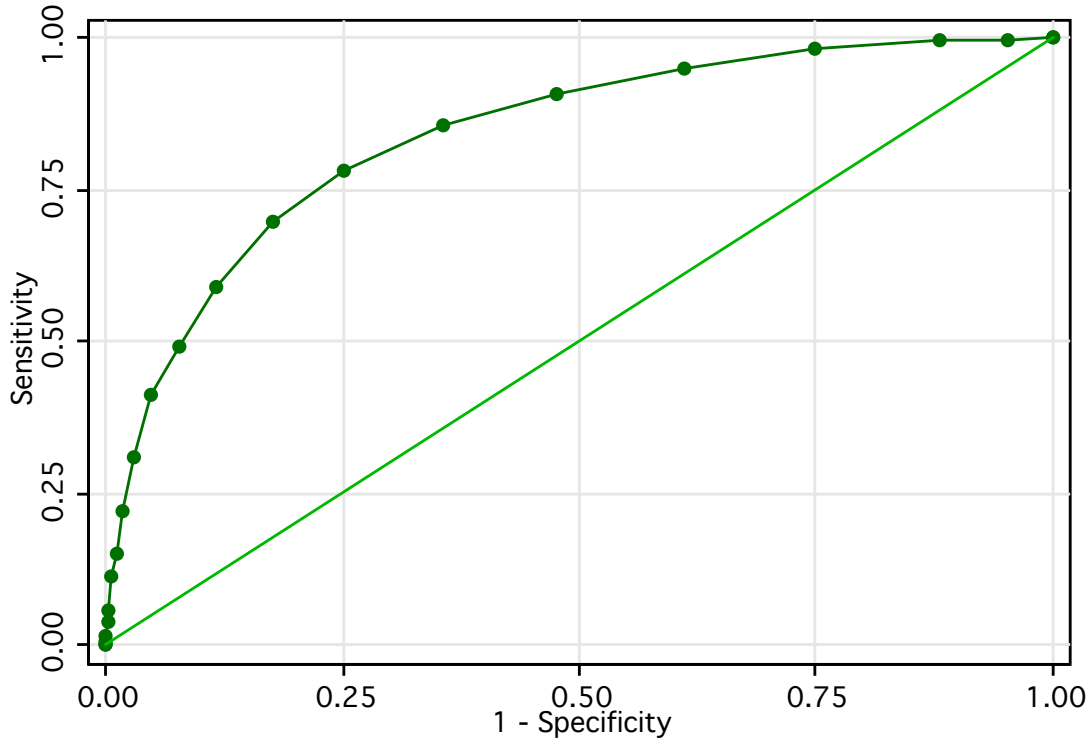
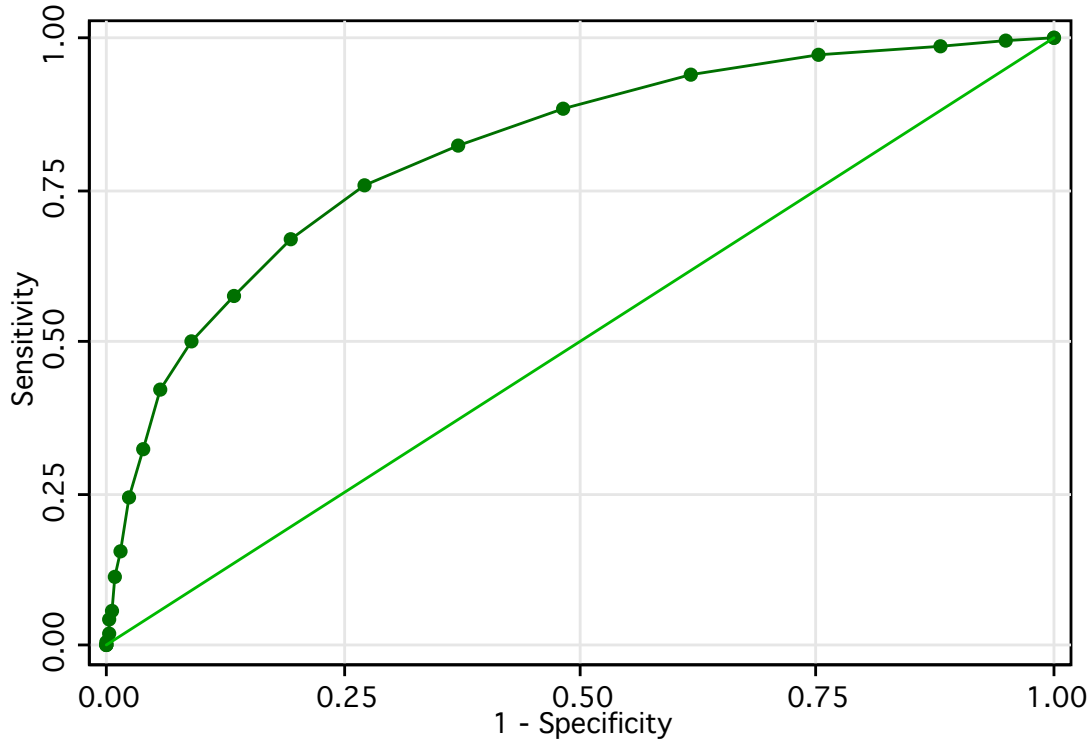

Variable	Training Coef.	Valid Coef.	Ratio
Age 60-64	0.63	0.43	0.68
Age 65-69	1.04	0.81	0.78
Age 70-74	1.31	0.95	0.72
Age 75-79	1.69	1.41	0.84
Age 80-84	2.12	2.03	0.96
Age > 85	2.79	2.68	0.96
Male	0.71	0.77	1.09
Diabetes	0.58	0.57	0.98
Cancer	0.72	0.51	0.71
Lung Disease	0.82	0.46	0.56
Heart Failure	0.85	0.76	0.89
BMI < 25	0.50	0.55	1.10
Smoking	0.72	0.47	0.65
Bathing	0.67	0.39	0.58
Money	0.64	0.35	0.54
Walking	0.74	0.69	0.94
Pushing	0.42	0.40	0.94

ROC Lee Model: Learn



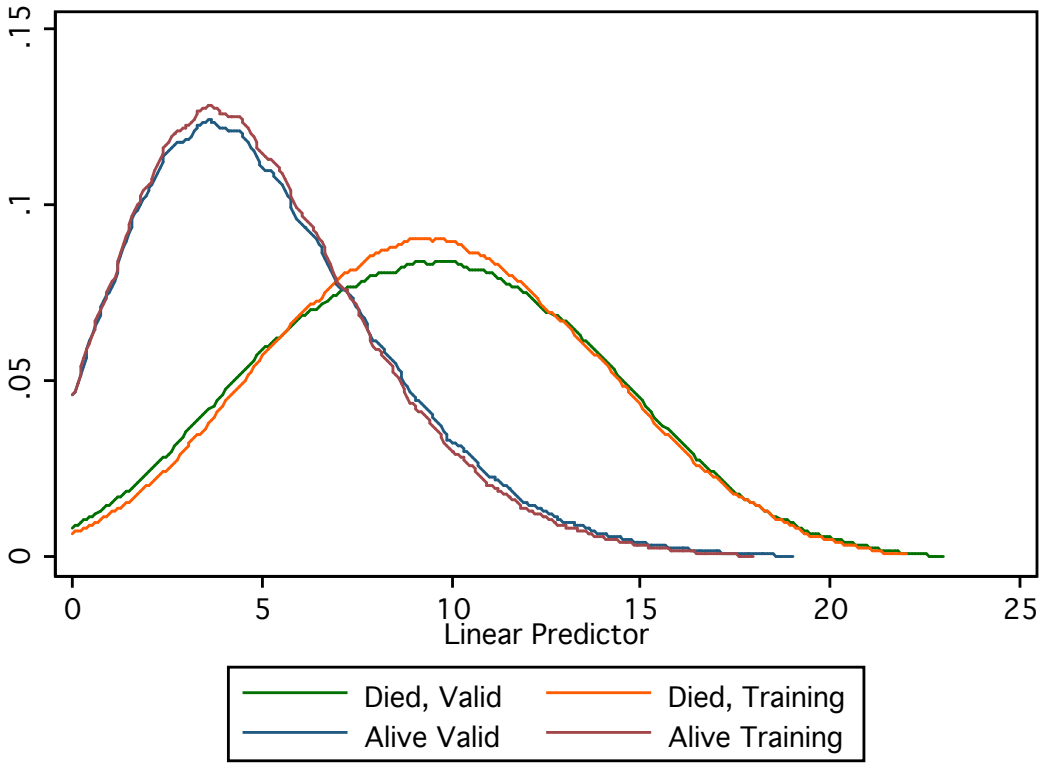
Area under ROC curve = 0.8405

ROC: Test



Area under ROC curve = 0.8175

Density



Calibration

Calibration

- Model: predicted probability
- Does predicted probability match truth
- Important for individual decision making
- Classic model-checking problem
- Incorporate statistical tests and descriptions of magnitude
- Find model misspecified in some way

Lee Model

validation set

```
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  _Iagecat_5 |   7.608406    1.14937    13.43   0.000     5.658564    10.23013
  _Iagecat_6 |   14.62968   2.233807    17.57   0.000     10.84588    19.73353
        male |   2.166964   .1712289     9.79   0.000     1.856058    2.529949
       hrsdm |   1.761092   .1639891     6.08   0.000     1.467303    2.113704
       hrsca |   1.664218   .1689592     5.02   0.000      1.36393    2.030617
     hrslung |   1.587757   .2179521     3.37   0.001     1.213219     2.07792
      hrschf |   2.136519   .3437415     4.72   0.000     1.558685    2.928569
      bmicat |   1.736036   .1361793     7.03   0.000     1.488635    2.024553
       smoke |   1.594206   .1606953     4.63   0.000     1.308409    1.942429
        bath |   1.474593   .1711166     3.35   0.001     1.174616    1.851179
       money |   1.420027   .1534369     3.25   0.001     1.149006    1.754974
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        push |    1.49066   .1350732     4.41   0.000     1.248098    1.780363
------------------------------------------------------------------------------
```

Aspects of Model Checking

- Involves
  - wrong functional form
  - undetected interaction

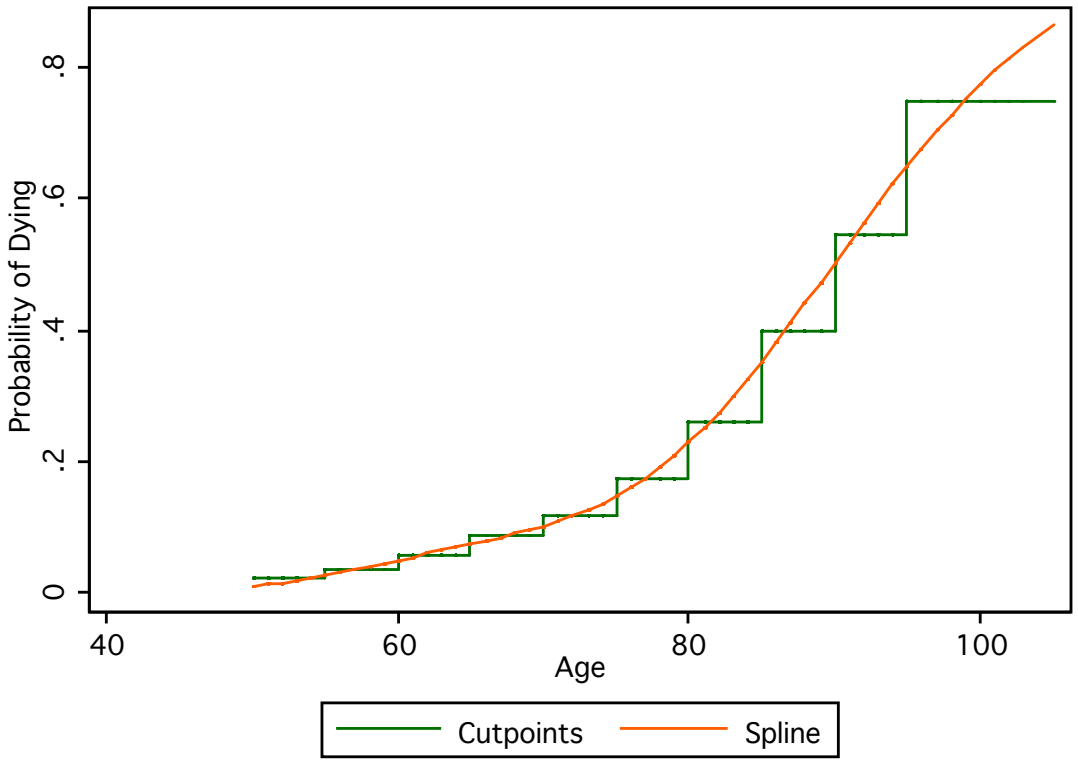
Function Form

- Involves continuous covariate: e.g., age
- Maybe assumed linear when it is not
- Can be too few cutpoints
  - choice of two
- Result: some ages systematically predicted wrong

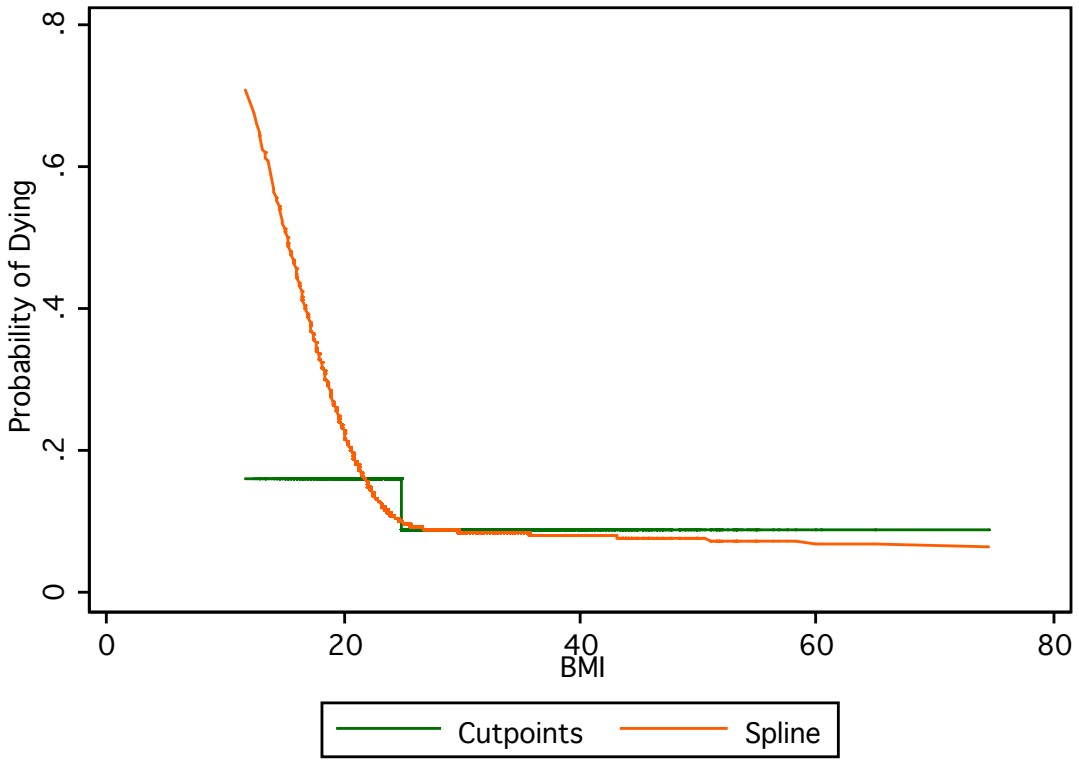
Functional Form

- Fit the spline model to continuous predictor(s)
- Fit the other model
- Plot the difference in fit
- Tests not useful with large or small n

Age



BMI



Functional Form

- Look basically fine
- Age has enough cutpoints
  - and we can afford them
  - underestimate mortality for 90yo
- BMI: only 5% of values less than 20
  - apparent diff => minimal for prediction

Undetected Interaction

- Effect of one variable depends on another
- For an someone with cancer, is diabetes important predictor?
- If modeled, points depend on two variables
- Just 17 predictors => 66 possible interactions
- That's just two-way interactions

Interactions with Age

Variable	Age < 60	Age 60-80	Age > 80
Male	2.69	2.15	1.81
Diabetes	2.03	1.94	1.22
Cancer	5.29	2.40	1.34
Lung Disease	3.96	2.08	1.72
Heart Failure	5.26	2.11	2.10
BMI < 25	1.38	1.78	1.95
Smoking	1.71	1.81	1.50
Bathing	1.04	2.06	2.33
Money	1.38	1.87	2.18
Walking	2.41	2.48	1.79
Pushing	1.24	1.60	1.44

Inspecting Calibration

- Form series of groups based on risk
- Compare expected mortality with observed
- Expected: average(probs)
- Observed: average(death)
  = *proportion who died*
- Within each group

  our point total furnishes such a group

Test Set

```
. table pts2 if valid==0, c(n dead mean dead mean prob)

-------------------------------------------
Points
          |        N     mean(dead)  mean(prob)
----------+--------------------------------
        0 |          486    .0041152    .0073689
        1 |          739    .0067659    .0126751
        2 |        1,366    .0095168     .017859
        3 |        1,474    .0325645    .0277967
        4 |        1,445    .0394464    .0407945
        5 |        1,330    .0503759    .0589664
        6 |        1,162    .0886403    .0849856
        7 |          886    .1230248    .1186558
        8 |          758    .1912929    .1653885
        9 |          551    .2413793    .2242661
       10 |          407    .2727273    .2922267
       11 |          320      .43125     .384265
       12 |          244    .4836065    .4604374
       13 |          174    .5344828    .5641574
       14 |          107    .5046729    .6368158
      15+|          203    .7438424    .7780437
-------------------------------------------
```

Validation Set

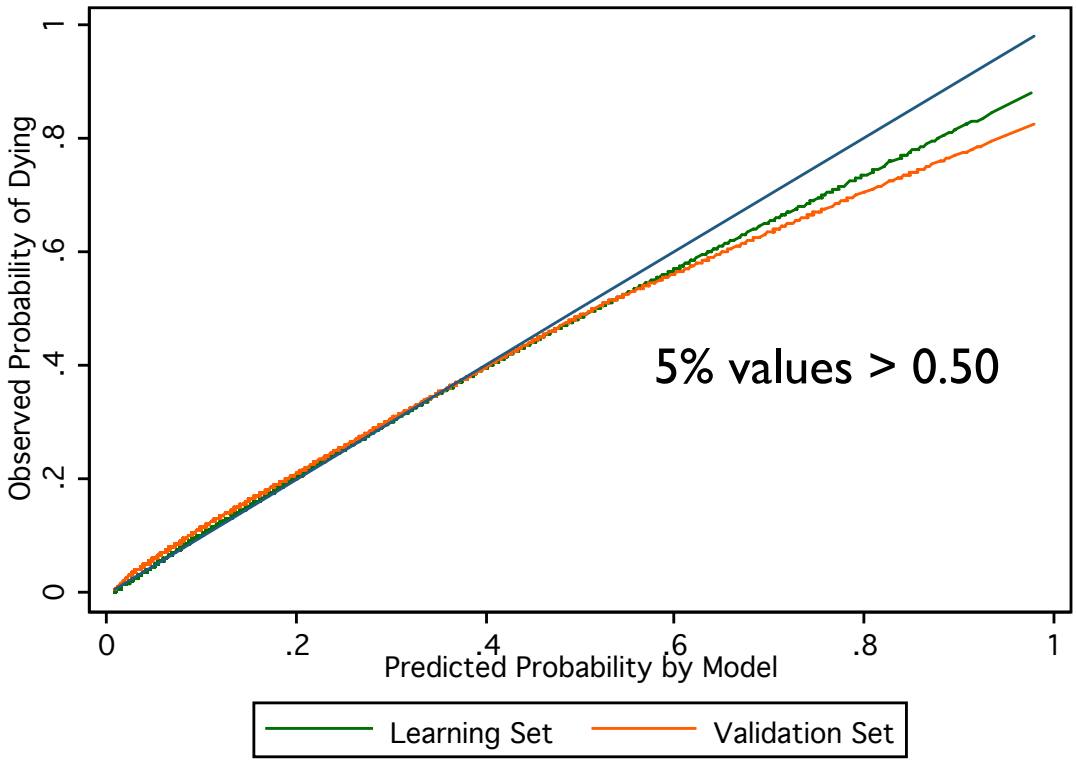
```
. table pts2 if valid==1, c(n dead mean dead mean prob)

-------------------------------------------
Points
          |       N     mean(dead)  mean(prob)
----------+--------------------------------
        0 |         354    .0056497    .0073689
        1 |         489    .0204499    .0127029
        2 |         889    .0168729    .0174934
        3 |         971    .0360453    .0275617
        4 |         986    .0598377    .0408728
        5 |         842    .0771971    .0585236
        6 |         758    .0936675    .0851236
        7 |         637    .1507064    .1188091
        8 |         498    .1967871    .1658374
        9 |         401      .19202    .2229141
       10 |         308     .275974    .2988213
       11 |         232    .4568965    .3782594
       12 |         192    .4427083    .4634854
       13 |         159     .591195    .5602076
       14 |          77    .5584416    .6408411
      15+ |         180    .6777778     .789714
-------------------------------------------
```

Smoothing

- Alternative graphical approach
- Draw smooth curve of mortality by predicted probability
- Should look like straight line
- Assessed in test and validation set

Smoothed Calibration



Summary

- See classic overfitting pattern
  - mortality lower than pred at high end
  - mortality higher than pred at low end
- Large calibrations at high end
  - represent 5% of the prediction
  - fixing would complicate model

Validation

- Calibration: examination of model spec
  - largely internal
- Discrimination: measure separation
  - largely external
- Calibration can be fixed
- Discrimination dictates utility of model