# Conceptualizing and testing moderated effects

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Epi 222: Health Disparities Research Methods
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# What is moderation?

. When the relationship between an *X* variable and a *Y* variable changes as a function of a third variable (i.e., another *X* variable)

. When the effect of one explanatory variable depends on the level of another explanatory variable

. When two (or more) variables considered in combination have a joint effect on the outcome that is greater than the sum of their individual effects

. A statistical interaction

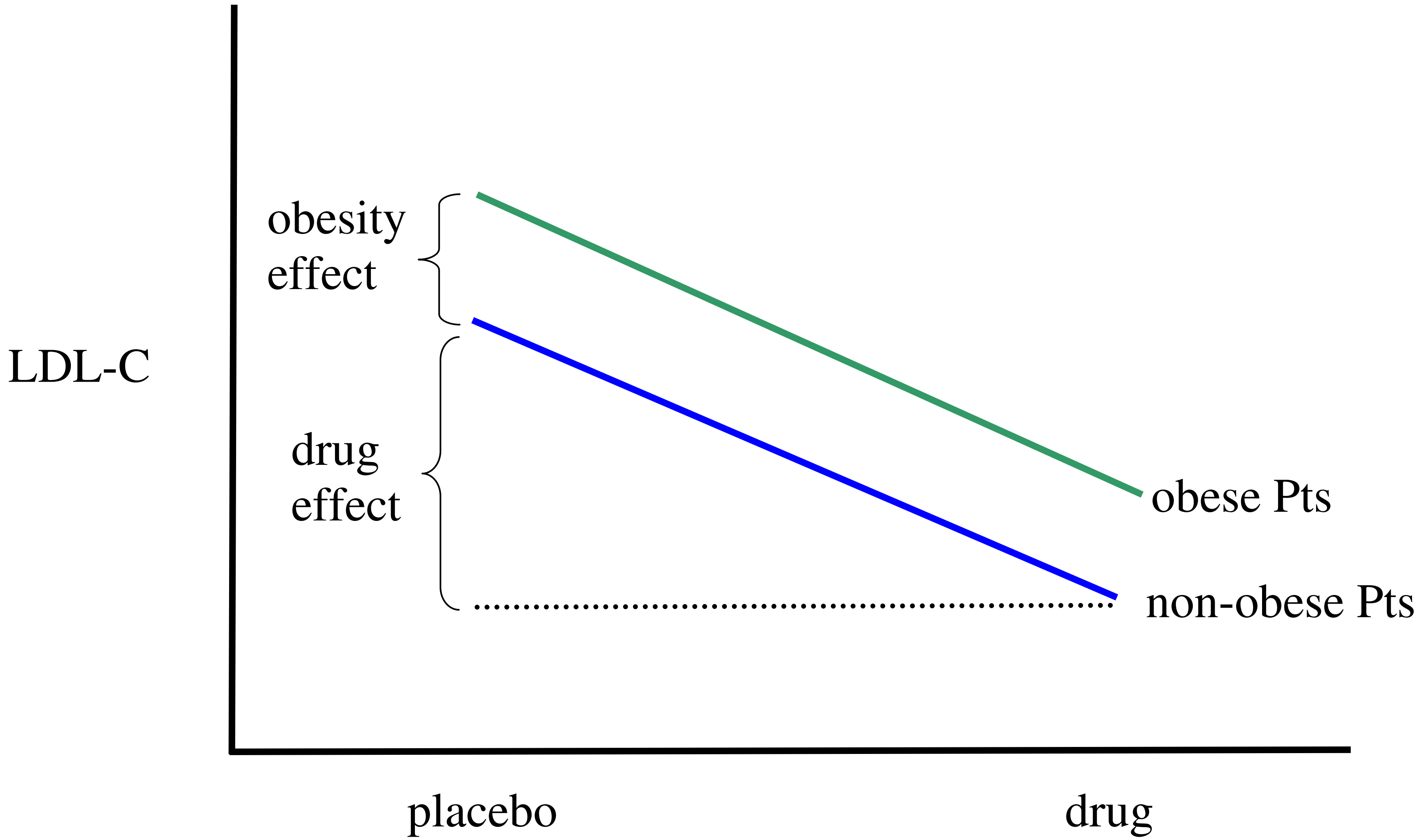
# Why bother with moderation?

Most empirical models of health disparities focus exclusively on main effects

If an interaction effect exists, then a main effects model is misspecified,
leading to biased effect estimates

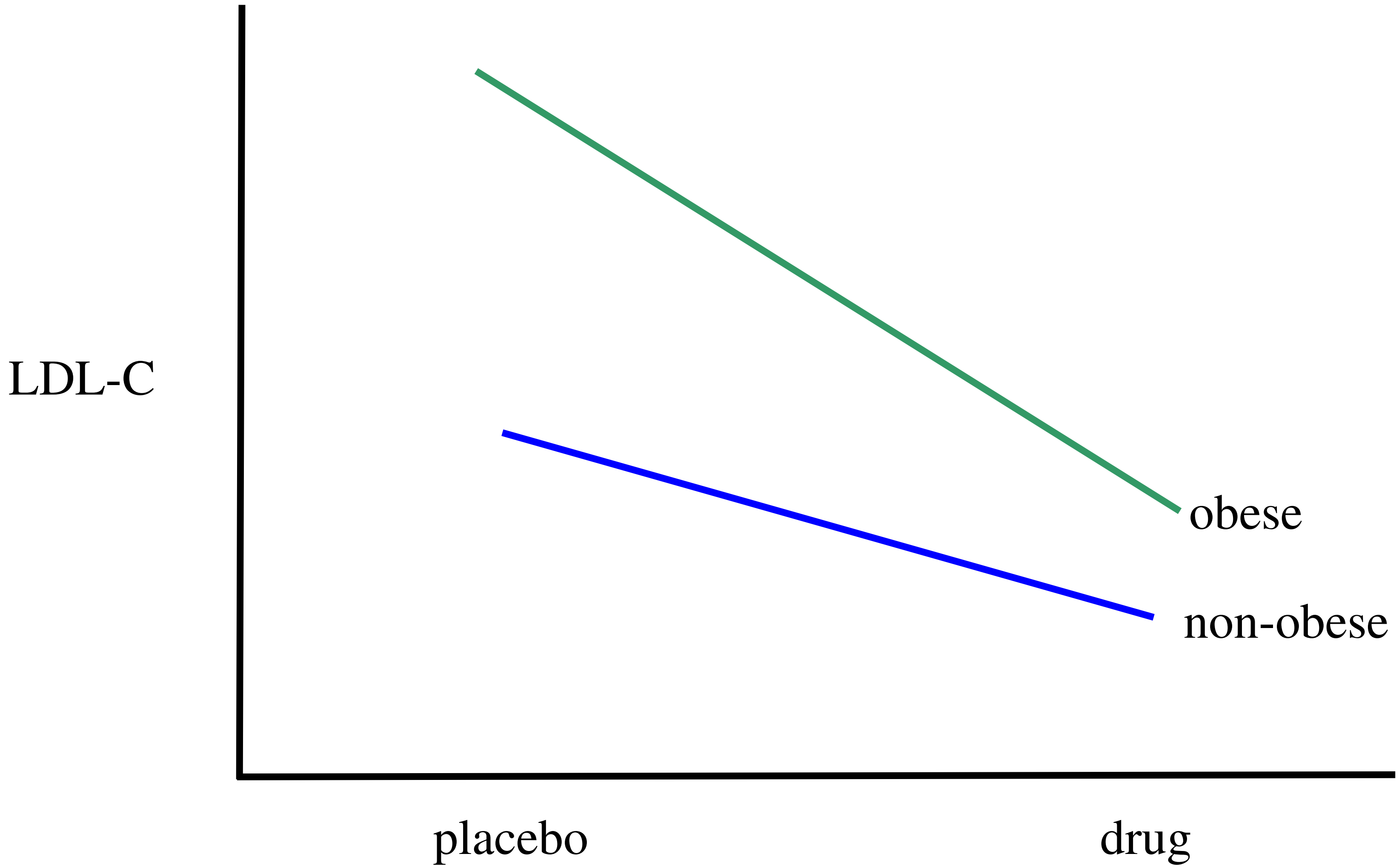
Little empirical progress has been made toward explaining health disparities.
A focus on potential interaction effects may help to ameliorate…

# Graphical examples: main effects only, no interaction




. What can be said about the effect of the drug? the effect of obesity?

# Graphical examples: interaction

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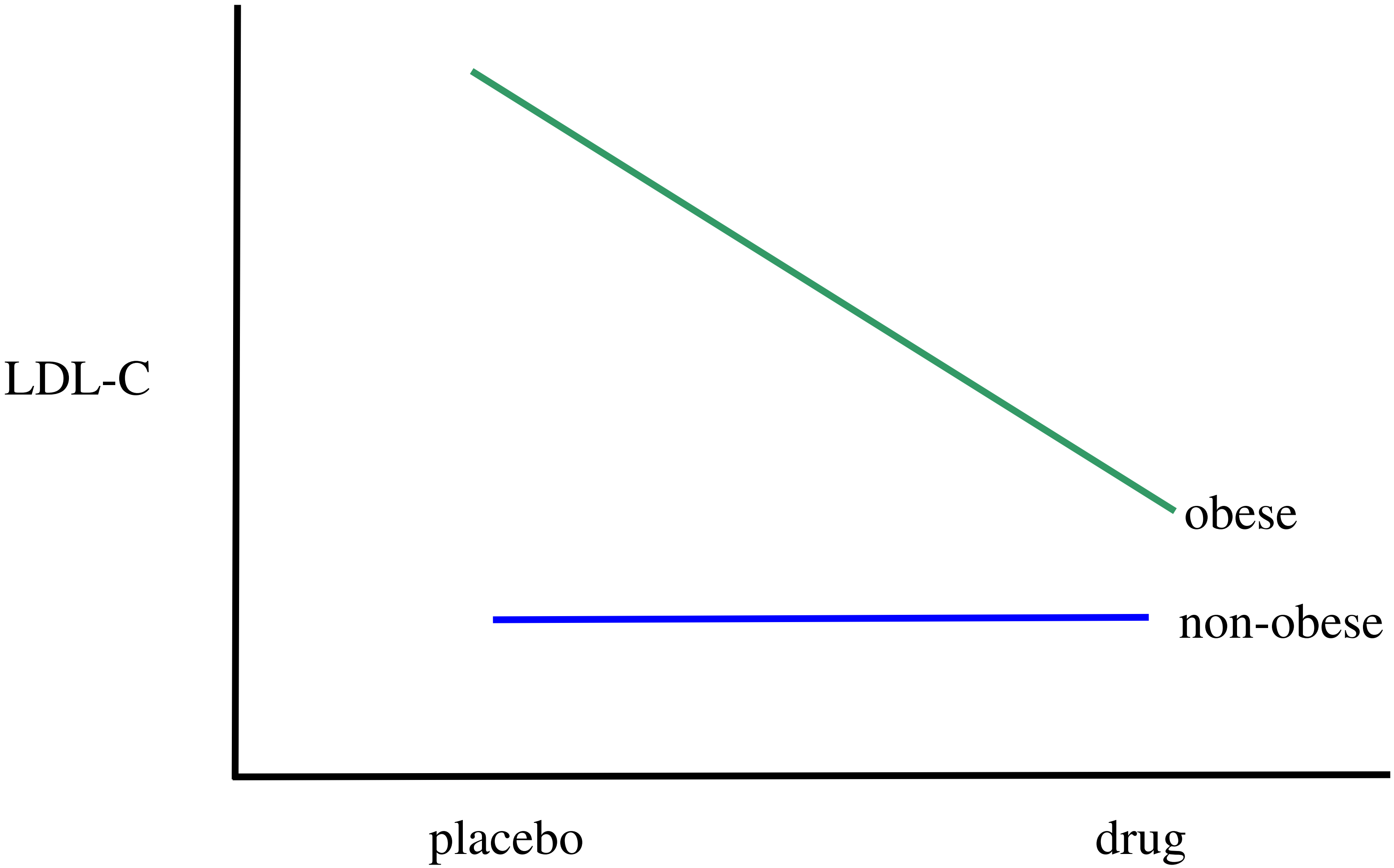
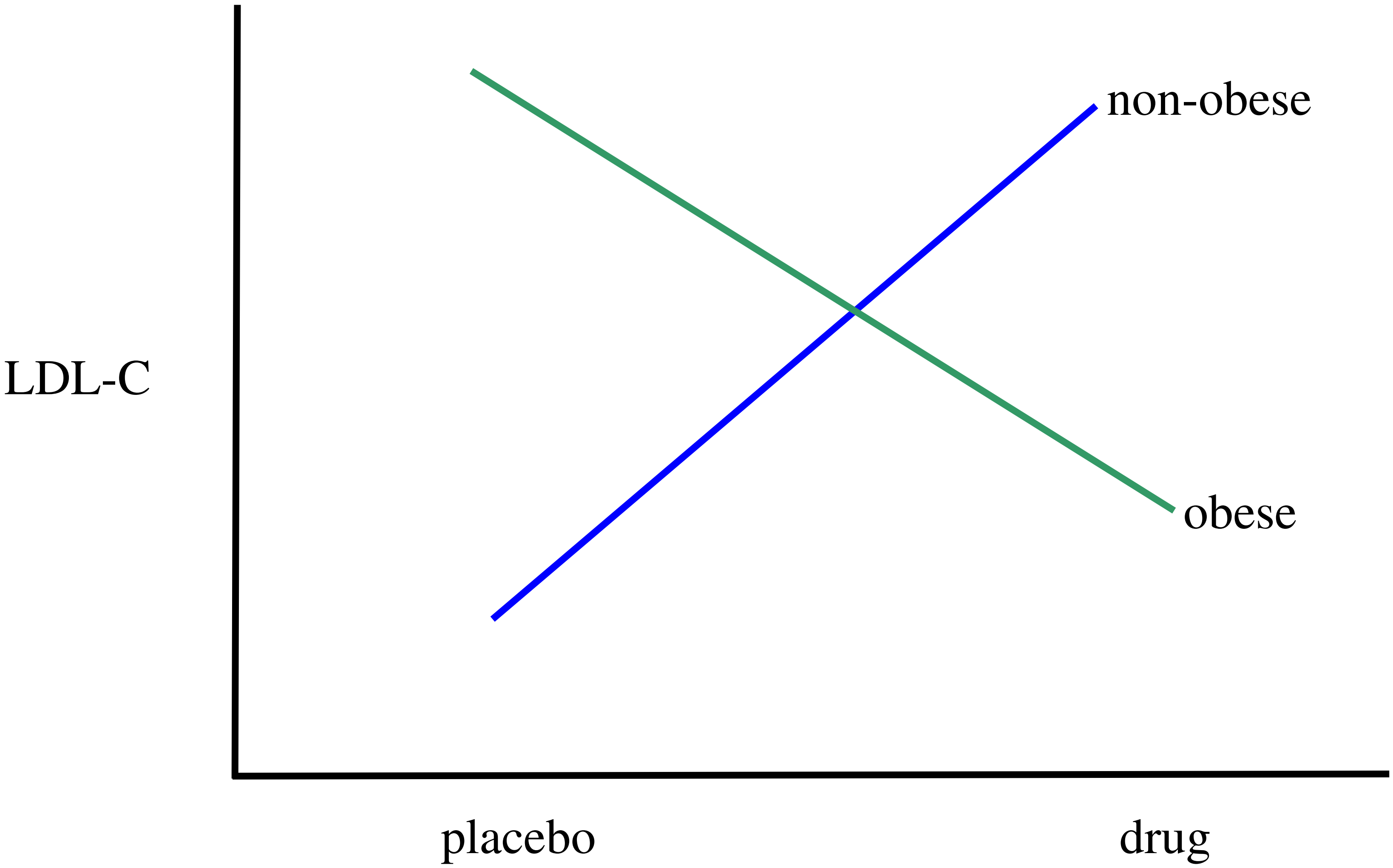
# Graphical examples: other possibilities



# Graphical examples: other possibilities



# Graphical examples: other possibilities

<u>Summary</u>

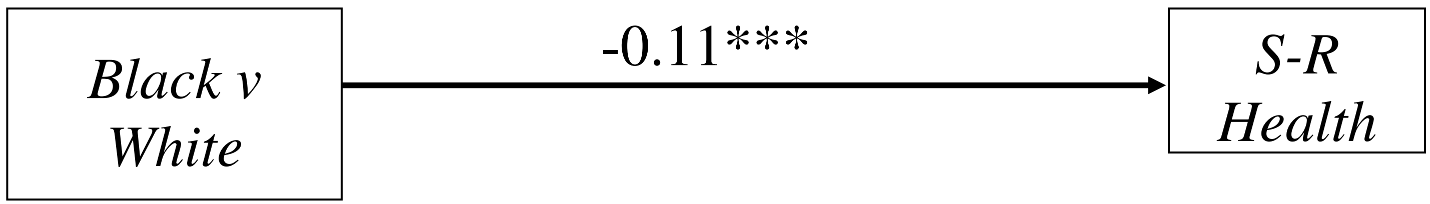
Interactions can take many forms, but the shared characteristic is that

the association between *X* and *Y* is non-constant

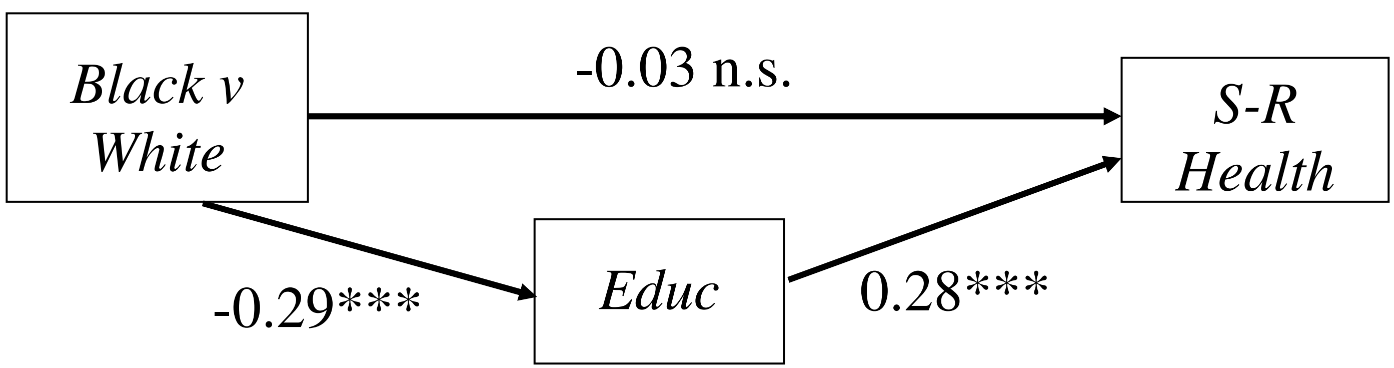
the magnitude of the association between *X* and *Y*
significantly differs as a function of one or morel additional variables

# Introduction: *Mediation* models are main effects models

Consider the following total effect model

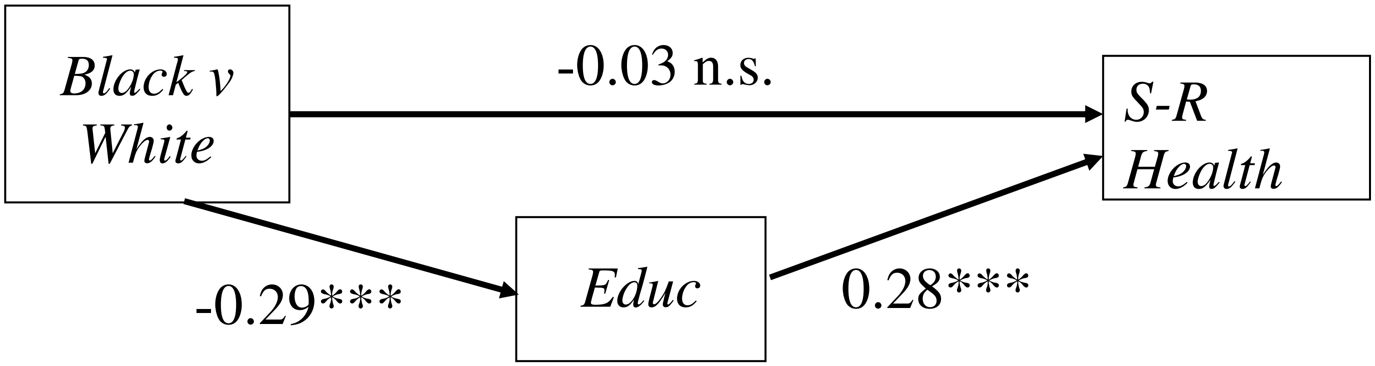



And one possible corresponding mediation model



# Introduction: *Mediation* models are main effects models

- The mediation model...




...assumes that the effect of Race is constant at all education levels

. Defending the estimated conditional effect of Race rests upon this assumption

. This assumption can be tested by estimating and testing interaction effects.

My goal is to provide a conceptual introduction to testing interactions.

# Example data: EPESE

Established Populations for Epidemiologic Studies of the Elderly (EPESE)

- Duke site
- Probability sample
- Baseline data collected in 1982
- 65 years and older
- 54% African American
- $N \approx 3900$ (with complete data on key variables)

# Example data: EPESE
# Outcome: self-rated health

*Compared to other people your own age,*
*would you say that your general health is*
*excellent, good, fair, poor?*

*How would you rate your health at the present time?*

| code | label | frequency | |
|---|---|---|---|
| | | 4-category | binary |
| 1 | poor | 570 | 1855 |
| 2 | fair | 1285 | |
| 3 | good | 1546 | 2086 |
| 4 | excellent | 540 | |

For demonstration,
*self-rated health* can be treated as continuous, categorical, or binary.

# Example data: EPESE
## Explanatory variables

Race

| code | label | frequency |
|---|---|---|
| 0 | White | 1820 |
| 1 | Black | 2121 |

Education

| code | label | frequency | |
|---|---|---|---|
| | | 4-category | binary |
| 0 | <8 | 1715 | 3030 |
| 1 | 8-11 | 1315 | |
| 2 | = HS | 335 | 911 |
| 3 | > HS | 576 | |

For demonstration,
*education* can be treated as continuous, categorical, or binary.

# Types of moderation models covered

Example 1: two binary X variables with continuous Y: pooled data

Example 1A: two binary X variables with continuous Y: stratified analyses

Example 2: two binary X variables with a binary Y: pooled data

Example 3: a binary & a continuous X with a continuous Y

Example 4: a binary & a categorical X with a continuous Y

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# Example 1: Two binary X variables with a continuous Y

Self-rated health means & mean differences as a function of two binary variables

| race | education | | Δ |
|---|---|---|---|
| | <HS (code=0) | ≥HS (code=1) | |
| White (code=0) | **2.422** | **2.969** | **0.547** |
| Black (code=1) | **2.396** | **2.777** | **0.381** |
| Δ | **-0.026** | **-0.192** | **-0.166** |

. The simple effects (pink and gray) represent main effects w/in sub-groups.

. The difference between the simple effects (yellow) is the interaction effect.

*Does the education effect differ across the races?*

*Does the race effect differ across education levels?*

# Example 1: Two binary X variables with a continuous Y

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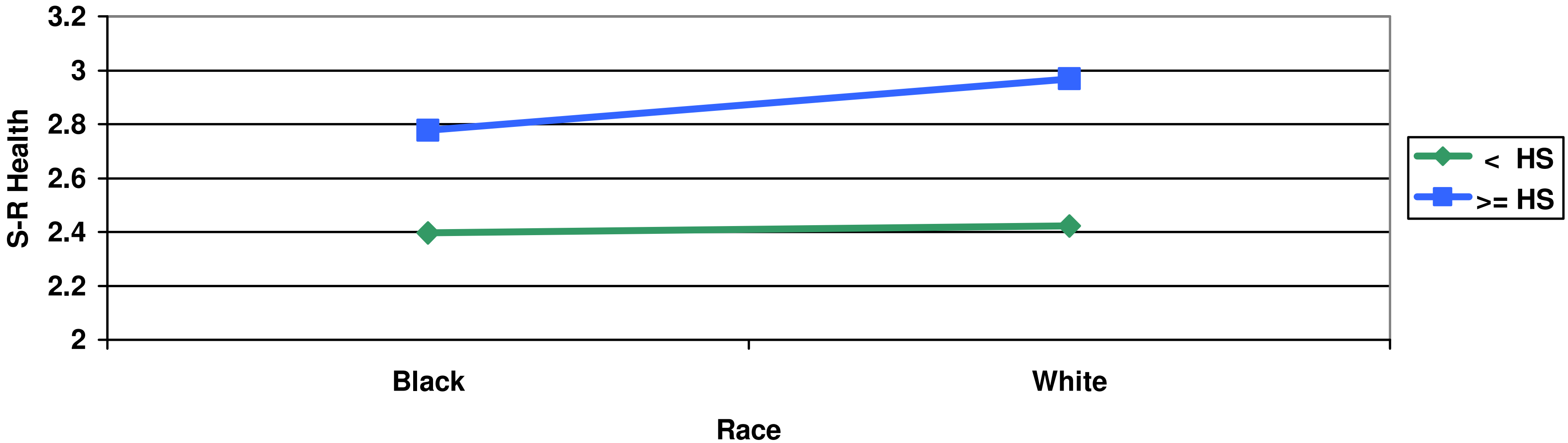

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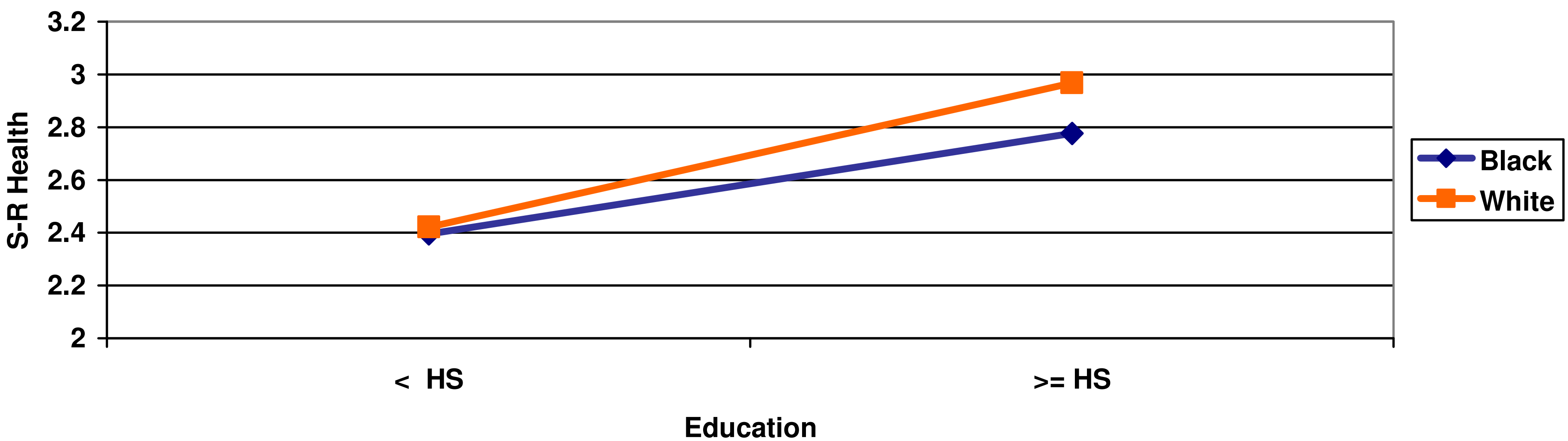

# Example 1: Two binary X variables with a continuous Y

$$\text{S-R\_Hx} = B_0 + B_1\times\text{Race} + B_2\times\text{Educ} + B_3\times\text{Race}\times\text{Educ} + \text{resid.}$$

| Parameter | DF | Estimate | StdErr | t | p |
|---|---|---|---|---|---|
| B0 (int) | 1 | 2.4218 | 0.0252 | 96.18 | <.0001 |
| B1 (race) | 1 | -0.0259 | 0.0325 | -0.80 | 0.4252 |
| B2 (educ) | 1 | 0.5471 | 0.0435 | 12.59 | <.0001 |
| B3 (r×e) | 1 | -0.1664 | 0.0697 | -2.39 | 0.0171 |
| *(custom test)* | | | | | |
| educ Blacks | 1 | 0.3807 | 0.0546 | 6.99 | <.0001 |

Self-rated health means & mean differences as a function of two binary variables

| race | education | | Δ |
|---|---|---|---|
| | <HS (code=0) | ≥HS (code=1) | |
| White (code=0) | **2.4218** | **2.9689** | **0.5471** |
| Black (code=1) | **2.3959** | **2.7766** | **0.3807** |
| Δ | **-0.0259** | **-0.1923** | **-0.1664** |

# Example 1: Two binary X variables with a continuous Y

Summary

. among Black respondents, those with ≥HS averaged 0.381 points higher on the self-rated health outcome compared to those with <HS, $p < .0001$.

. among White respondents, those with ≥HS averaged 0.547 points higher on the self-rated health outcome compared to those with <HS, $p < .0001$.

. A significant interaction existed between race and education: the effect of education was significantly stronger for Whites than for Blacks, $p < .02$.

## The simple approach

If the interaction is significant, then

report the p-value for the interaction effect and

report the effect of education within each race, or

report the effect of race within each education level.

Be cautious when reporting main effects.

Make sure you are very clear about them.

# Types of moderation models covered

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Example 4: A binary & a categorical X with a continuous Y

# Example 1A: Two binary X variables with a continuous Y

*Stratified analyses (fit the following model within each race-specific sample)*

**S-R_Hx = $B_0$ + $B_2 \times$Educ + resid.**

Whites ($n$=1820)

| Parameter | DF | Estimate | StdErr | t | p |
|---|---|---|---|---|---|
| $B0_W$ (int) | 1 | 2.4218 | 0.0252 | 96.04 | <.0001 |
| $B2_W$ (educ) | 1 | 0.5471 | 0.0435 | 12.57 | <.0001 |

Blacks ($n$=2121)

| Parameter | DF | Estimate | StdErr | t | p |
|---|---|---|---|---|---|
| $B0_B$ (int) | 1 | 2.3959 | 0.0205 | 116.93 | <.0001 |
| $B2_B$ (educ) | 1 | 0.3807 | 0.0545 | 6.99 | <.0001 |

*t-test of interaction effect* $(df = N1 + N2 - 4)$

$$t = (B2_B - B2_W) / \mathrm{SQRT}(SE_B^2 + SE_W^2)$$

$$= (0.3807 - 0.5471) / \mathrm{SQRT}(.0545^2 + .0435^2)$$

$$= -2.39$$

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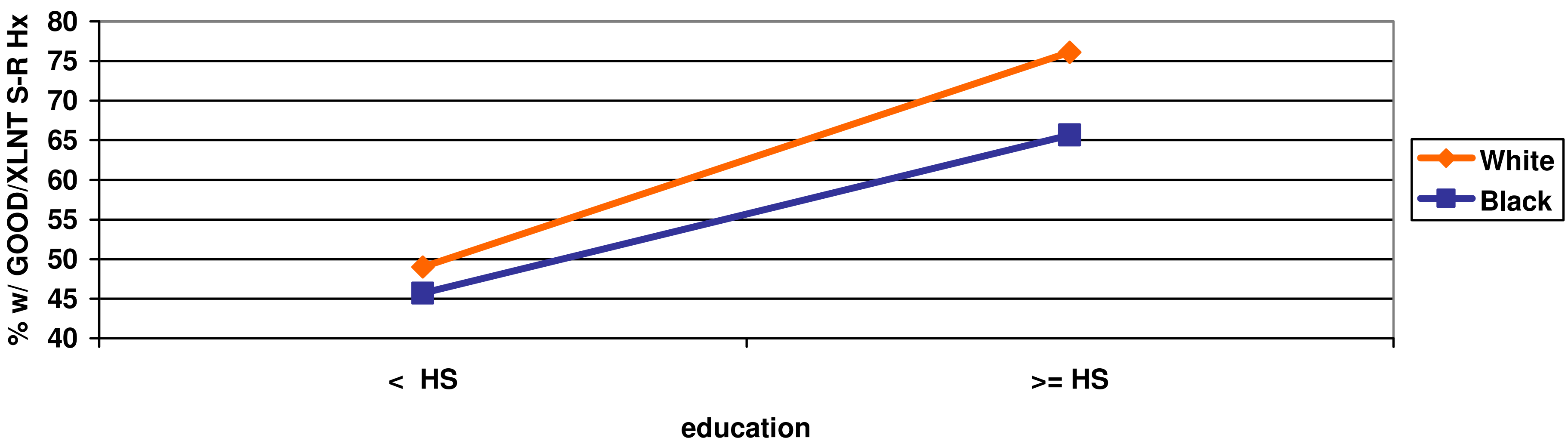
# Example 2: Two binary X variables with a binary Y

% with good/excellent self-rated health as a function of two binary variables

| race | education | | OR |
|---|---|---|---|
| | <HS | ≥HS | |
| White | **48.97%** | **76.10%** | **3.319** |
| Black | **45.69%** | **65.67%** | **2.274** |
| OR | **0.877** | **0.601** | **0.685 (OR ratio)** |



# Example 2: Two binary X variables with a binary Y

% with good/excellent self-rated health as a function of two binary variables

| race | education | | Δ logit |
|---|---|---|---|
| | <HS | ≥HS | |
| White | **48.97%**<br>**(-0.0414)** | **76.10%**<br>**(1.1584)** | **1.1998** |
| Black | **45.69%**<br>**(-0.1729)** | **65.67%**<br>**(0.6484)** | **0.8213** |
| Δ logit | **-0.1315** | **-.5100** | **-.3785** |

logit?

logit = ln(π/(1-π)), where π is the response probability

E.g., the logit of .4897 = ln(0.4897 ÷ (1 – 0.4987)) = **–.0414**

# Example 2: Two binary X variables with a binary Y

% with good/excellent self-rated health as a function of two binary variables

| race | education <HS | education ≥HS | Δ logit |
|---|---|---|---|
| White | **48.97%** **(-0.0414)** | **76.10%** **(1.1584)** | **1.1998** |
| Black | **45.69%** **(-0.1729)** | **65.67%** **(0.6484)** | **0.8213** |
| Δ logit | **-0.1315** | **-.5100** | **-.3785** |

| Parameter | DF | Estimate | Error | t | p |
|---|---|---|---|---|---|
| B0 | 1 | -0.0414 | 0.0575 | 0.72 | 0.4722 |
| B1 (race) | 1 | -0.1315 | 0.0743 | 1.77 | 0.0768 |
| B2 (educ) | 1 | 1.1998 | 0.1109 | 10.81 | <.0001 |
| B3 (race*educ) | 1 | -0.3785 | 0.1712 | 2.21 | 0.0271 |

**OR[educ for Whites] = exp(1.1998) = 3.3194, p < .0001**

**OR[educ for Blacks] = exp(0.8213) = 2.2735, p < .0001**

# Example 2: Two binary X variables with a binary Y

Summary

. among Black respondents, those with ≥HS had 2.27 higher odds of good/excellent self-rated health compared to those with <HS, $p < .0001$.

. among White respondents, those with ≥HS had 3.32 higher odds of good/excellent self-rated health compared to those with <HS, $p < .0001$.

. A significant interaction existed between race and education: the effect of education was significantly stronger for Whites than for Blacks, $p < .03$.

## The simple approach

If the interaction is significant, then

- report the p-value for the interaction effect, and
- report on the effect (OR) of education within each race, or
- report on the effect (OR) of race within each education level.

Be cautious when reporting main effects.

Make sure you are very clear about them.

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# Example 3: A binary & continuous X with a continuous Y

<u>Mean self-rated health</u>
Whites: 2.605
Blacks: 2.450

<u>Education</u>
*Treated as a continuous variable with values 0, 1, 2, and 3*
mean = 0.942
std dev = 1.050

# Example 3: A binary & continuous X with a continuous Y

**S-R_Hx = $B_0$ + $B_1$×Race + $B_2$×Educ + $B_3$×Race×Educ + resid.**

| Parameter | DF | Estimate | StdErr | t | p |
|---|---|---|---|---|---|
| B0 (int) | 1 | 2.2863 | 0.0315 | 72.64 | <.0001 |
| B1 (race) | 1 | 0.0509 | 0.0391 | 1.30 | 0.1928 |
| B2 (educ) | 1 | 0.2530 | 0.0190 | 13.33 | <.0001 |
| B3 (r×e) | 1 | −0.0845 | 0.0276 | 3.06 | 0.0022 |
| *(custom test)* | | | | | |
| educ(Black) | 1 | 0.1685 | 0.0200 | . 8.41 | <.0001 |
| | | (0.1685 = 0.2530 + −0.0845) | | | |

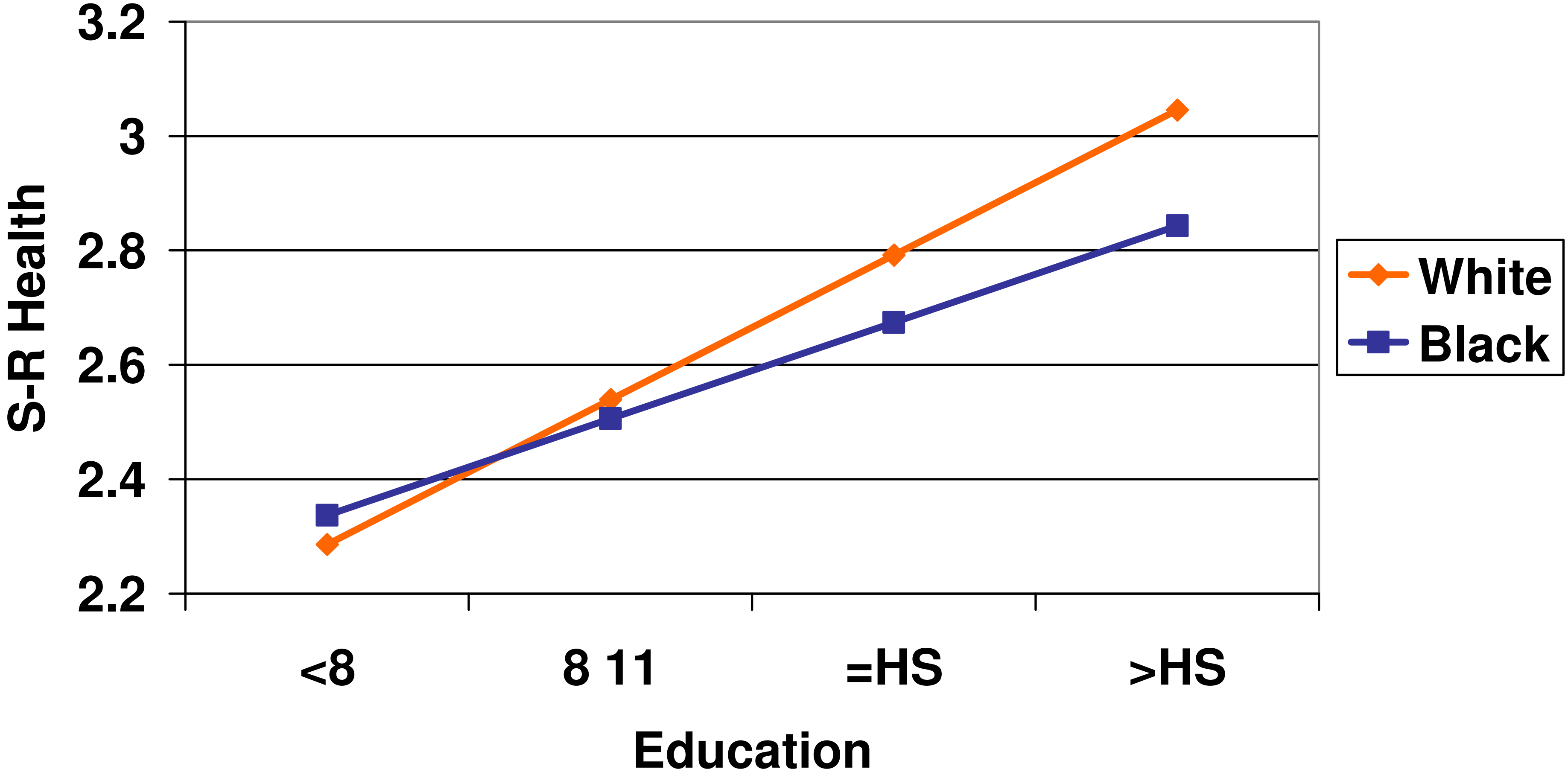
*Model-predicted* values of self-rated health

| | education level | | | |
|---|---|---|---|---|
| | <8 (code=0) | 8-11 (code=1) | =HS (code=2) | >HS (code=3) |
| White (code=0) | 2.2863 | **2.5393** | 2.7923 | 3.0453 |
| Black (code=1) | 2.3372 | **2.5057** | 2.6742 | 2.8427 |
| Δ | 0.0509 | -0.0336 | -0.1181 | -0.2026 |

0.2530 = **2.5393** – 2.2863: effect of a one-unit increase in education for Whites

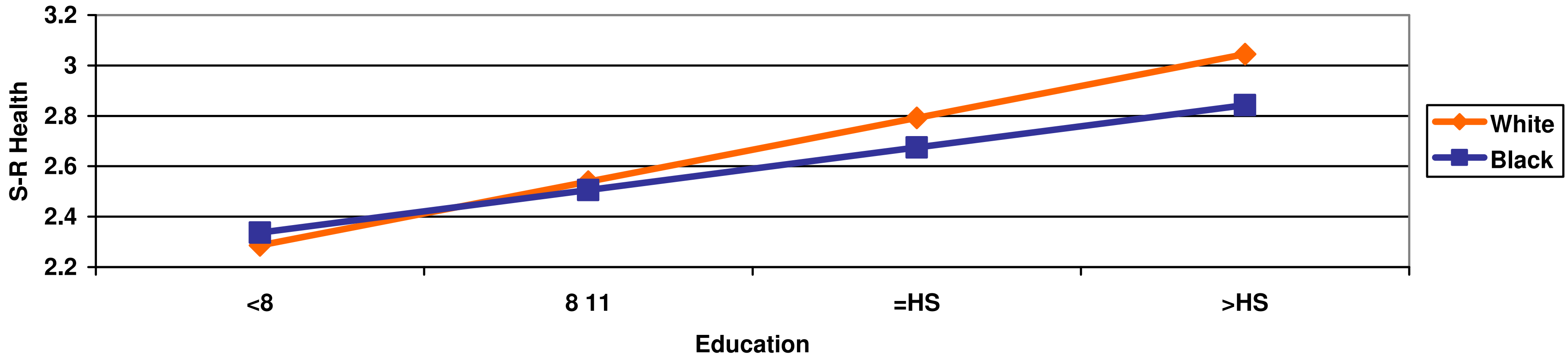
0.1685 = **2.5057** – 2.3372: effect of a one-unit increase in education for Blacks

# Example 3: A binary & continuous X with a continuous Y




What can be said about the effect of race? the effect of education?

# Example 3: A binary & continuous X with a continuous Y




Summary

. among White respondents, for every one-category increase in education the expected value of self-rated health increased by 0.2530 points, $p < .0001$.

. among Black respondents, for every one-category increase in education the average self-rated health increased by 0.1685 points, $p < .0001$.

. A significant interaction existed between race and education: the effect of education was significantly stronger for Whites than for Blacks, $p < .01$.

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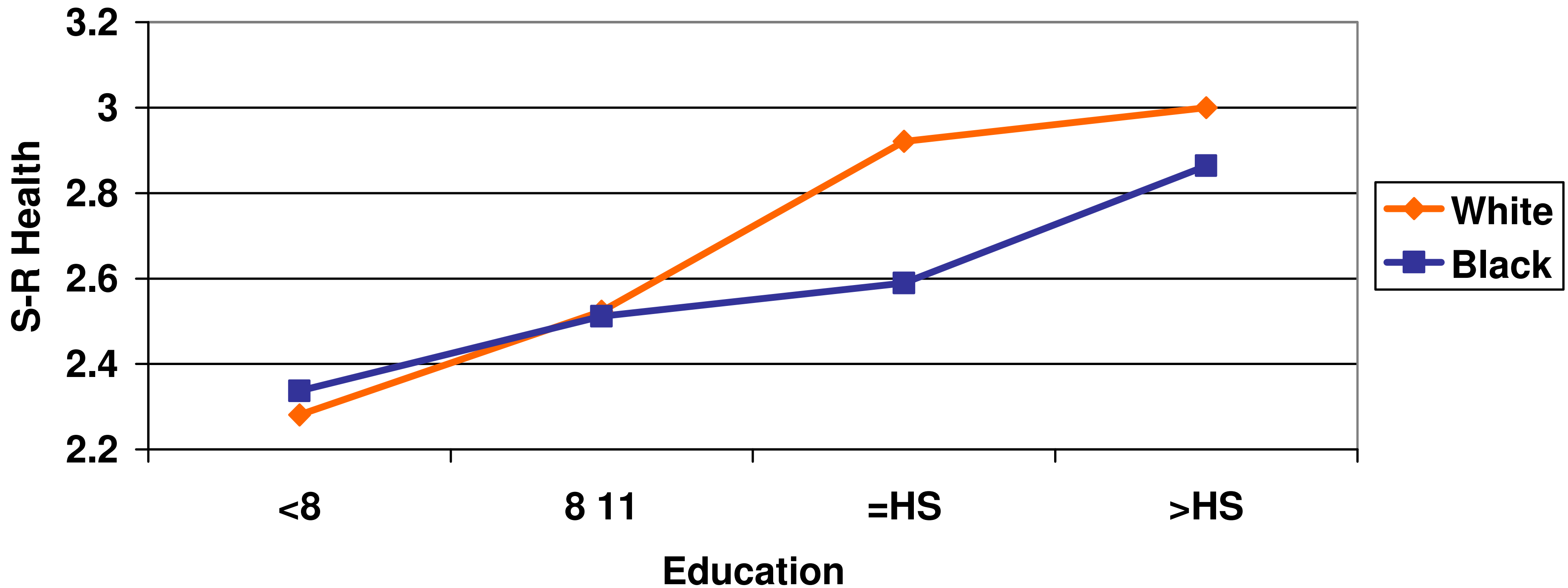
# Example 4: A binary & categorical X with a continuous Y

*Observed* mean values of self-rated health

| | education level | | | |
|---|---|---|---|---|
| | <8 | 8-11 | =HS | >HS |
| White (code=0) | 2.2806 | 2.5235 | 2.9208 | 3.0000 |
| Black (code=1) | 2.3375 | 2.5114 | 2.5895 | 2.8634 |



# Example 4: A binary & categorical X with a continuous Y

This example has a binary race indicator and a 4-category education indicator.

Therefore, the effects of education and the race-by-education interaction will each have 3 degrees of freedom.

In such cases, I first look at the omnibus test of each effect.

| Source | DF | $\chi^2$ | p |
|---|---|---|---|
| educ | 3 | 180.28 | <.0001 |
| race*educ | 3 | 13.80 | 0.0032 |

The interaction is significant

To describe the nature of the interaction, the choices are to

(a) report race differences within each level of education, or

(b) report education differences within each race

# Example 4: A binary & categorical X with a continuous Y

*Observed* mean values of self-rated health

| | education level | | | |
|---|---|---|---|---|
| | <8 | 8-11 | =HS | >HS |
| White (code=0) | 2.2806 | 2.5235 | 2.9208 | 3.0000 |
| Black (code=1) | 2.3375 | 2.5114 | 2.5895 | 2.8634 |
| Δ (*p*) | -.0568, *p*=.217 | .0120. *p*=.803 | .3314, p=.001 | .1366, p=.071 |

*Race differences within each level of education (custom tests)*

| Label | Estimate | Error | t | p |
|---|---|---|---|---|
| WvB: <8 | -0.0568 | 0.0461 | 1.23 | 0.2174 |
| WvB: 8-11 | 0.0120 | 0.0481 | 0.24 | 0.8025 |
| WvB: =HS | 0.3314 | 0.1055 | 3.14 | 0.0017 |
| WvB: >HS | 0.1366 | 0.0757 | 1.80 | 0.0713 |

*Some education-level differences within each race*

| | | | | |
|---|---|---|---|---|
| W: =HS v 8-11 | 0.3974 | 0.0651 | 6.10 | <.0001 |
| W: >HS v =HS | 0.0792 | 0.0721 | 1.10 | 0.2722 |
| B: =HS v 8-11 | 0.0780 | 0.0960 | 0.81 | 0.4162 |
| B: >HS v =HS | 0.2739 | 0.1080 | 2.54 | 0.0112 |

## *Race differences within each level of education (custom tests)*

| Label | Estimate | Error | t | p |
|---|---|---|---|---|
| WvB: <8 | -0.0568 | 0.0461 | 1.23 | 0.2174 |
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| WvB: =HS | 0.3314 | 0.1055 | 3.14 | 0.0017 |
| WvB: >HS | 0.1366 | 0.0757 | 1.80 | 0.0713 |

## *Some education-level differences within each race*

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| W: =HS v 8-11 | 0.3974 | 0.0651 | 6.10 | <.0001 |
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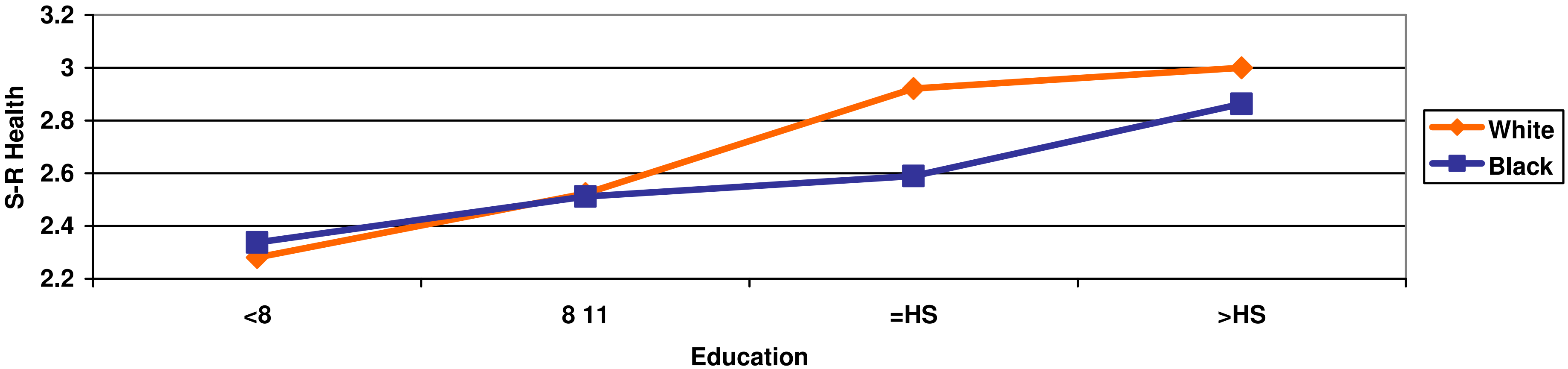
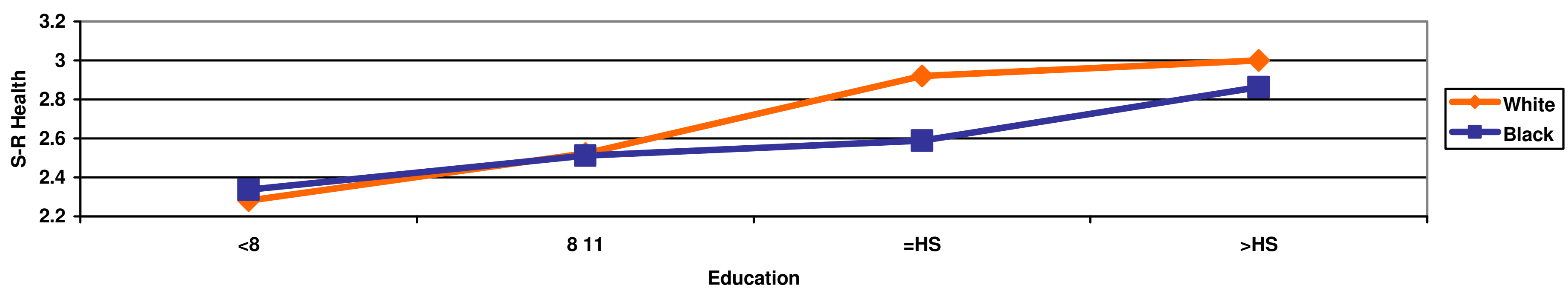


# Example 4: A binary & categorical X with a continuous Y

Summary

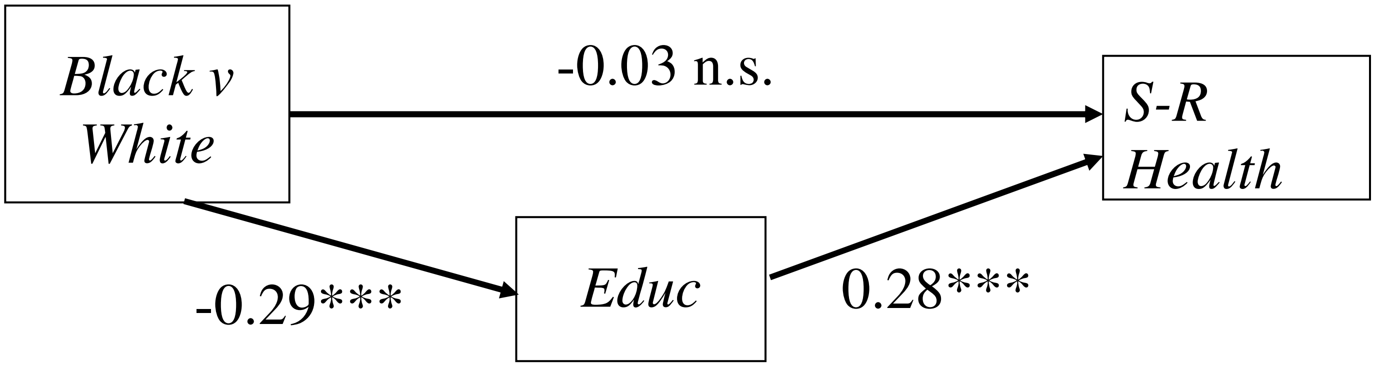
. A significant interaction existed between race and categorical education level. There were no significant differences in self-rated health between Black and White respondents who had less than a HS education. Among those with a HS education, Whites had significantly higher levels of self-rated health, compared to Blacks, $p < .01$. Among those with more than HS education, there was a trend for Whites to have higher self-rated health than Blacks, $p = .071$.

. Mostly, self-rated health significantly increased with each increase in education level. There were two exceptions: among Blacks, the increase from 8-11 years to HS education; and among Whites, the increase from HS to >HS education.



# Revisiting the *mediation* model

- The mediation model...




...assumes that the effect of Race is constant at all education levels

. Defending the estimated conditional effect of Race rests upon this assumption

# Revisiting the *mediation* model

- The effect of Race was not constant at all Educ levels
  Conversely the effect of Educ is not constant for each Race

- Therefore, the *mediation* model is misspecified, misleading, indefensible.
  It estimates the effect of Race conditional on a single effect of Educ
  But the effect of Educ is not constant across the races
  The mediation model suggests that conditional on Educ,
  Race has no direct effect

- The *moderation* model…
  showed a significant Race effect among those with a HS education and
  a marginal effect among those with more than a HS education

- Interpretation: Race does directly affect General Health,
  but only at higher levels of Educ.

*There is a price to be paid for ignoring potential interaction effects.*

# Extensions: interaction effects

We have covered interactions between

2 binary variables,

A binary and a continuous variable, and

A binary and a categorical variable

It is also possible to have interactions between

2 categorical variables

2 continuous variables

Aiken, LS & West, SG (1991).

*Multiple Regression: Testing and Interpreting Interactions.* Sage.

3 or more variables

# Parting thoughts

. Whenever an interaction involves a categorical variable consult the omnibus, multi-df, test of the interaction

If significant, explore simple effects within each level of the categorical explanatory variable

If there are 2 categorical variables, you have a choice:
you can explore the effects of X1 on Y within each category of X2, or
you can explore the effects of X2 on Y within each category of X1

. Usually, interacting variables are conceptualized as being contemporaneous
That is, one variable is not assumed to cause the other.

Testing interactions may provide important insights into health disparities

**Thank you**