

Interactions in regression

Binary Variable Interactions

Outcome = Systolic BP

No interaction

	Drink = 0	Drink = 1
Smoke = 1	$\mu + a$	$\mu + a + b$
Smoke = 0	μ	$\mu + b$

μ = average BP in reference group (smoke=0 and drink=0)
a = Smoking effect, b = Drinking effect

Binary Variable Interactions

Outcome = Systolic BP

Interaction

	Drink = 0	Drink = 1	
Smoke = 1	$\mu + a$	$\mu + a + b + c$	$c \neq 0$
Smoke = 0	μ	$\mu + b$	

a = Smoking effect,
c = interaction

b = Drinking effect,
c > 0: synergistic;
c < 0: antagonistic

Binary Variable Interaction with a Continuous Variable

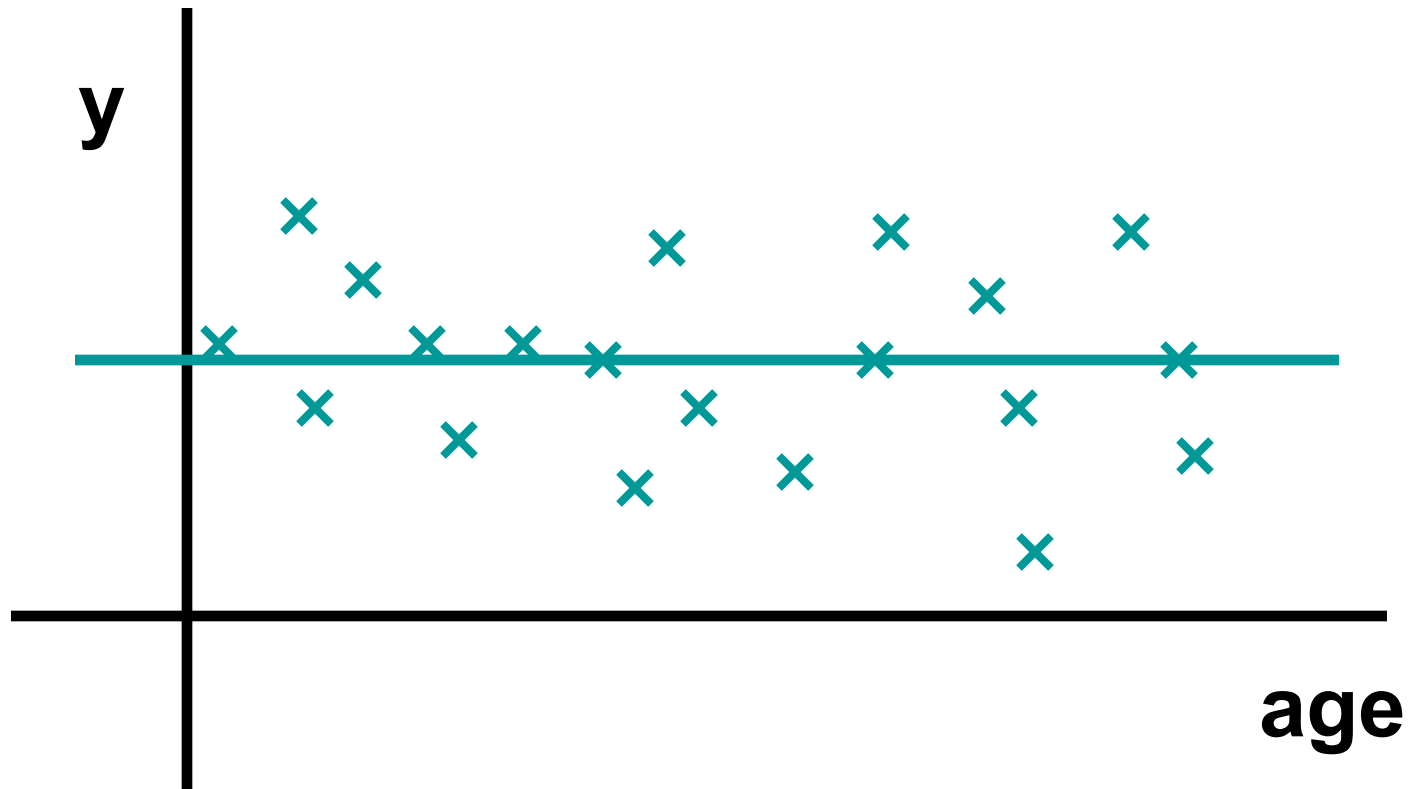
Case 1

y = cognition score - outcome

$x_{Dx} = 1$, AD (Alzheimer's)

$x_{Dx} = 0$, HC (Healthy Control)

$$y = \alpha + \epsilon$$



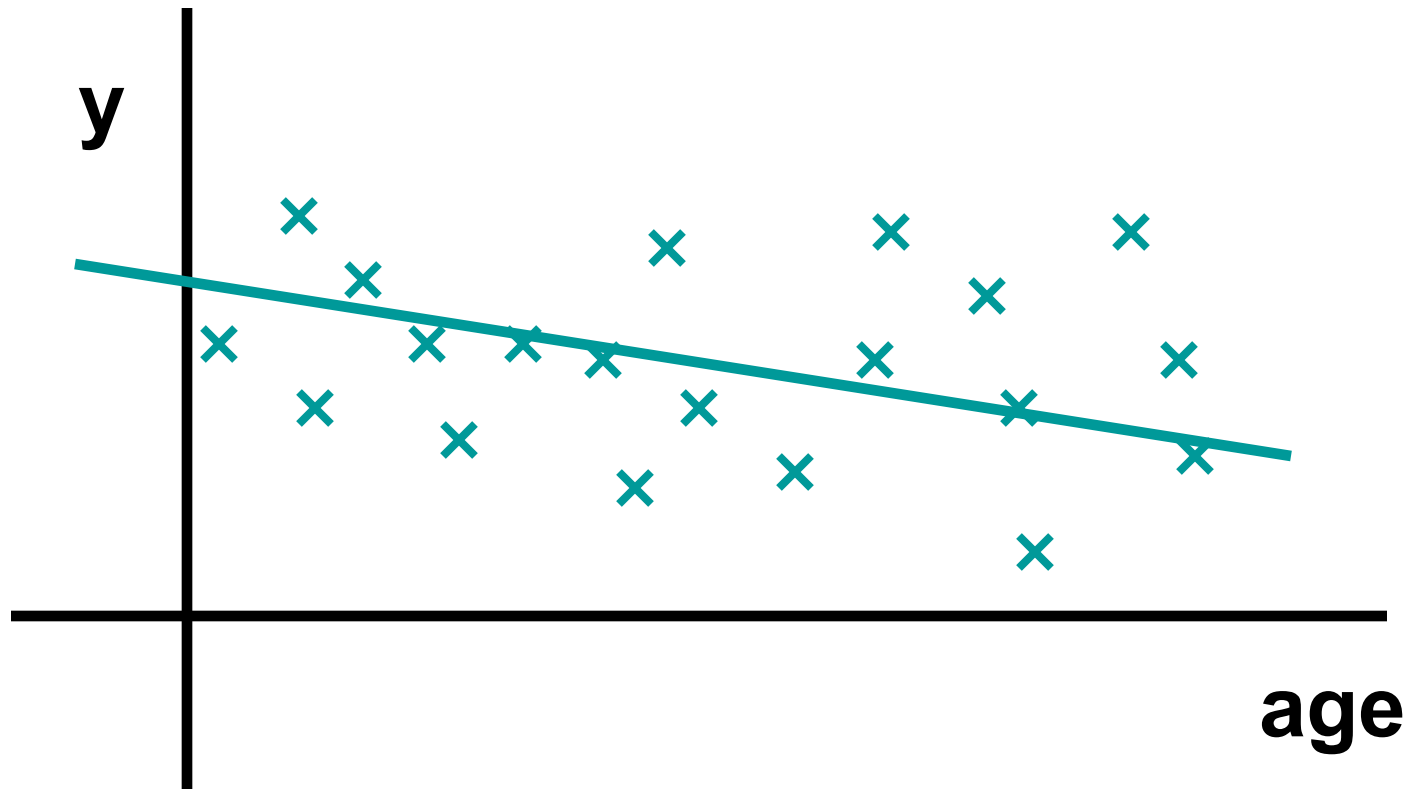
Case 2

y = cognition score - outcome

$x_{Dx} = 1$, AD (Alzheimer's)

$x_{Dx} = 0$, HC (Healthy Control)

$$y = \alpha + \beta_{age} x_{age} + \epsilon$$



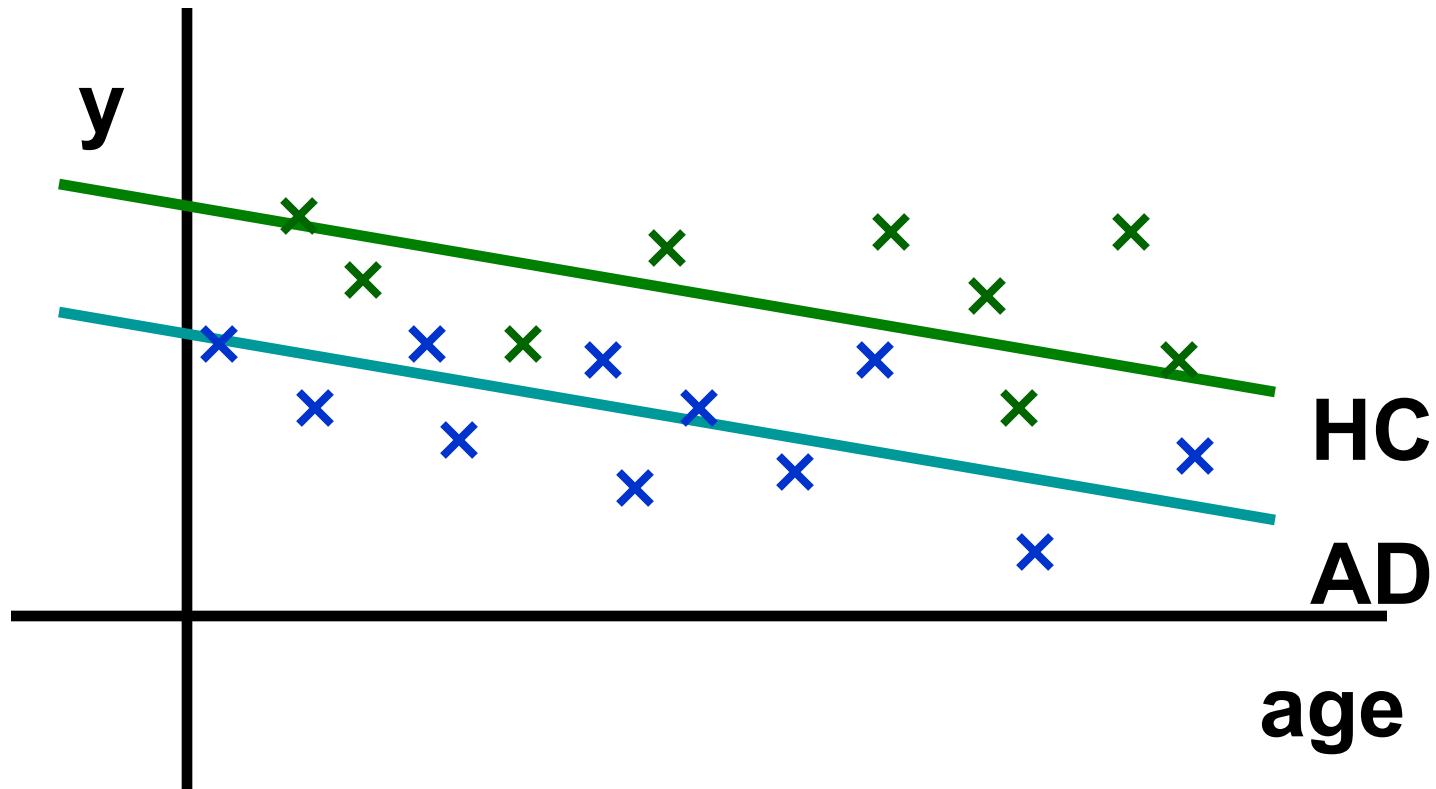
Case 3

y = cognition score - outcome

$x_{Dx} = 1$, AD (Alzheimer's)

$x_{Dx} = 0$, HC (Healthy Control)

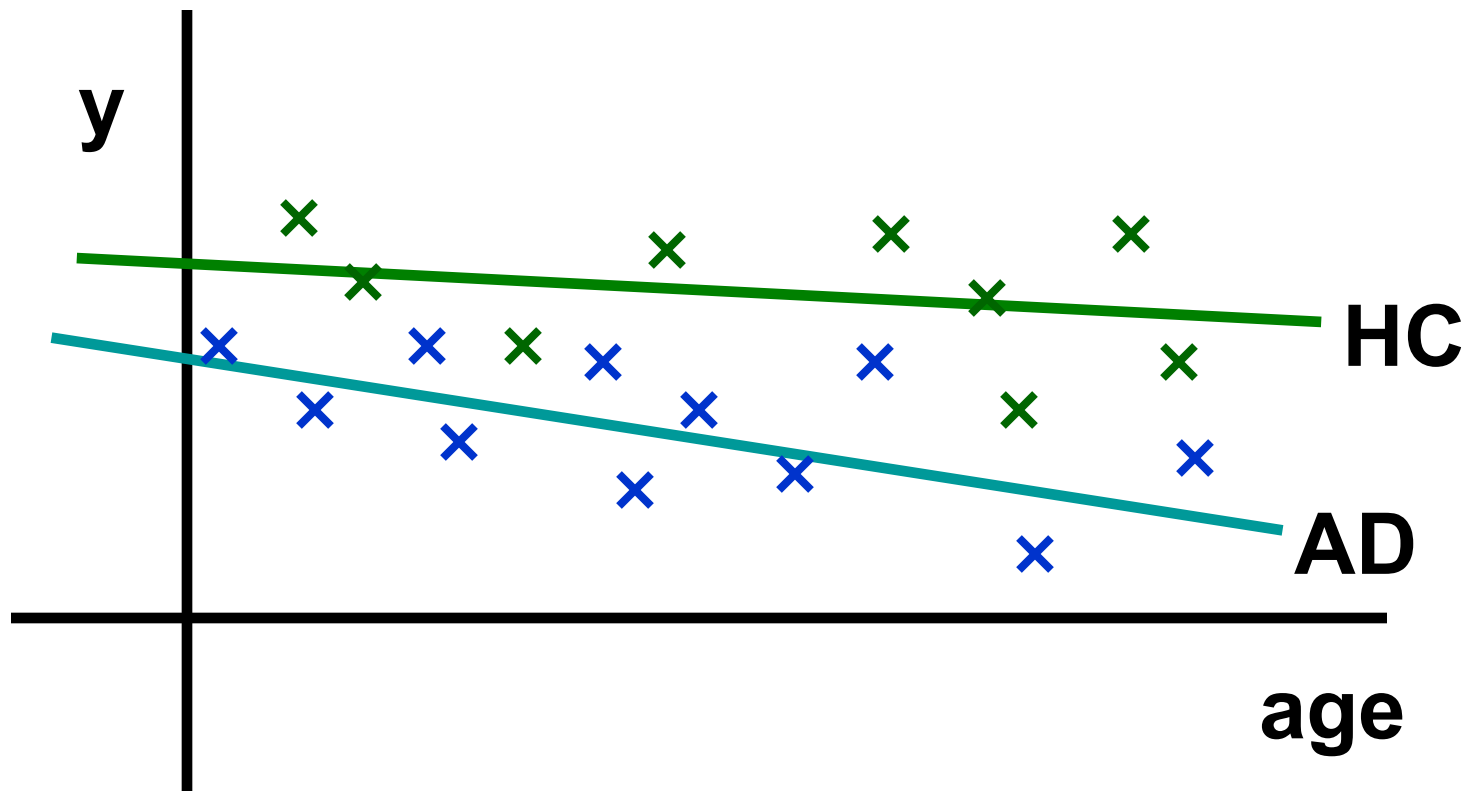
$$y = \alpha + \beta_{age} x_{age} + \beta_{Dx} x_{Dx} + \epsilon$$



Case 4

$$y = \alpha + \beta_{age} x_{age} + \beta_{Dx} x_{Dx} + \beta_{inter} x_{age}x_{Dx} + \epsilon$$

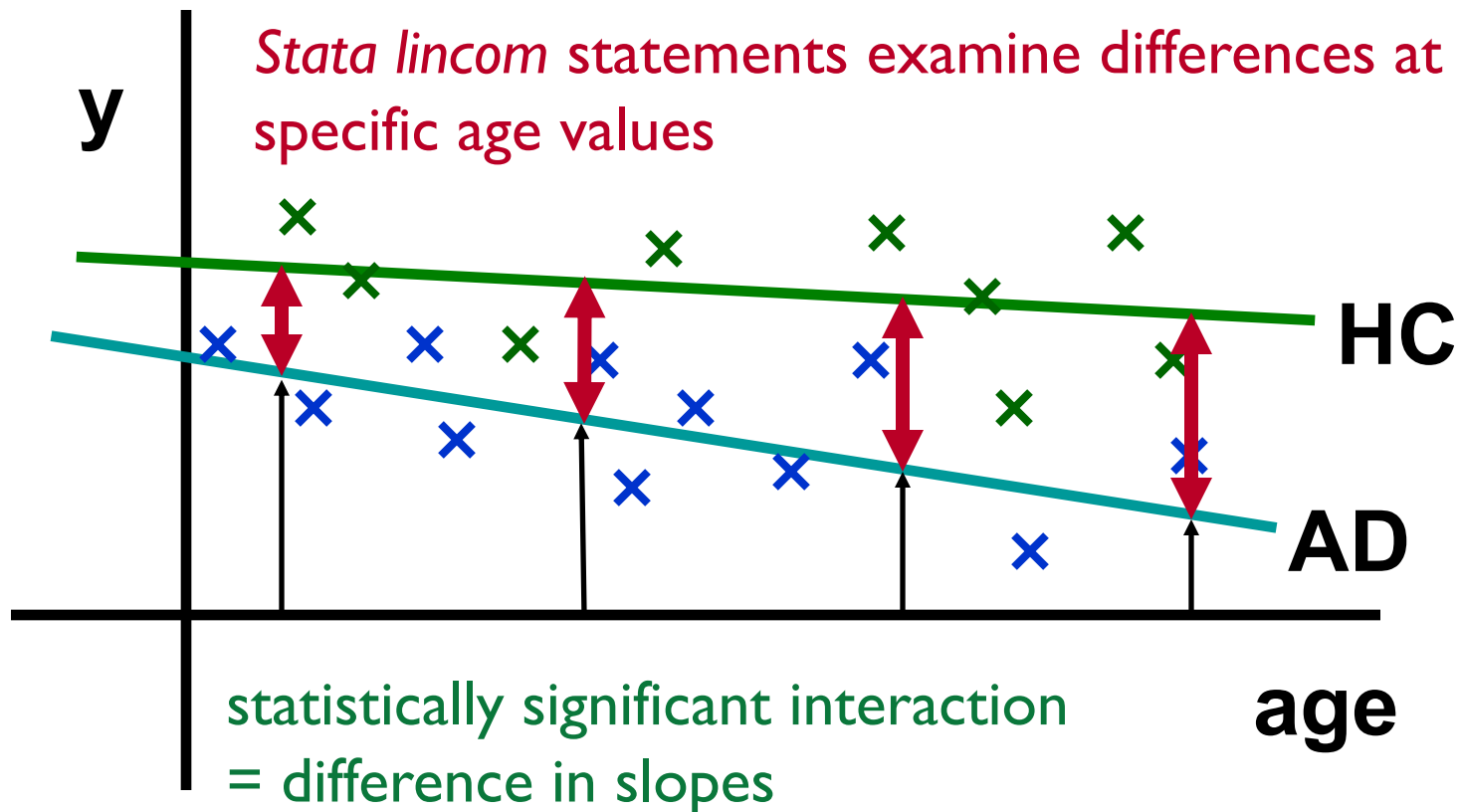
$$\beta_{age} \neq 0, \beta_{Dx} \neq 0, \beta_{inter} \neq 0$$



Case 4

$$y = \alpha + \beta_{age} x_{age} + \beta_{Dx} x_{Dx} + \beta_{inter} x_{age}x_{Dx} + \epsilon$$

$$\beta_{age} \neq 0, \beta_{Dx} \neq 0, \beta_{inter} \neq 0$$



Note: continuous variable by continuous variable interaction even trickier to interpret – we won't go there