Chi-squared test

Details of the test

- In general it tests for an association between two categorical variables
- Where each variable has only two categories this is equivalent to the z test for two proportions
- The test is based on the **chi-squared distribution with *n* degrees of freedom** where *n* is given by (no. of rows – 1) × (no. of columns – 1)
- It gives a P value but no direct estimate or confidence interval for the estimates unlike the z test, which gives both (📖 z test for two independent proportions, p. 260)
- For more details of test see Bland, Chapter 13[1]

Null hypothesis

- There is no association between the two variables in the population from which the samples come

Rationale of test

- It calculates the frequencies that would be expected if there were no association (i.e. null hypothesis is true)
- It compares the observed frequencies with these expected values
- If the observed frequencies are very different to the expected values this provides evidence that there is an association
- The test uses a formula based on the chi-squared distribution to give a P value

Assumptions of test

- Large sample test
- Rule of thumb for test to be valid:
  at least 80% of expected frequencies must be greater than 5
- **For a 2x2 test this means all expected values must be >5**
- If assumptions don't hold, consider collapsing the table if multi-category, use chi-squared with continuity correction (see 📖 Yates' correction, p. 263) or Fisher's exact test (📖 Fisher's exact test, p. 266)

Doing a chi-squared test

- ❶ **Always use with frequencies, never use percentages for calculations**
- The formula used works for a chi-squared test for all size tables
- The test is usually done with a computer program – the calculations are done to show how the test works

Yates' correction

The chi-squared test is based on frequencies which are discrete whilst the chi-squared distribution is continuous. The fit is good enough for large samples but breaks down when this is not so. Yates' correction is a modification of the chi-squared formula which makes the test statistic fit the continuous chi-squared distribution better.

Yates' correction is sometimes is given as an option for chi-squared tests in statistical programs and is worth using unless the sample is very large when it will make no difference to the P value. Some programs give both a Yates' corrected and the ordinary chi-squared P value. In such cases the corrected test may give a slightly bigger P value than the ordinary chi-squared test and this larger P value should be reported.

Note that using Yates' correction does not remove the need for the assumptions regarding the expected values.

Reference

1 Bland M. *An introduction to medical statistics.* 3rd ed. Oxford: Oxford University Press, 2000.

Chi-squared test: calculations

Doing a chi-squared test

These data in Table 8.1 come from a large US study of delayed time to defibrillation after in-hospital cardiac arrest.[1]

Table 8.1 Delayed time to defibrillation after in-hospital cardiac arrest[1]

		Event occurred after hours	
		Yes	**No**
Time to	>2 min	2094 (44%)	836 (41%)
defibrillation	≤2 min	2650 (56%)	1209 (59%)
Total		4744	2045

- Overall proportion with time >2 min:
  = (2094+836)/(4744+2045) = 2930/6789 = 0.431580
- Overall proportion with time ≤2 min:
  = (2650+1209)/(4744+2045) = 3859/6789 = 0.568420
- Expected values are given by multiplying these by each column total:
  4744x0.431 580=2047.4155, 2045x0.431 580=882.5811
  4744x0.568 420=2696.5845, 2045x0.568 420=1162.4189
- The chi-squared test statistic is given by the following formula where *O and E* are the observed and expected frequencies for all cells of table:

$$\sum_{all\ cells} \frac{(O-E)^2}{E}$$

$$= \frac{(2094-2047.4155)^2}{2047.4155} + \frac{(836-882.5811)^2}{882.5811} + \frac{(2650-2696.5845)^2}{2696.5845} + \frac{(1209-1162.4189)^2}{1162.4189}$$

$$= 6\ 19$$

With degrees of freedom: (2–1)×(2–1)=1

This has a P value of 0.013 which is statistically significant. Hence we have good evidence of a relationship between out of hours occurrence of cardiac events and delayed time to defibrillation.

Reference

1 Chan PS, Krumholz HM, Nichol G, Nallamothu BK. Delayed time to defibrillation after in-hospital cardiac arrest. *N Engl J Med* 2008; **358**(1):9–17.