

Multiple Imputation

VGSM, Chapter 11

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Last Week

- Missing data grouped by relationship between missingness and its value
- MCAR/MAR and NMAR
- MAR is the assumption where most action is
- Motivates multiple imputation
- Model-based guesses at missing data

UNOS Dataset

- Data on 9775 pediatric kidney transplants from 1990-2002
- Followed for transplant outcomes
465 deaths
- Predictors: age, age of donor, year, HLA loci, txtype
- Cold ischemia time: missing in 23%

Multiple Imputation

1. Assume data MAR
2. Develop predictive model for missing data
3. Sample from predictive model
4. Make multiple complete datasets
5. Analyze datasets
6. Synthesize results

Selecting MAR Variables

- Bias is the biggest threat
 - More predictors make MAR plausible
 - Include the outcome
- Unimportant covariates: little cost
 - probably not much gained
- Any predictor in model -- include impute
 - possible some that aren't

Modeling Choice

- Specify a parametric model for imputations
- cold_ischemia: continuous variable but also has many "0" s
- Many missing values like to be "0"
- Difficult distribution to model
- Try Normal distribution

Predictive Model

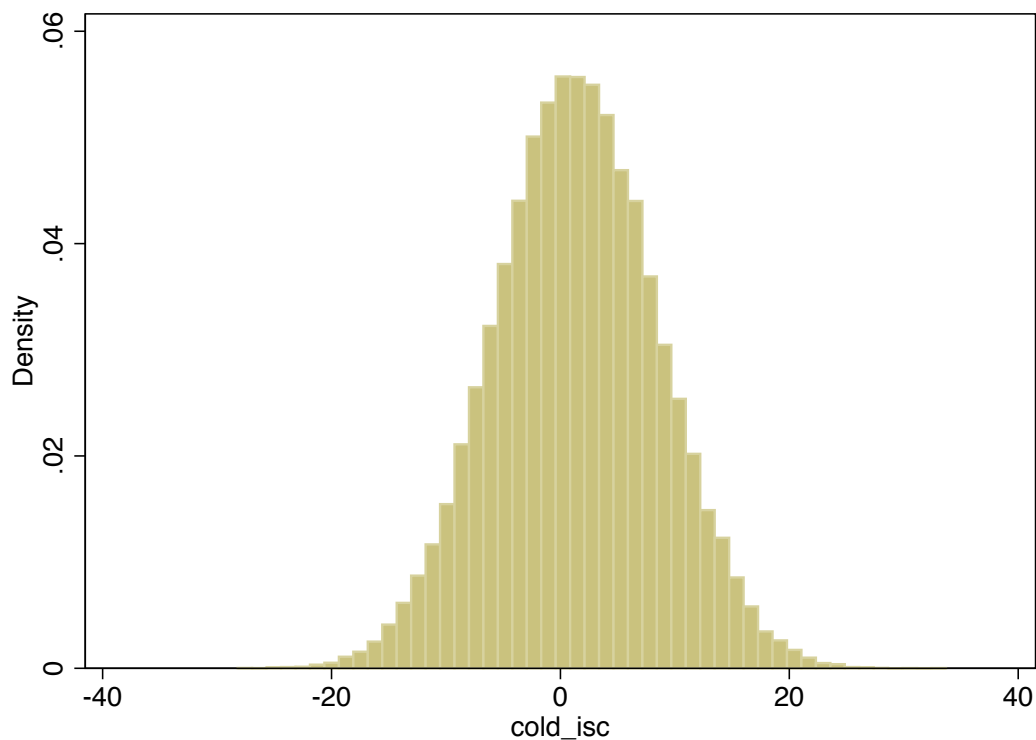
```
reg cold_isc txttype death i.hla age_don age i.hlamat  
_ihlamat_0-6 (naturally coded; _ihlamat_0 omitted)
```

Source	SS	df	MS	Number of obs =	7347
Model	606123.292	10	60612.3292	F(10, 7336) =	1208.63
Residual	367898.979	7336	50.1498062	Prob > F =	0.0000
				R-squared =	0.6223
				Adj R-squared =	0.6218
Total	974022.271	7346	132.592196	Root MSE =	7.0817

cold_isc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
txttype	18.62832	.2249336	82.82	0.000	18.18738	19.06925
death	.6670558	.388197	1.72	0.086	-.0939219	1.428034
hlamat						
1	.9133429	.3598258	2.54	0.011	.2079808	1.618705
2	.7184059	.3568126	2.01	0.044	.0189507	1.417861
3	1.232046	.3520939	3.50	0.000	.5418412	1.922252
4	1.377349	.395934	3.48	0.001	.6012044	2.153493
5	1.955531	.5342044	3.66	0.000	.9083367	3.002725
6	2.253536	.5631313	4.00	0.000	1.149637	3.357436
age_don	.0124535	.0068079	1.83	0.067	-.000892	.0257989
age	.028418	.0162956	1.74	0.081	-.003526	.0603619
_cons	-.5646455	.4347184	-1.30	0.194	-1.416818	.2875275

Normal Imputations

Mean 1.4. SD =7



Misspecified Distribution

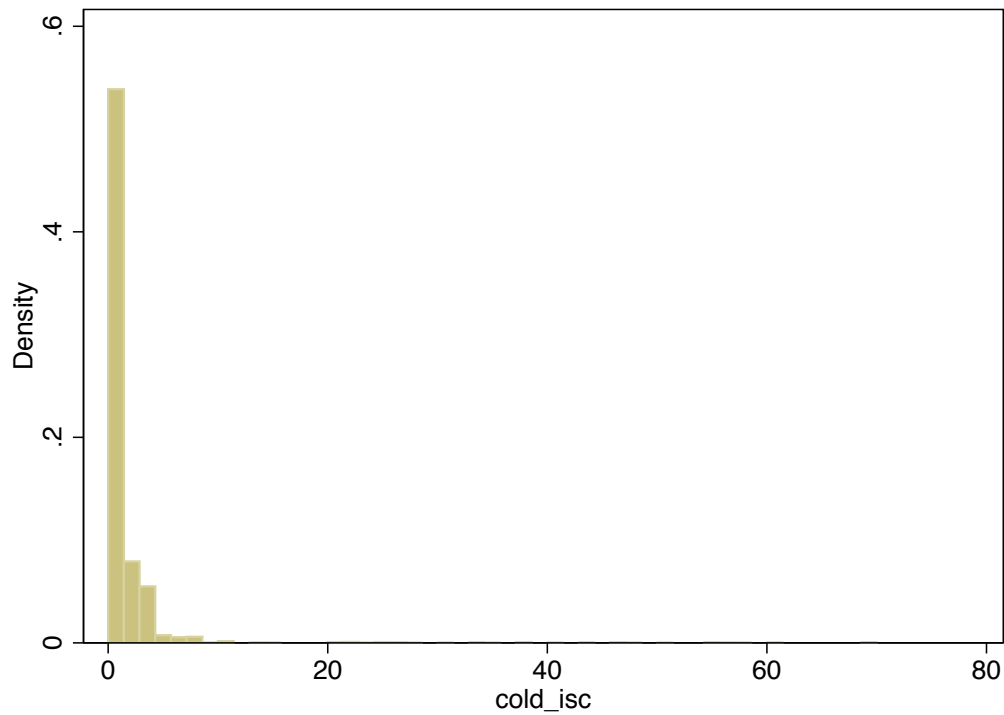
- Minimal effect
- Most important to get moments correct
means, SD, covariance of imputed data
- Unappealing to get implausible values
- Option to force plausible value

Predictive Means Matching

- Linear regression fit to missing data
$$E(Y_{\text{mis}}) = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p$$
- Sample from “posterior” of β | Data
- With each beta calculate predictions
for all observations
- Impute Y_{mis} from observed values k
nearest neighbors
- Forces plausible value
must specify k — *Stata default, $k=1$*

PMM Imputations

Mean 1.4, SD =4



Missing Data

Variable	Obs	Mean	Std. Dev.	Min	Max
hlamat	9541	2.57363	1.378919	0	6
age_don	9662	31.29952	13.38866	0	73
age	9766	11.64653	5.291385	0	18
cold_isc	7525	10.85967	11.51735	0	72
death	9775	.0475703	.2128662	0	1
prevtx	9702	.1443001	.3514119	0	1
txtype	9775	.4733504	.4993148	0	1

Multiple Missing Variable

- Can be handled by multiple imputation
- Means developing a model for several variable simultaneous
- Two main ways to do this
 - assume variables are normal
so called multivariate normal
 - “chained” regression equal
so called iterative chained equations

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Multivariate Normal Imputation

- Treats all variables as normal
- Imputes using linear regression
imputes well for linear
- Can be coaxed to do binary and ordinal
- Can't impute for unordered categorical

Chained Imputation

- Regression model for each missing variable
- “ICE”: iterative chained equations
- Combined for multivariate imputations
- Builds a multivariate distribution
- Separate models: gives big flexibility

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Possible Imputation Models

- Continuous: regress, pmm
- Binary: logit
- Ordinal: ologit
- Nominal: mlogit
- Discrete: poisson, nbreg
- Continuous+threshold: truncreg

separate models for each missing variable

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3. Make multiple complete datasets

Imputing in Stata

- Extensive suite of capabilities
- Data management
- Creating imputations
- Analyzing and synthesizing

mi package

- Powerful package for missing data
- Extensive capabilities
lots of choices, appear esoteric
- Can handle multiple missing predictors
- Presents results in easy to read format

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mi set

- Must declare data to be “mi set”
- `mi set mlong`
- Stores the imputations in long format
just governs how imputations are stored
- There is also `mi stset` & `mi xtset`

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mi register

- Must declare variable as needing or not needing imputation
- `mi register imputed cold_isc`
- `mi register regular death hlamat age_don age`
- Says `cold_isc` will be imputed and others not

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mi impute

- This command actually does the imputations
- Can impute single or multiple variables
- Specifies:
 - predictors used for imputation
 - statistical model for imputation
 - number of imputations

imputing and analysis happen in separate steps

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- Specifies:
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imputing and analysis happen in separate steps

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Imputing Cold Ischemia

- `mi impute regress cold_isc = death txtype
i.hlamat age_don age, add(100) rseed(123)`
- Use linear regression to impute cold_isc
- Predictors: death, txtype, etc
- `add(100) = 100 imputations`
- `rseed( ) = sets a random seed`
*means you can replicate imputations
very very useful*

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Imputation Output

```
. mi impute regress cold_isc = hlamat age_don age death prevtx txttype  
, add(100) rseed(123)
```

```
Univariate imputation          Imputations =      100  
Linear regression              added  =      100  
Imputed: m=1 through m=100    updated =         0
```

Variable	Observations per m			Total
	Complete	Incomplete	Imputed	
cold_isc	7347	2020	2020	9367

(complete + incomplete = total; imputed is the minimum across m of the number of filled-in observations.)

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Predictive Mean Matching

```
. mi impute pmm cold_isc = hlamat age_don age death prevtx txttype,  
add(100) rseed(123) dots knn(1)
```

```
Univariate imputation          Imputations =      100  
Predictive mean matching        added  =      100  
Imputed: m=1 through m=100    updated =         0
```

Variable	Observations per m			Total
	Complete	Incomplete	Imputed	
cold_isc	7347	2020	2020	9367

(complete + incomplete = total; imputed is the minimum across m of the number of filled-in observations.)

MI Dataset — 20 Imputations

tabulate _mi_m

_mi_m	Freq.	Percent	Cum.	
0	9,367	18.82	18.82	← original data
1	2,020	4.06	22.88	
2	2,020	4.06	26.94	
3	2,020	4.06	31.00	
4	2,020	4.06	35.06	
5	2,020	4.06	39.12	
6	2,020	4.06	43.18	
7	2,020	4.06	47.23	
8	2,020	4.06	51.29	
9	2,020	4.06	55.35	
10	2,020	4.06	59.41	← imputation "10"
11	2,020	4.06	63.47	
12	2,020	4.06	67.53	
13	2,020	4.06	71.59	
14	2,020	4.06	75.65	
15	2,020	4.06	79.71	
16	2,020	4.06	83.76	
17	2,020	4.06	87.82	
18	2,020	4.06	91.88	
19	2,020	4.06	95.94	
20	2,020	4.06	100.00	
Total	49,767	100.00		

9,347 record dataset has 49,767 records from 20 impute

Chained Syntax

- mi register imputed hlamat age_don age cold_isc prevtx death txttype
register everything
- mi impute chained (pmm, knn(1)) cold_isc (ologit) hlamat (regress) age_don (regress) age (logit) prevtx = death txttype, add(100) rseed(897)
- (method) variable = nonmissing
- Very flexible syntax

Conditional Models

```
mi impute chained (pmm, knn(1)) cold_isc (ologit) hlamat (regress) age_don ///
> (regress) age (logit) prevtx = death txttype, add(100) rseed(897) replace dots
```

Conditional models:

```
age: regress age i.prevtx age_don i.hlamat cold_isc death txttype
prevtx: logit prevtx age age_don i.hlamat cold_isc death txttype
age_don: regress age_don age i.prevtx i.hlamat cold_isc death txttype
hlamat: ologit hlamat age i.prevtx age_don cold_isc death txttype
cold_isc: pmm cold_isc age i.prevtx age_don i.hlamat death txttype , knn(1)
```

Performing chained iterations:

imputing m=1 through m=100

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Imputations

```
Multivariate imputation          Imputations =      100
Chained equations                 added =      100
Imputed: m=1 through m=100       updated =       0
```

```
Initialization: monotone         Iterations =      100
                                burn-in =       10
```

```
cold_isc: predictive mean matching
hlamat: ordered logistic regression
age_don: linear regression
age: linear regression
prevtx: logistic regression
```

Variable	Observations per m			Total
	Complete	Incomplete	Imputed	
cold_isc	7525	2250	2250	9775
hlamat	9541	234	234	9775
age_don	9662	113	113	9775
age	9766	9	9	9775
prevtx	9702	73	73	9775

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Multiple Imputation

1. Assume data MAR
2. Develop predictive model for missing data
3. Sample from predictive model
4. Make multiple complete datasets
5. Analyze datasets
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Analyze and Synthesize

Source of Uncertainty

- Predictive distn of cold_isc
 - $\text{cold_isc}_i = \alpha + \beta_1 * x_1 + \beta_2 * x_2 + \dots + \beta_p * x_p + \epsilon_i$
- Sources of uncertainty
 1. prediction: ϵ_i
 2. parameters: $\beta_1, \beta_2, \dots, \beta_p$
 3. form of model: *predictors, transformation*
- Imputation deals with 1 & 2

Result of 1st Imputation

```

Logistic regression                                Number of obs   =      9,775
                                                    LR chi2(11)     =     103.06
                                                    Prob > chi2     =      0.0000
Log likelihood = -1818.4095                        Pseudo R2      =      0.0276
  
```

death	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
cold_isc	1.013205	.0060088	2.21	0.027	1.001497	1.025051
age_don	.9901152	.0038156	-2.58	0.010	.982665	.9976219
age	.9485558	.0084687	-5.92	0.000	.932102	.9653002
prevtx	1.353609	.1697865	2.41	0.016	1.058583	1.730858
txtype	1.184901	.2099091	0.96	0.338	.8373152	1.676777
hlamat						
1	1.039716	.1861719	0.22	0.828	.7319779	1.476832
2	.8916278	.1633056	-0.63	0.531	.6227049	1.276688
3	.8342293	.1559687	-0.97	0.332	.5782872	1.203448
4	.7531256	.1692512	-1.26	0.207	.4848144	1.169929
5	.8627274	.2608962	-0.49	0.625	.476939	1.560574
6	.915812	.2973173	-0.27	0.786	.4846908	1.730405
_cons	.1004935	.0226025	-10.22	0.000	.0646681	.1561658

Imputations

Imputation #	OR txttype	SE(OR)
1	1.18	0.20
2	1.12	0.20
3	1.17	0.21
4	1.20	0.21
5	1.20	0.21
mean	1.17	0.21
SD	0.03	0.01

Combining Imputations

K = Number of Imputations = 5

$$\begin{aligned} \text{MI Estimate} &= (OR_1 + OR_2 + \dots + OR_K) / K \\ &= 1.17 \end{aligned}$$

$$\begin{aligned} \text{MI Variance} &= (1 + 1/K) * \{ SE(OR_k) \}^2 + \\ &\quad \{ \text{mean}(\{ SE_k(OR) \}^2) \} \\ &= (1 + 1/5) * \{ 0.033 \}^2 + \\ &\quad (0.206)^2 \\ &= .044 \end{aligned}$$

$$\text{MI SE} = \text{sqrt}(\text{MI Variance}) = 0.21$$

Imputed Regression

the analysis step

- mi estimate, or : logistic death cold_isc
txty i.hlam age_don age
- Fits a logistic regression with imputations
based on the last commands
- After “:” usual commands
- Need “or” to get odds ratios

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Results

```
mi estimate, eform: logistic death cold_isc age_don age prevtx txtype i.hlamat
```

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.012858	.0062722	2.06	0.039	1.00062	1.025246
age_don	.9904244	.0038148	-2.50	0.012	.9829756	.9979296
age	.9485306	.0084643	-5.92	0.000	.9320852	.9652662
prevtx	1.357085	.1709674	2.42	0.015	1.060149	1.73719
txtype	1.17499	.2110693	0.90	0.369	.8262195	1.670985
hlamat						
1	1.035961	.1870476	0.20	0.845	.7271112	1.475999
2	.8521283	.1601151	-0.85	0.395	.5894205	1.231926
3	.80732	.1519525	-1.14	0.256	.558221	1.167576
4	.6858873	.1610081	-1.61	0.109	.4326734	1.08729
5	.8363426	.2538196	-0.59	0.556	.4613647	1.516087
6	.913864	.2937117	-0.28	0.779	.4867319	1.715826
_cons	.1035627	.0233885	-10.04	0.000	.066521	.1612307

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PMM Results

```
mi estimate, or: logistic death cold_isc age_don age year prevtx txttype i.hlamat
```

Multiple-imputation estimates	Imputations	=	5
Logistic regression	Number of obs	=	9775

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.010375	.0071917	1.45	0.154	.9959816	1.024976
age_don	.9905783	.0038174	-2.46	0.014	.9831245	.9980887
age	.9480711	.0084704	-5.97	0.000	.9316139	.964819
prevtx	1.366319	.174341	2.45	0.014	1.063889	1.754719
txttype	1.233898	.2356188	1.10	0.272	.8466503	1.798269
hlamat						
1	1.043439	.1886803	0.24	0.814	.7319728	1.487438
2	.8503009	.1644444	-0.84	0.402	.5813449	1.243688
3	.8090594	.1524061	-1.12	0.261	.5592625	1.170429
4	.6948287	.160795	-1.57	0.116	.4413473	1.093893
5	.8621357	.2653442	-0.48	0.630	.4713913	1.576775
6	.9053687	.294839	-0.31	0.760	.4781864	1.71417
_cons	.1033468	.0233963	-10.03	0.000	.0663109	.1610677

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Data Management

- Imputing creates phantom observations
- Data is only good for imputations
- Useful command -- “mi extract”
creates a dataset based on one
“imputation”
- *mi extract 0, clear*
back to regular incomplete data
- *mi extract l0, clear*
complete dataset using imputation #l0
allows you to check imputation set

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Imputation I0

```
. mi extract 10, clear

. logistic death cold_isc age_don age prevtx txttype i.hlamat

Logistic regression                                Number of obs   =       9775
                                                    LR chi2(11)        =      109.38
                                                    Prob > chi2         =       0.0000
Log likelihood = -1815.2502                        Pseudo R2          =       0.0292
```

death	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
cold_isc	1.013805	.0059296	2.34	0.019	1.00225	1.025494
age_don	.9906357	.0037988	-2.45	0.014	.9832182	.9981092
age	.9483539	.0084602	-5.94	0.000	.9319163	.9650814
prevtx	1.463773	.1797112	3.10	0.002	1.15072	1.861993
txttype	1.124474	.1991213	0.66	0.508	.7947287	1.591035
hlamat						
1	1.048589	.1855962	0.27	0.789	.7412181	1.483422
2	.835029	.1530105	-0.98	0.325	.5830792	1.195847
3	.7858572	.1462116	-1.30	0.195	.5457258	1.131652
4	.6671839	.1515587	-1.78	0.075	.4274497	1.041372
5	.8654381	.255724	-0.49	0.625	.4849743	1.544377
6	.858816	.2780721	-0.47	0.638	.4552957	1.619969
_cons	.1043656	.0233851	-10.09	0.000	.0672712	.1619143

but this dropped all other imputations₄₁

mi xeq simpler approach

- mi xeq I0: summ cold_isc
summarize cold_isc from imputation I0

```
m=10 data:
-> summ cold_isc
```

Variable	Obs	Mean	Std. Dev.	Min	Max
cold_isc	9,775	10.14958	11.4468	0	72

mi xeq

examining results

```
mi xeq 10: logistic death cold_isc age_don age prevtx txttype i.hlamat
```

```
-> logistic death cold_isc age_don age prevtx txttype i.hlamat
```

```
Logistic regression                                Number of obs   =      9775
                                                    LR chi2(11)    =     109.38
                                                    Prob > chi2    =      0.0000
Log likelihood = -1815.2502                        Pseudo R2      =      0.0292
```

death	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]	
cold_isc	1.013805	.0059296	2.34	0.019	1.00225	1.025494
age_don	.9906357	.0037988	-2.45	0.014	.9832182	.9981092
age	.9483539	.0084602	-5.94	0.000	.9319163	.9650814
prevtx	1.463773	.1797112	3.10	0.002	1.15072	1.861993
txttype	1.124474	.1991213	0.66	0.508	.7947287	1.591035
hlamat						
1	1.048589	.1855962	0.27	0.789	.7412181	1.483422
2	.835029	.1530105	-0.98	0.325	.5830792	1.195847
3	.7858572	.1462116	-1.30	0.195	.5457258	1.131652
4	.6671839	.1515587	-1.78	0.075	.4274497	1.041372
5	.8654381	.255724	-0.49	0.625	.4849743	1.544377
6	.858816	.2780721	-0.47	0.638	.4552957	1.619969
_cons	.1043656	.0233851	-10.09	0.000	.0672712	.1619143

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mi extract

- Useful command
- Analysis not supported in mi estimate:
- Use “mi extract, `k’” looped for k
- Save results
- Analyze using published formulae
not optimal, but possible

How Many Imputations

- Feasibly many:
"50% missingness 5 draws is highly efficient"
- True but gives poor standard errors
better guideline 20-40
- Chained Imputation on UNOS data
 - 20 imputations in 2 min.
 - 50 imputations in 5 min.
 - 500 imputations in 50 min.

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50 Imputations

```
. mi estimate, eform: logistic death cold_isc age_don age prevtx txttype i.hlamat
```

```
Multiple-imputation estimates      Imputations      =      50
Logistic regression                Number of obs    =     9775
                                   Average RVI        =     0.0371
                                   Largest FMI         =     0.1946
DF adjustment: Large sample        DF: min         =    1310.53
                                   avg          =   894264.52
                                   max          =   5923449.56
Model F test: Equal FMI           F( 11,351411.5)=     9.02
Within VCE type: OIM              Prob > F         =     0.0000
```

death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc	1.009856	.0066926	1.48	0.139	.9968112	1.023071
age_don	.9905272	.0038156	-2.47	0.013	.983077	.9980339
age	.9482437	.0084732	-5.95	0.000	.9317811	.9649971
prevtx	1.376873	.1752271	2.51	0.012	1.072907	1.766954
txttype	1.237584	.2299089	1.15	0.251	.8598254	1.781308
hlamat						
1	1.048506	.1886219	0.26	0.792	.7369546	1.491768
2	.8519965	.1592486	-0.86	0.391	.5906521	1.228977
3	.8045399	.1525117	-1.15	0.251	.5548634	1.166565
4	.6962644	.1608806	-1.57	0.117	.4426748	1.095125
5	.8605673	.2608053	-0.50	0.620	.4751321	1.558674
6	.9041763	.2940166	-0.31	0.757	.4780357	1.710196

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500 Imputations

```
. mi estimate, eform: logistic death cold_isc age_don age prevtx txttype i.hlamat
```

Multiple-imputation estimates	Imputations	=	500
Logistic regression	Number of obs	=	9775
	Average RVI	=	0.0381
	Largest FMI	=	0.1853
DF adjustment: Large sample	DF: min	=	14556.70
	avg	=	1.19e+07
	max	=	7.27e+07
Model F test: Equal FMI	F(11, 3.4e+06)=		9.02
Within VCE type: OIM	Prob > F	=	0.0000

	death	exp(b)	Std. Err.	t	P> t	[95% Conf. Interval]	
cold_isc		1.010071	.0066465	1.52	0.128	.9971269	1.023184
age_don		.9905008	.0038143	-2.48	0.013	.9830531	.9980049
age		.9482384	.0084729	-5.95	0.000	.9317765	.9649913
prevtx		1.373944	.1765813	2.47	0.013	1.067999	1.767533
txttype		1.2328	.2287713	1.13	0.259	.856902	1.773593
hlamat							
1		1.049783	.1892282	0.27	0.788	.7373369	1.494627
2		.8522425	.1586011	-0.86	0.390	.5917734	1.227357
3		.8053358	.1526196	-1.14	0.253	.5554756	1.167587
4		.6966721	.159863	-1.58	0.115	.44433	1.092323
5		.8545295	.2592478	-0.52	0.604	.4715075	1.548694
6		.9090107	.2962998	-0.29	0.770	.4798573	1.721971

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Approach to Imputation Model

- Inclusivity over parsimony
more predictor: MAR more plausible
- Will remove biases -- primary objective
- Two things to include: outcome, predictor in your analytic model
- Functional form tends not to be critical

Making Up Data?

- No.
- Making a model for missingness
- Making a guess at missing data
- Fully representing uncertainty
 - this is the key to it's validity

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Maximum Likelihood (ML)

Based on implication #1

- Maximum likelihood valid for MAR data
as long as CMAR variables incorporated
- Works especially well for missing outcome
especially in longitudinal analysis
where attrition depends on previous values
- ML often fit with the EM algorithm
EM also iterative fills in missing data
- Imputation & analysis models consistent
MI and maximum like very similar

Disadvantages of ML

- Cumbersome for missing predictors don't usual model their distribution
- Awkward if CMAR variables are not a part of desired
- Lacks flexibility of MI

Inverse Weighting

Based on implication #1

- Model $\text{pr}(\text{non missingness} | C_{\text{MAR}})$
weight by chance of being observed
- Two step approach
not implemented in software
easy to mess up (my experience)
- Clear connection to complete data analysis
- Inefficient

3 Methods

- Same MAR assumption
- Inverse weight:
model missingness, data model flexible
- MI/ML:
model data, missingness model unspecified