Derivation of No Treat – Test and Test – Treat Probability Thresholds

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B = cost of failing to treat a D+ individual
C = cost of treating a D- individual unnecessarily
T = cost of test
P = Probability of D+

$p[- \mid D+]$ = probability of negative test given D^+ = 1 - sensitivity
$p[+ \mid D-]$ = probability of positive test given D^- = 1 - specificity

Expected Cost of No Treat strategy: $P \times B$

Expected Cost of Treat strategy: $(1-P) \times C$

Expected Cost of Test strategy:

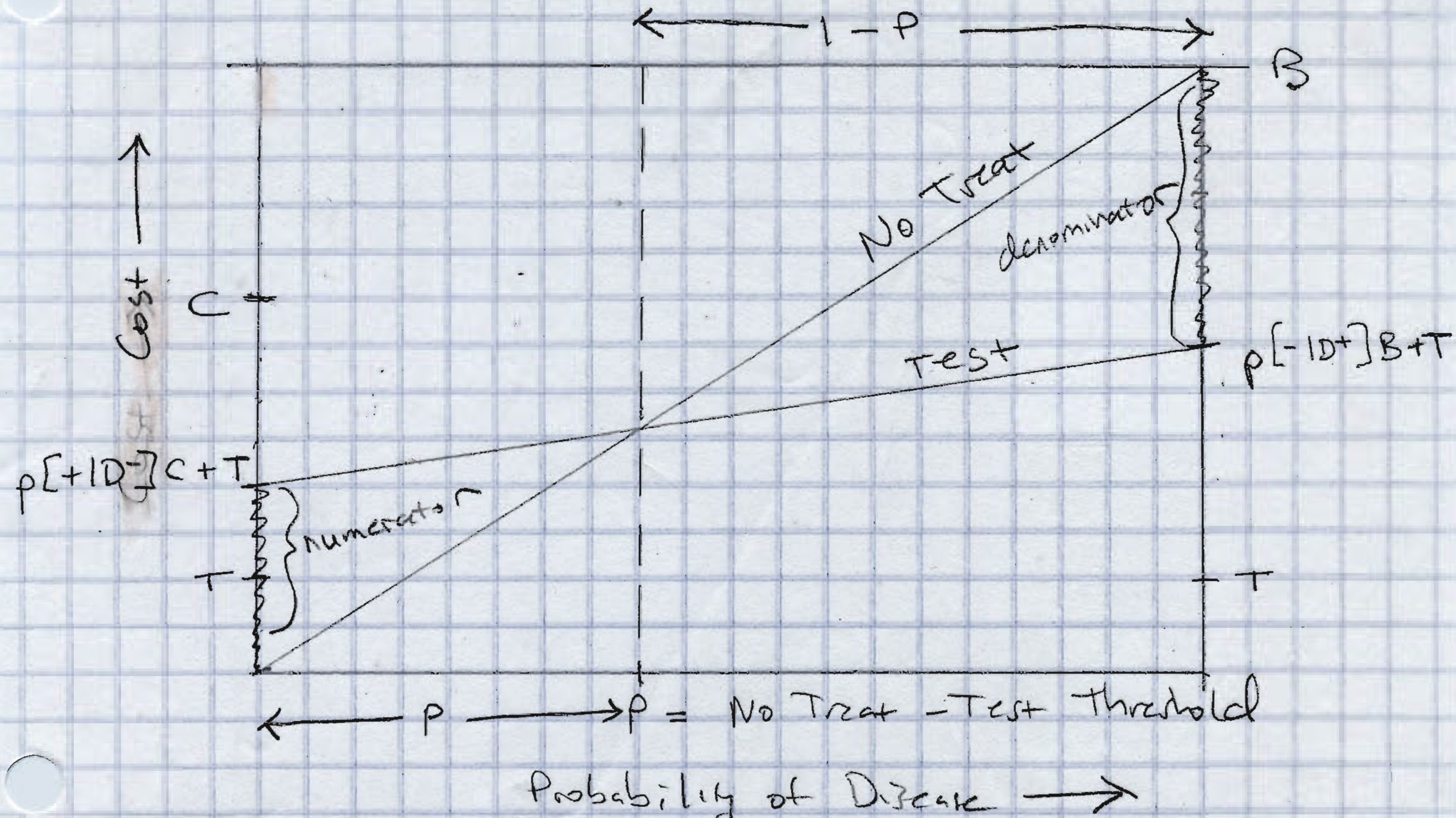
$$P(p[-\mid D+])B + (1-P)(p[+\mid D^-])C + T$$

Reminder about odds and probability:

You can convert odds to probability by adding the numerator to the denominator.

If threshold odds are C/B, then threshold probability is $C/(B+C)$

No Treat - Test Threshold - Graphical Derivation



Convince yourself that the ratio of the line ~~[illegible]~~ labelled "numerator" to the line labelled "denominator" is equal to the threshold odds P/(1-P).

$$P/(1-P) = \frac{p[+|D^-]C + T}{B - p[-|D^+]B - T}$$

$$= \frac{p[+|D^-]C + T}{B(1-p[-|D^+]) - T}$$

Substitute $p[+|D^+]$ for $1 - p[-|D^+]$

$$= \frac{p[+|D^-]C + T}{p[+|D^+]B - T}$$

= No Treat - Test Threshold Odds

(add numerator to denominator for probability)

No Treat – Test Threshold – Algebraic Derivation

No Treat – Test threshold is where the expected cost of the "No Treat" strategy equals the expected cost of the "Test" strategy.

$$(p)(B) = p\,(p[-|D+])\,B + (1-p)(p[+|D-])\,C + T$$

Substitute $(p)(T) + (1-p)T$ for T

$$(p)(B) = p\,(p[-|D+])B + (p)(T) + (1-p)(p[+|D-])C + (1-p)T$$

$$(p)(B) = p(p[-|D+]B + T) = (1-p)(p[+|D-]C + T)$$

Subtract this

$$(p)(B) - p(p[-|D+]B + T) = (1-p)(p[+|D-]C + T)$$

$$(p)[(B)(1 - p[-|D+]) - T] = (1-p)(p[+|D-]C + T)$$

Substitute $p[+|D+]$ for $1 - p[-|D+]$

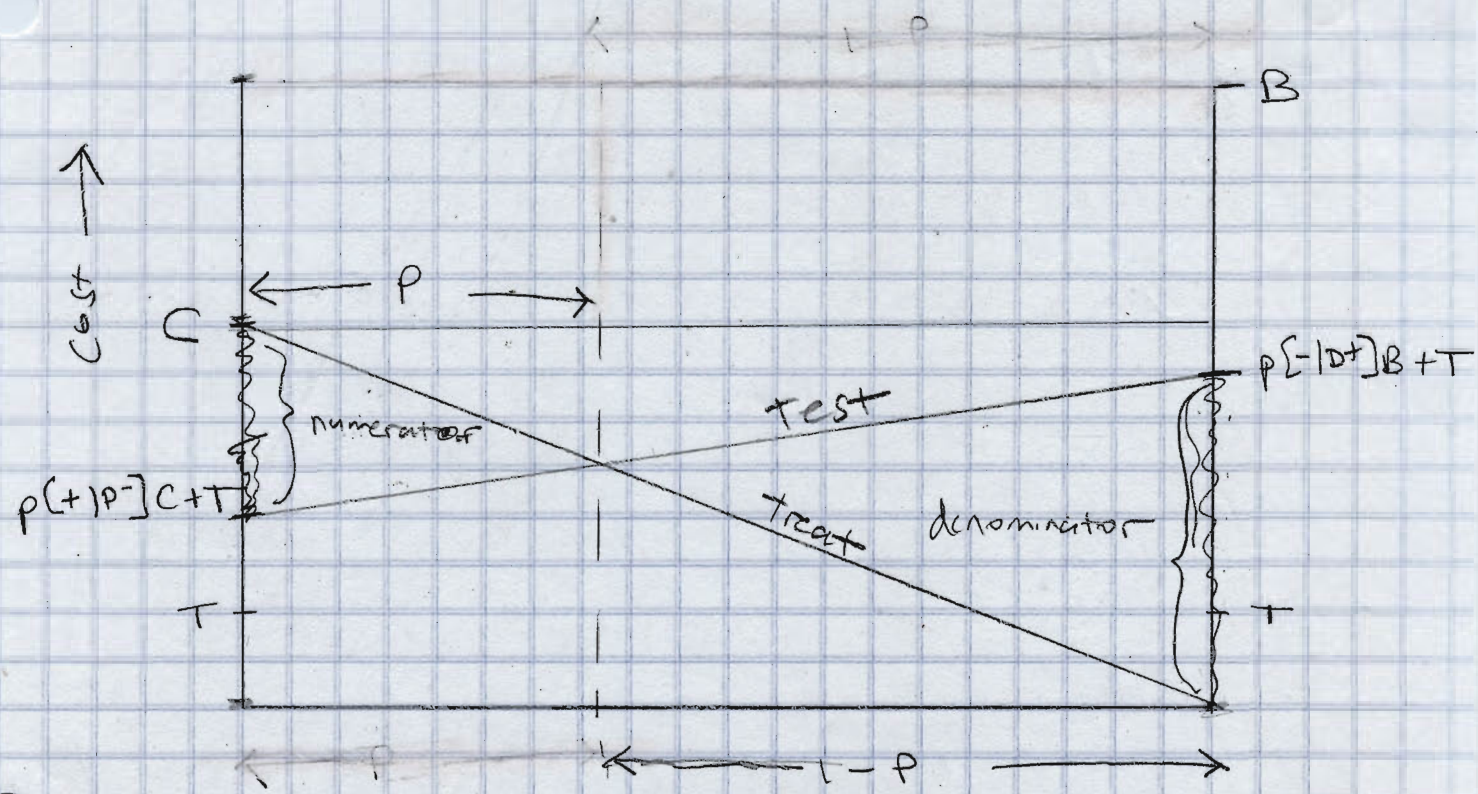
$$(p)[p[+|D+](B) - T] = (1-p)(p[+|D-]C + T)$$

$$\frac{p}{(1-p)} = \frac{p[+|D-]C + T}{p[+|D+]B - T}$$

This is threshold odds. To get threshold probability add the numerator to the denominator.

$$p = \frac{p[+|D-]C + T}{p[+|D+]B + p[+|D-]C}$$

Test – Treat Threshold – Graphical Derivation



Convince yourself that

$$\frac{p}{1-p} = \frac{C - (p[+|D^-]C + T)}{p[-|D^+]B + T}$$

$$= \frac{C(1 - p[+|D^-]) - T}{p[-|D^+]B + T}$$

substitute $p[-|D^-]$ for $1 - p[+|D^-]$

$$= \frac{C\,p[-|D^-] - T}{B\,p[-|D^+] + T}$$

= Test – Treat threshold odds

(add numerator to denominator to get threshold probability)

Test-Treat Threshold - Algebraic Derivation

The Test-Treat threshold is where the expected cost of the "Test" strategy equals the expected cost of the "Treat" strategy.

$$P(p[-|D+])B + (1-P)p[+|D-]C + T = (1-P)C$$

substitute $(P)(T) + (1-P)T$ for T

$$P(p[-|D+])B + (P)(T) + (1-P)p[+|D-]C + (1-P)T = (1-P)C$$

$$P(p[-|D+]B + T) + \underline{(1-P)(p[+|D-]C + T)} = (1-P)C$$

subtract this

$$P(p[-|D+]B + T) = (1-P)C - (1-P)(p[+|D-]C + T)$$

rearrange terms

$$P(p[-|D+]B + T) = (1-P)(1 - p[+|D-]C - (1-P)T$$

substitute $p[-|D-]$ for $1 - p[+|D-]$

$$P(p[-|D+]B + T) = (1-P)(p[-|D-]C - T)$$

$$\frac{P}{1-P} = \frac{p[-|D-]C - T}{p[-|D+]B + T}$$

This is threshold odds. To get threshold probability add the numerator to the denominator.

$$P = \frac{p[-|D-]C - T}{p[-|D+]B + p[-|D-]C}$$