Prediction Models

Biostatistics 210

Model Fit and Complexity

- Log-likelihood (LL) measures closeness of data to model
- -2*Log-likelihood: like a sum of squares *smaller means more variation explained*
- Always goes down with more predictors
- LR test: must go down by 3.84 for 1 predictor to be significant

Aikake Info. Criterion

- Turns out -2*log-likelihood is biased
- Assessed on same data use to minimize
- Unbiased version:
- -2*log-likelihood - 2*K
- K is # parameters in model
- Call the Aikake Info. Criterion (AIC)
- p-value criterion $p < 0.16$

Bayesian Info. Criteria

- A more stringent version
- -2*LL - K*log(n)
- n is number of predictors
- bigger penalty for adding predictor *increases with sample size*

BIC Table

N	Chi2 >	p-value
20	3	0.083
100	4.6	0.032
200	5.3	0.021
500	6.2	0.013
2000	7.6	0.006

Information Criteria

- Choose model with lowest AIC/BIC
- Very big difference
- AIC will allow some non-sig predictors
- BIC will exclude some non-sig predictors
  BIC can lead to underfitting

Building Prediction Models

- Balance utility, prior knowledge
- Predictors powerful, portable, interpretable, accessible
- Mitigate overfitting: BIC, cross-validation, Construct simple score
- Context dependent cutpoints

Prediction w/ Logistic Regression

Predictor $x_1, \ldots\ldots, x_p$

$\downarrow$

Linear Pred. $x_1\beta_1 + \ldots\ldots + x_p\beta_p \longrightarrow$ Score

$\downarrow$

Predicted Probability $$\frac{\exp(\beta_0 + x_1\beta_1 + \ldots\ldots + x_p\beta_p)}{1+\exp(\beta_0 + x_1\beta_1 + \ldots\ldots + x_p\beta_p)}$$

17 df predictor model

coefficients

```
. xi: logistic dead i.agecat male hrsdm hrsca hrslung hrschf bmi smoke bath money walkjog push, coef
i.agecat          _Iagecat_0-6        (naturally coded; _Iagecat_0 omitted)

Logistic regression                               Number of obs   =      11652
                                                  LR chi2(17)     =    2080.12
                                                  Prob > chi2     =     0.0000
Log likelihood = -3132.1774                       Pseudo R2       =     0.2493

------------------------------------------------------------------------------
        dead |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   _Iagecat_1 |   .6345948   .1454731     4.36   0.000     .3494727    .9197169
   _Iagecat_2 |   1.039148   .1413782     7.35   0.000     .7620513    1.316244
   _Iagecat_3 |   1.314136   .1370846     9.59   0.000     1.045455    1.582817
   _Iagecat_4 |   1.689447   .1371779    12.32   0.000     1.420583    1.958311
   _Iagecat_5 |    2.11972   .1429917    14.82   0.000     1.839461    2.399979
   _Iagecat_6 |   2.789075   .1451113    19.22   0.000     2.504662    3.073487
        male |    .712165   .0704824    10.10   0.000     .5740221    .8503079
       hrsdm |   .5781861   .0848568     6.81   0.000     .4118699    .7445023
       hrsca |   .7208027   .0851805     8.46   0.000      .553852    .8877534
     hrslung |   .8225759   .1244037     6.61   0.000     .5787491    1.066403
      hrschf |    .851144    .145293     5.86   0.000     .5663751    1.135913
      bmicat |   .5017212   .0698006     7.19   0.000     .3649147    .6385278
       smoke |   .7204122   .0927079     7.77   0.000     .5387081    .9021163
        bath |   .6721684   .1055242     6.37   0.000     .4653448    .8789919
       money |    .643781   .0961402     6.70   0.000     .4553498    .8322122
     walkjog |   .7358055   .0786537     9.36   0.000     .5816471    .8899639
        push |   .4231833   .0791656     5.35   0.000     .2680215     .578345
       _cons |  -4.903092   .1321857   -37.09   0.000    -5.162171   -4.644013
------------------------------------------------------------------------------
```

Creating a Score

- Mimics the linear predictor (lp)
  if model is right, lp summarizes risk
- Simple score requires categorical predictor
- Recode them so 1=high risk, 0 = low
- Score is sum of points for each predictor
- Score for jth predictor: round($\beta_j/\min(\beta)$)

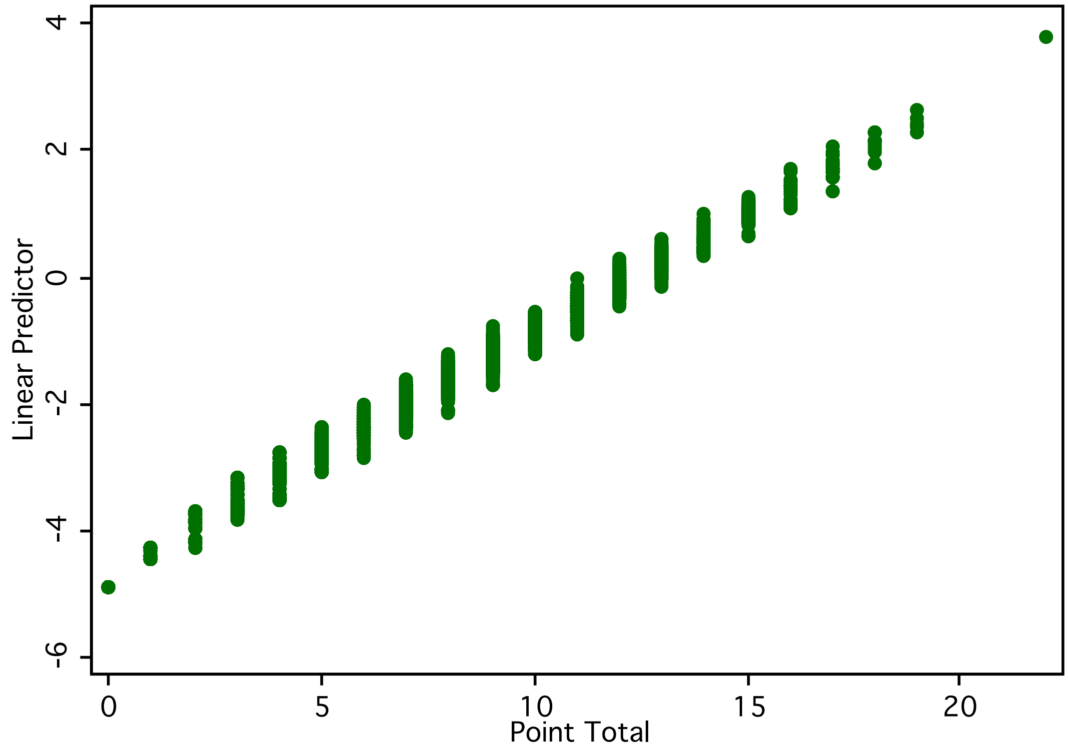
Calculating Points

Push: HR = 1.53
coef = 0.42 (smallest coef)
points = +1 0.42/0.42

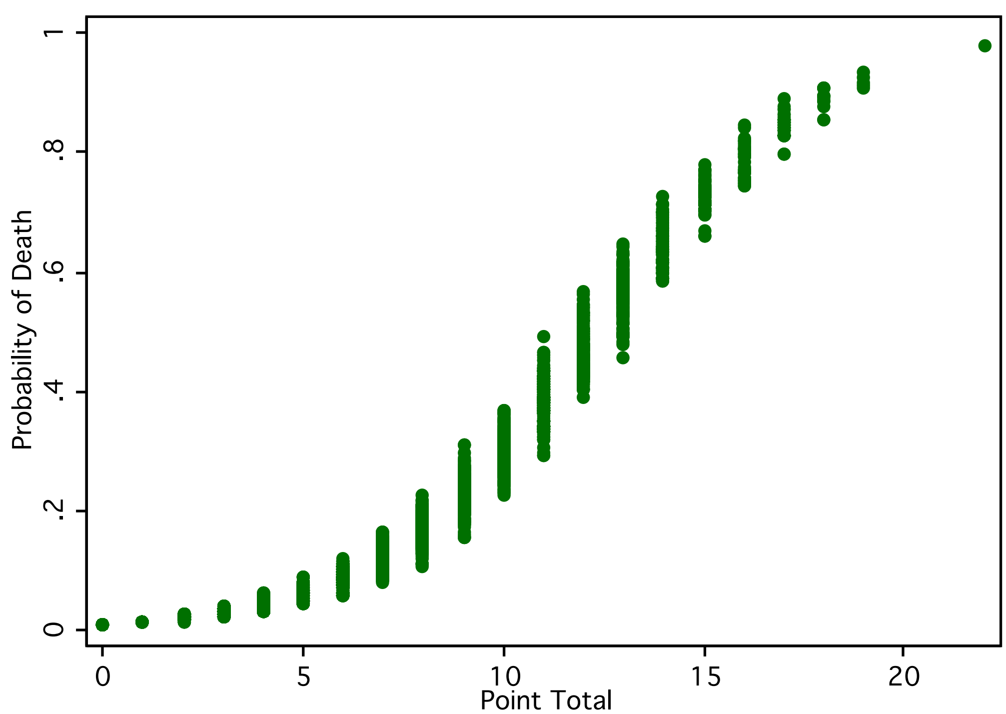
Walk/Jog HR = 2.1
coef = 0.736
points = +2 $0.736/0.42 \approx 2$

Variable	Lee Points
Age 60-64	+1
Age 65-69	+2
Age 70-74	+3
Age 75-79	+4
Age 80-84	+5
Age > 85	+7
Male	+2
Diabetes	+1
Cancer	+2
Lung Disease	+2
Heart Failure	+2
BMI < 25	+1
Smoking	+2
Bathing	+2
Money	+2
Walking	+2
Pushing	+1

Points v. Linear Predictor



Probability of Death



Last Step

- The score be used directly
- $p_{death} \approx \exp(-4.9+0.42*\text{score})$
- Can also be used predict/stratify
- Would need a cutpoint
  Lee uses quantiles 0-5, 6-9, 10-13, ≥14

  What groups to form gets to fundamental issues of why you are making a predictive model

End Result

- Score from 0-23
- Lower score -> lower risk of death
- 90% of people have score of 10 or less
- Is it a good summary of risk?
- Can it predict who will die?
- What does a "7" on the score mean?

Model Validation

Model Validation

Altman and Royston

- Statistical and clinical considerations
  - best model for predictors?
  - accurate enough for purposes?
- Interplay between statistic and context

Statistical Fit

- Discrimination
  - Is it a strong model?
  - Can we effectively predict?
- Calibration
  - Are predictions accurate?

Discrimination

- Difference between groups
- Various measures
  - mean difference in score
  - c statistic

Validation Set

- Total dataset 19,710 people available
- 11,701: east, west, central region US
  learning set
- 8009: south
  test set
- Deliberately non-comparable
- Rationale: *assess portability*

Lee Model
validation set

```
. xi: logistic dead i.agecat male hrsdm hrsca hrslung hrschf bmi smoke bath money walkjog push
i.agecat          _Iagecat_0-6        (naturally coded; _Iagecat_0 omitted)

Logistic regression                               Number of obs   =       7973
                                                  LR chi2(17)     =    1413.63
                                                  Prob > chi2     =     0.0000
Log likelihood = -2423.8528                       Pseudo R2       =     0.2258

------------------------------------------------------------------------------
        dead | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  _Iagecat_1 |   1.537822   .2259016     2.93   0.003       1.1531    2.050903
  _Iagecat_2 |   2.248315   .3291624     5.53   0.000     1.687476    2.995549
  _Iagecat_3 |   2.586506    .381196     6.45   0.000     1.937602     3.45273
  _Iagecat_4 |   4.109389   .5813912     9.99   0.000     3.114228    5.422558
  _Iagecat_5 |   7.608406    1.14937    13.43   0.000     5.658564    10.23013
  _Iagecat_6 |   14.62968   2.233807    17.57   0.000     10.84588    19.73353
        male |   2.166964   .1712289     9.79   0.000     1.856058    2.529949
       hrsdm |   1.761092   .1639891     6.08   0.000     1.467303    2.113704
       hrsca |   1.664218   .1689592     5.02   0.000      1.36393    2.030617
     hrslung |   1.587757   .2179521     3.37   0.001     1.213219     2.07792
      hrschf |   2.136519   .3437415     4.72   0.000     1.558685    2.928569
      bmicat |   1.736036   .1361793     7.03   0.000     1.488635    2.024553
       smoke |   1.594206   .1606953     4.63   0.000     1.308409    1.942429
        bath |   1.474593   .1711166     3.35   0.001     1.174616    1.851179
       money |   1.420027   .1534369     3.25   0.001     1.149006    1.754974
     walkjog |   1.995648   .1817115     7.59   0.000      1.66947    2.385553
        push |    1.49066   .1350732     4.41   0.000     1.248098    1.780363
------------------------------------------------------------------------------
```

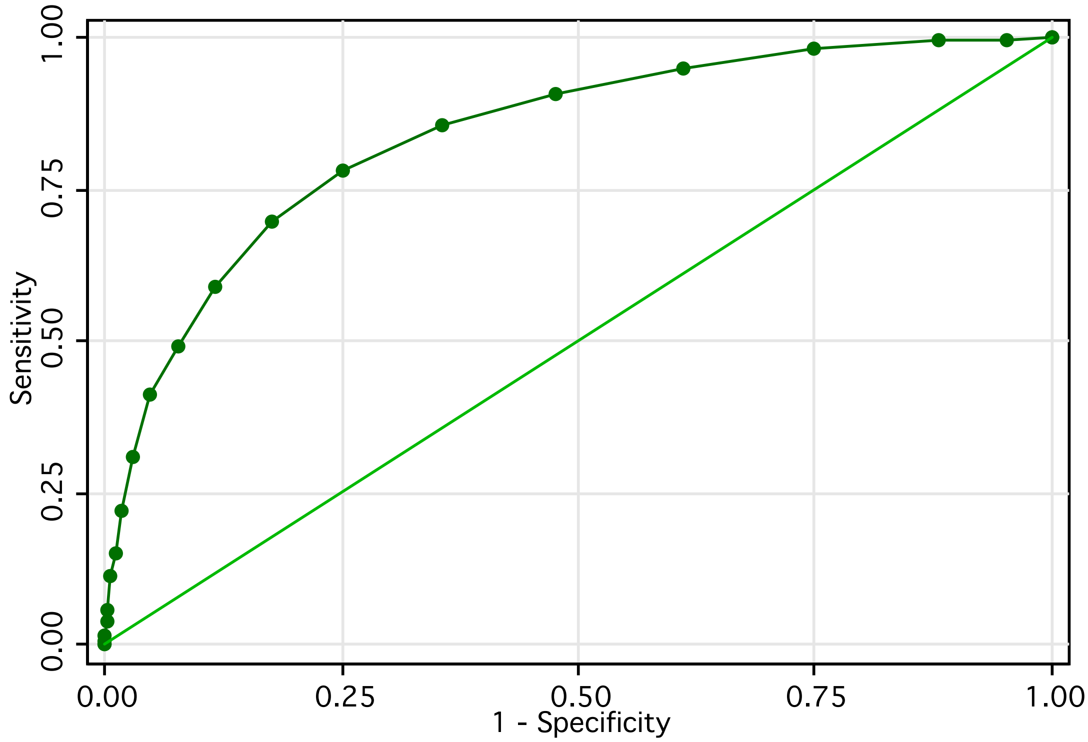
Variable	Training Coef.	Valid Coef.	Ratio
Age 60-64	0.63	0.43	0.68
Age 65-69	1.04	0.81	0.78
Age 70-74	1.31	0.95	0.72
Age 75-79	1.69	1.41	0.84
Age 80-84	2.12	2.03	0.96
Age > 85	2.79	2.68	0.96
Male	0.71	0.77	1.09
Diabetes	0.58	0.57	0.98
Cancer	0.72	0.51	0.71
Lung Disease	0.82	0.46	0.56
Heart Failure	0.85	0.76	0.89
BMI < 25	0.50	0.55	1.10
Smoking	0.72	0.47	0.65
Bathing	0.67	0.39	0.58
Money	0.64	0.35	0.54
Walking	0.74	0.69	0.94
Pushing	0.42	0.40	0.94

Area Under ROC

- Also known as C-statistic
- Summary measure of AUC curve *between 0 and 1*
- Probability that dead person has more points than a living person
- Measures purity of groups
- Clearly related to discrimination

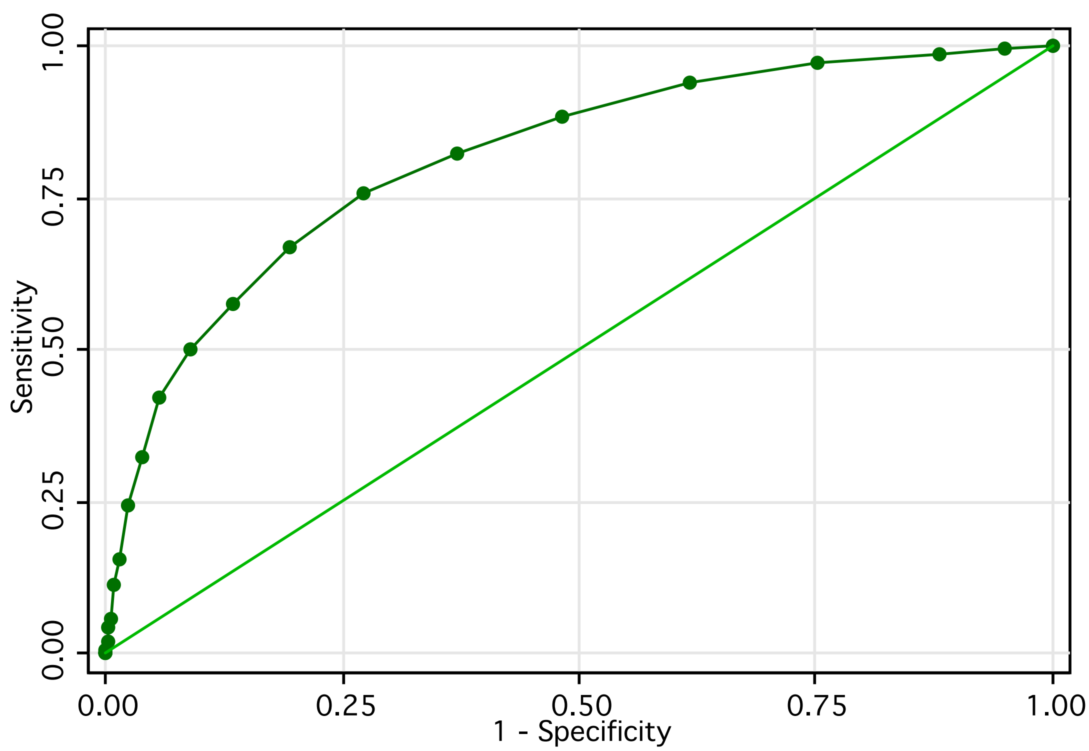
C-statistic (test):
0.8424 (linear predictor), 0.8405 (points)

ROC Lee Model: Learn



Area under ROC curve = 0.8405

ROC: Test

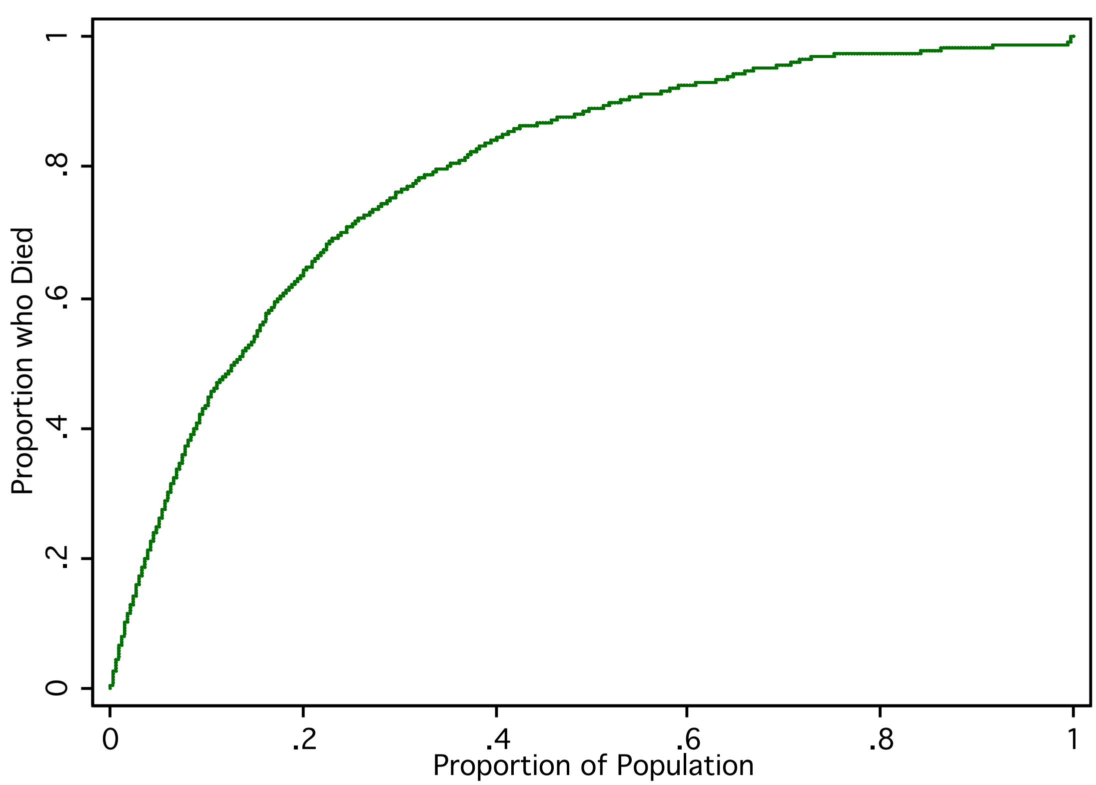


Area under ROC curve = 0.8175

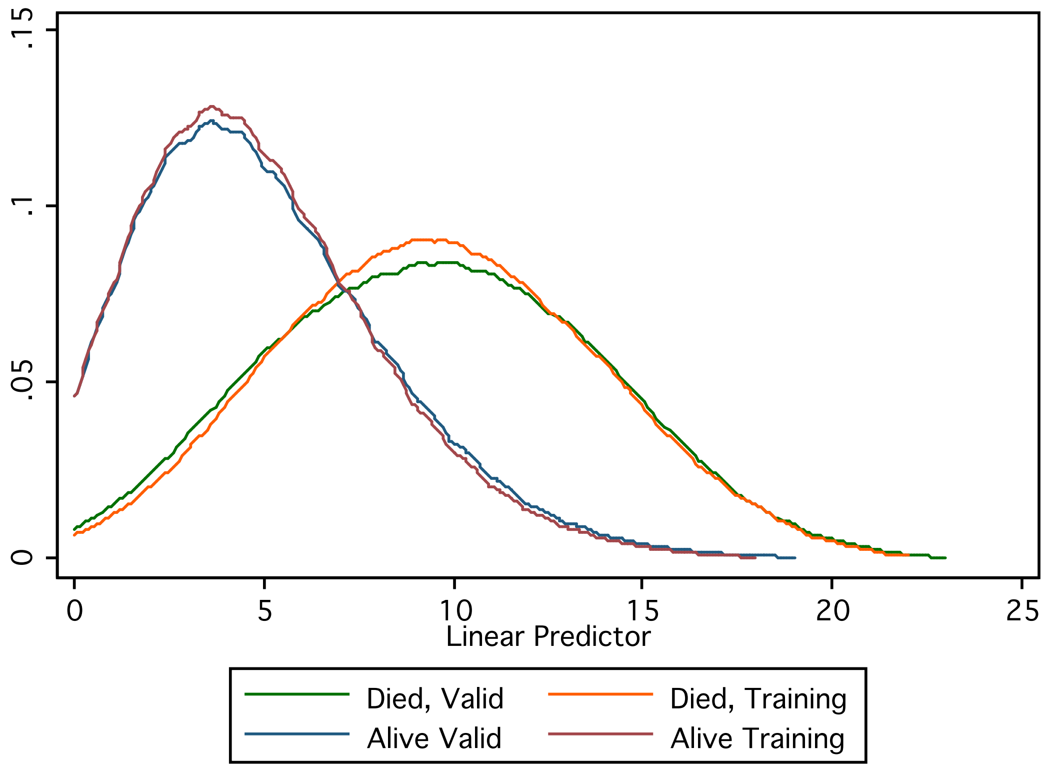
Lorenz Curve

- Cumulative proportion of outcome by cumulative positive classifications
- Works in prevalence information
- Gives a sense of screening performance
- Also used for income distributions
  10% of pop^n has 40% of mortality risk

Lorenz



Density



Calibration

Calibration

- Model: predicted probability
- Does predicted probability match truth
- Important for individual decision making
- Classic model-checking problem
- Incorporate statistical tests
  and descriptions of magnitude
- Find model misspecified in some way

Lee Model

validation set

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------------------------------------------------------------------------------
```

Aspects of Model Checking

- Involves
  - wrong functional form
  - undetected interaction

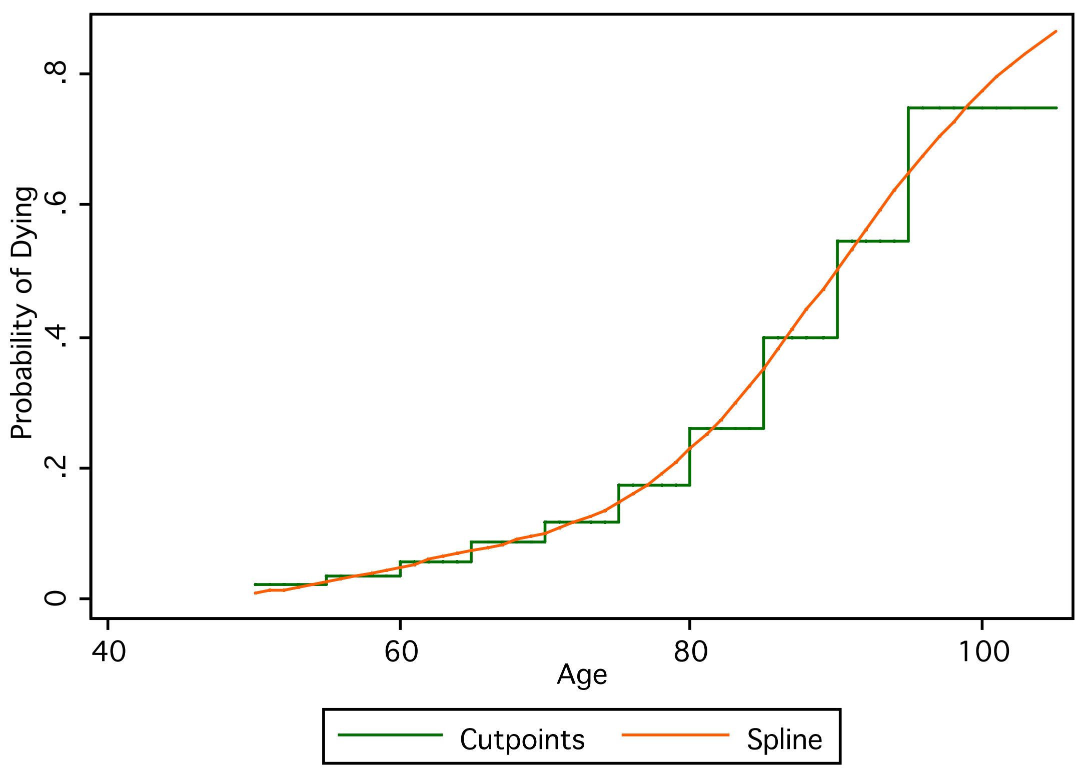
Functional Form

- Involves continuous covariate: e.g., age
- Maybe assumed linear when it is not
- Can be too few cutpoints
  - choice of two
- Result: some ages systematically predicted wrong

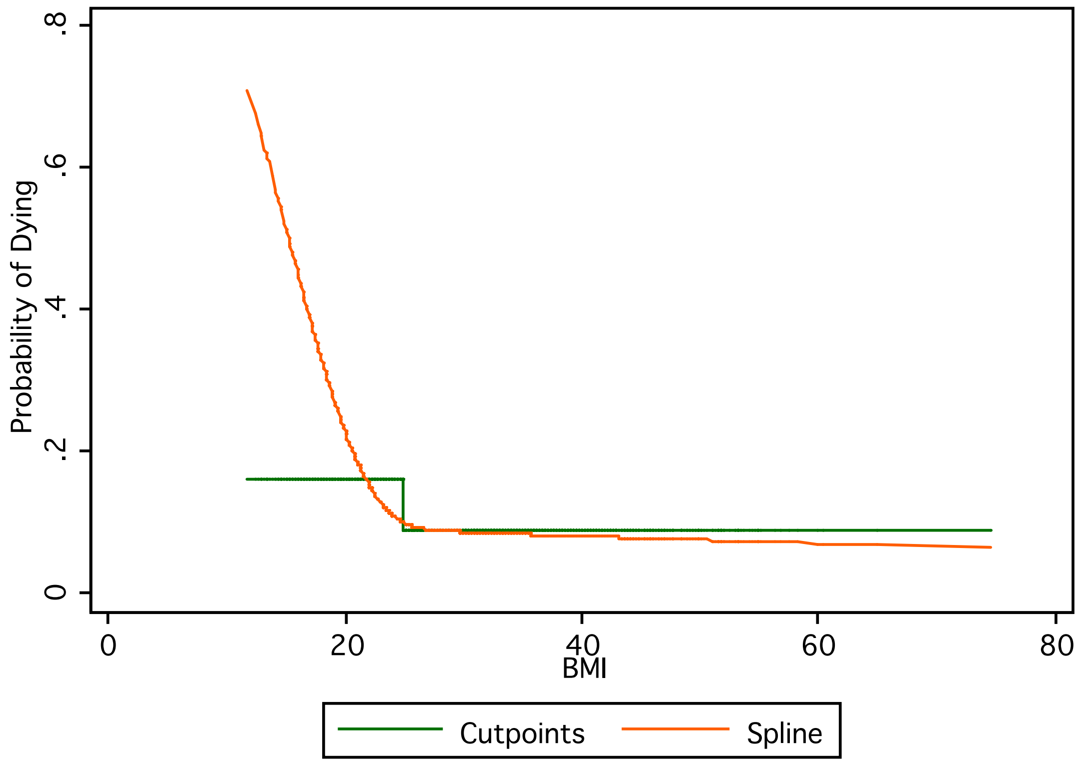
Functional Form

- Fit the spline model to continuous predictor(s)
- Fit the other model
- Plot the difference in fit
- Tests not useful with large or small n

Age



BMI



Functional Form

- Look basically fine
- Age has enough cutpoints
  - and we can afford them
  - underestimate mortality for 90yo
- BMI: only 5% of values less than 20
  - apparent diff => minimal for prediction

Undetected Interaction

- Effect of one variable depends on another
- For someone with cancer, is diabetes important predictor?
- If modeled, points depend on two variables
- Just 17 predictors => 66 possible interactions
- That's just two-way interactions

Inspecting Calibration

- Form series of groups based on risk
- Compare expected mortality with observed
- Expected: average(probs)
- Observed: average(death)
  = *proportion who died*
- Within each group

our point total furnishes such a group

Test Set

```
. table pts2 if valid==0, c(n dead mean dead mean prob)
```

Points

	N	mean(dead)	mean(prob)
0	486	.0041152	.0073689
1	739	.0067659	.0126751
2	1,366	.0095168	.017859
3	1,474	.0325645	.0277967
4	1,445	.0394464	.0407945
5	1,330	.0503759	.0589664
6	1,162	.0886403	.0849856
7	886	.1230248	.1186558
8	758	.1912929	.1653885
9	551	.2413793	.2242661
10	407	.2727273	.2922267
11	320	.43125	.384265
12	244	.4836065	.4604374
13	174	.5344828	.5641574
14	107	.5046729	.6368158
15+	203	.7438424	.7780437

Validation Set

```
. table pts2 if valid==1, c(n dead mean dead mean prob)
```

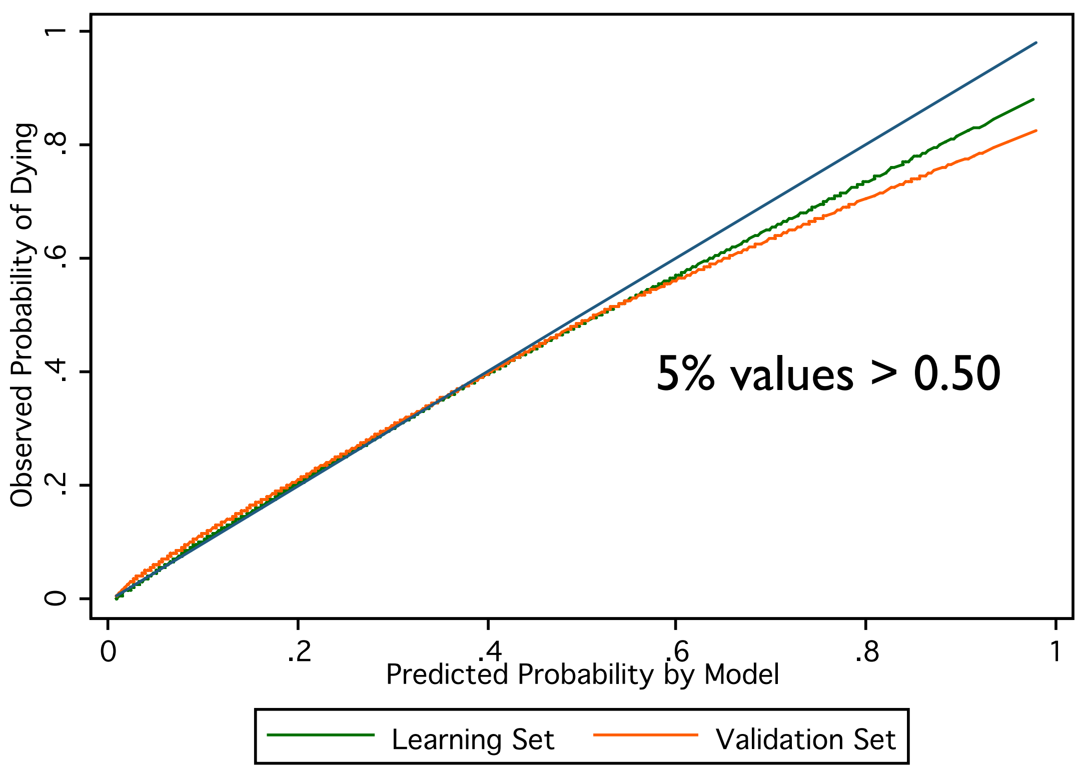
Points

	N	mean(dead)	mean(prob)
0	354	.0056497	.0073689
1	489	.0204499	.0127029
2	889	.0168729	.0174934
3	971	.0360453	.0275617
4	986	.0598377	.0408728
5	842	.0771971	.0585236
6	758	.0936675	.0851236
7	637	.1507064	.1188091
8	498	.1967871	.1658374
9	401	.19202	.2229141
10	308	.275974	.2988213
11	232	.4568965	.3782594
12	192	.4427083	.4634854
13	159	.591195	.5602076
14	77	.5584416	.6408411
15+	180	.6777778	.789714

Smoothing

- Alternative graphical approach
- Draw smooth curve of mortality by predicted probability
- Should look like straight line
- Assessed in test and validation set

Smoothed Calibration



Summary

- See classic overfitting pattern
  - mortality lower than pred at high end
  - mortality higher than pred at low end
- Large calibrations at high end
  - represent 5% of the prediction
  - fixing would complicate model

Validation

- Calibration: examination of model spec
  - largely internal
- Discrimination: measure separation
  - largely external
- Calibration can be fixed
- Discrimination dictates utility of model