Confidence Intervals & Hypothesis Testing

- Hypothesis testing of two means
- Hypothesis testing of proportions

Hypothesis Testing, two-sided hypothesis

Example: Do children with congenital heart disease walk at the same or a different age as compared to healthy children? Healthy children start walking at mean age of 11.4 months

- Null hypothesis: H_0: $\mu = 11.4$ months (μ_0)
- Alternative hypothesis: H_A: $\mu \neq 11.4$ months
- Significance level=0.05
- Data: Sample of children with congenital heart defects, n=9, sample mean=12.8, sample SD=2.4

Hypothesis Testing, two-sided hypothesis

- Calculate the test statistic and its associated p value

$$t_{stat} = \frac{\overline{X} - \mu_0}{s / \sqrt{n}}$$
$$= \frac{12.8 - 11.4}{2.4 / \sqrt{9}} = 1.75$$

- p- value is P(T>1.75) + P(T<-1.75) =
  2*P(T>1.75)

```
display ttail(8,1.75)*2
.11823278
```

Hypothesis Testing, two-sided hypothesis

Or run ttesti in Stata
** ttesti samplesize samplemean samplesd hypothesizedmean

```
ttesti 9 12.8 2.4 11.4

One-sample t test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |       9        12.8          .8         2.4     10.9552     14.6448
------------------------------------------------------------------------------
    mean = mean(x)                                                t =   1.7500
Ho: mean = 11.4                                  degrees of freedom =        8

   Ha: mean < 11.4             Ha: mean != 11.4               Ha: mean > 11.4
 Pr(T < t) = 0.9409       Pr(|T| > |t|) = 0.1182           Pr(T > t) = 0.0591
```

→ Fail to reject the null hypothesis

Hypothesis Testing, one or two-sided hypotheses

- For data already in Stata, the ttest command also works
- e.g. We want to test that the mean CD4 cell count <350 among persons newly diagnosed with HIV in Uganda
  - Null hypothesis: H_0: $\mu \geq 350$ cells/mm^3
  - Alternative hypothesis (the one we are worried about – that patients come in with low CD4 cell counts):

    H_A: $\mu < 350$ cells/mm^3
  - Significance level=0.05

Hypothesis Testing, one or two sided hypothesis

Use ttest varname== hypothesized value

```
. ttest cd4count==350

One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
cd4count |     999    329.2332    8.419592    266.1177    312.7111    345.7554
------------------------------------------------------------------------------
    mean = mean(cd4count)                                         t =  -2.4665
Ho: mean = 350                                   degrees of freedom =      998

   Ha: mean < 350             Ha: mean != 350               Ha: mean > 350
 Pr(T < t) = 0.0069      Pr(|T| > |t|) = 0.0138           Pr(T > t) = 0.9931
```

→ We reject the null hypothesis

Data set: vct_baseline_biostat200_v1.dta

Hypothesis Testing, one or two sided hypothesis

- What if you ran a test and got z_{stat} =1.83? If your hypothesis was one-sided, you'd reject. If your hypothesis was two-sided, you'd fail to reject.

- Therefore it is very important to specify your test a priori!

Hypothesis testing

- For clinical trials, most of the time we use a two-sided hypotheses
  - That way no one will suspect you of changing your hypothesis test after the fact
  - On the other hand, does it really make sense to have an alternative hypothesis that a new treatment is either better *or worse* than the old one?
  - Assuming your new therapy is better or 'not inferior' we may use a one-sided test

Hypothesis test of a proportion

- Variables that are measured as successes or failures, disease or no disease, etc., can be considered binomial random variables
  - x=number of successes
  - n=number of "trials"
  - x/n = proportion diseased
  - Use the z-statistic

$$z_{stat} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

Hypothesis test of a proportion

- Example: Micronutrient intake of black women in South Africa
- Study: Pre-menopausal women randomly selected based on geographic location
- Micronutrient intake was determined using a Quantitative Food Frequency Questionnaire developed for South Africans
- Question: Do more than 10% of the women have the micronutrient intake of less than the RDA?

Data adapted from Hattingh Z. et al, West Indian Med J 2008: 57 (5):431

Hypothesis test of a proportion

- Null hypothesis:
  - 10% or fewer of the women aged 25-34 have insufficient dietary folate levels (less than 268 µg, a cutoff based on RDA)
  - H_0: $p_0 \leq 0.10$
- Alternative hypothesis:
  - More than 10% of the women aged 25-34 have insufficient dietary folate levels
  - H_A: $p_0 > 0.10$
- Significance level set at = 0.05

Hypothesis test of a proportion

- The data:
  - 158/279=56.6% had folate levels<268 µg

- Using the normal approximation to the binomial distribution:

z_{stat} = (.566-0.10)/√(0.10*0.90/279) =25.9

$$z_{stat} = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

The chance of observing a z statistic of this large (25.9) is very small (<0.05)

→ So we reject the null hypothesis

Hypothesis testing of a proportion

The stata command prtest or prtesti gives you a test of a proportion using the normal approximation

** prtesti samplesize observedp hypothp

```
. prtesti  279 .566  .1

One-sample test of proportion                   x: Number of obs =       279
------------------------------------------------------------------------------
    Variable |       Mean   Std. Err.                     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |       .566   .0296723                      .5078434    .6241566
------------------------------------------------------------------------------
    p = proportion(x)                                             z =  25.9458
Ho: p = 0.1

     Ha: p < 0.1                 Ha: p != 0.1                 Ha: p > 0.1
 Pr(Z < z) = 1.0000         Pr(|Z| > |z|) = 0.0000          Pr(Z > z) = 0.0000
```

Hypothesis testing of a proportion

- Null hypothesis:
  - Fewer than 10% of the women aged 25-34 have dietary zinc levels of less than 5 mg
  - H_0: $p < 0.10$
- Alternative hypothesis:
  - 10% or more of the women aged 25-34 have dietary zinc levels of less than 5 mg
  - H_A: $p \geq 0.10$
- Significance level = 0.05

Hypothesis testing of a proportion

- The data: 31/279=11.1% had zinc levels of less than 5 mg

```
prtesti 279 .111 .1

One-sample test of proportion                    x: Number of obs =      279
------------------------------------------------------------------------------
    Variable |       Mean   Std. Err.                     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |       .111   .0188066                      .0741397    .1478603
------------------------------------------------------------------------------
    p = proportion(x)                                             z =   0.6125
Ho: p = 0.1

     Ha: p < 0.1                 Ha: p != 0.1                 Ha: p > 0.1
 Pr(Z < z) = 0.7299         Pr(|Z| > |z|) = 0.5402          Pr(Z > z) = 0.2701
```

- Using the normal approximation, we find that the data are NOT inconsistent with the null hypothesis, therefore we fail to reject the null

Using the binomial distribution

```
. bitesti 279 .111 .1

        N   Observed k   Expected k   Assumed p   Observed p
------------------------------------------------------------
      279        31          27.9       0.10000      0.11111

  Pr(k >= 31)            = 0.295225  (one-sided test)
  Pr(k <= 31)            = 0.767742  (one-sided test)
  Pr(k <= 24 or k >= 31) = 0.548577  (two-sided test)

Or we could have used:
. di binomialtail(279,31,.1)
.29522463
```

This is an exact test, rather than using the normal approximation. if n or p are small, you may need to use an exact test – we will examine other tests (Chi-squared) in future lectures

- The commands in Stata to do this are `display binomialtail(n,k,p)` or `bitesti samplesize observedp hypothp`