Multilevel Models

Problems with Traditional Approaches

Individual level analysis (ignore group)

- Violation of independence of data assumption leading to misestimated standard errors (standard errors are smaller than they should be).

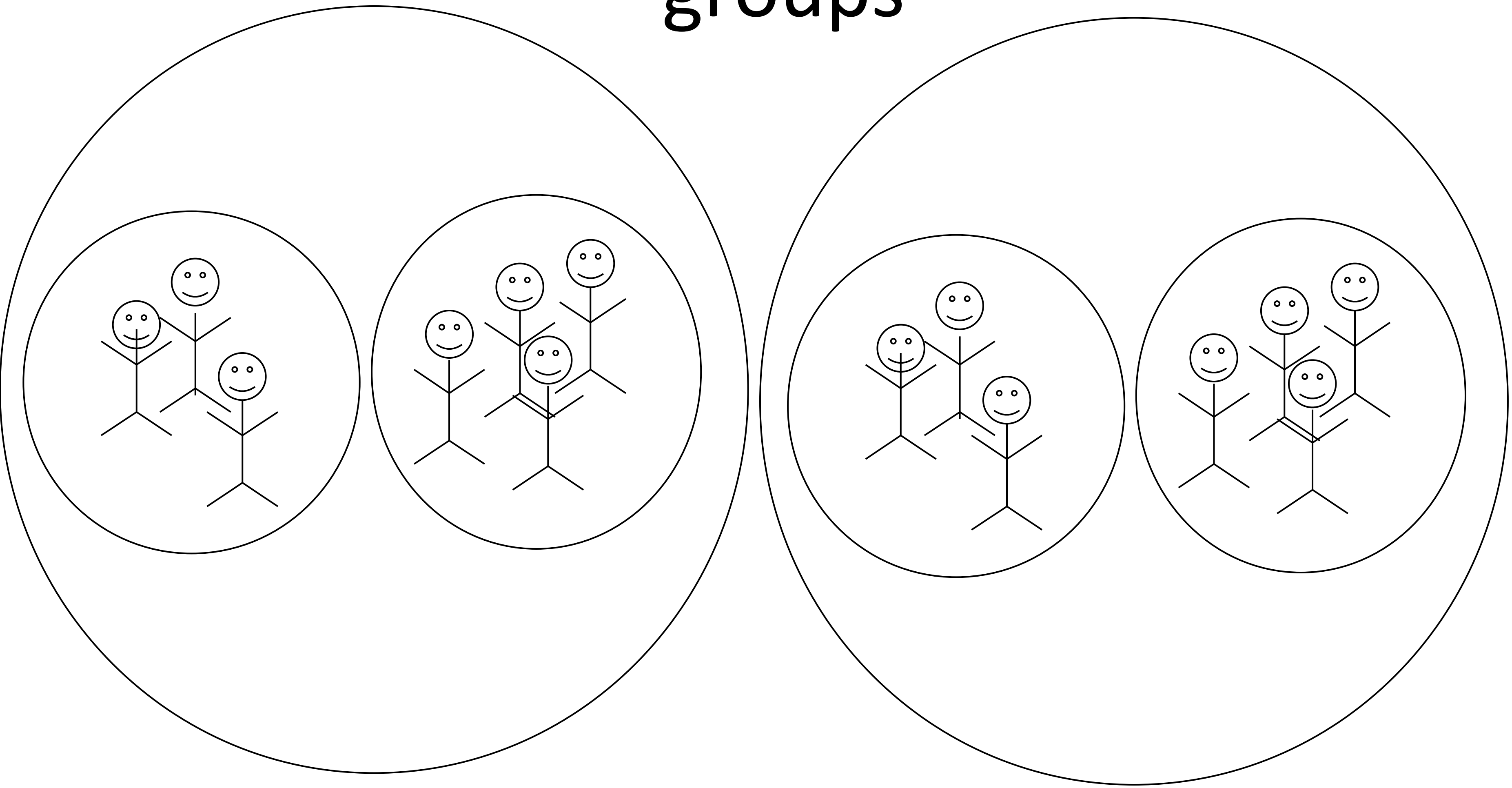
Group level analysis

- Aggregate data and ignore individuals
- Aggregation bias = the meaning of a variable at Level-1 (e.g., individual level SES) may not be the same as the meaning at Level-2 (e.g., school level SES)

Multilevel Analysis

- Simultaneous examination of group- and individual-level effects
- Both between group and within group variability
- Used for:
  - Understanding the causes of inter-individual variation
  - Inter-group variation

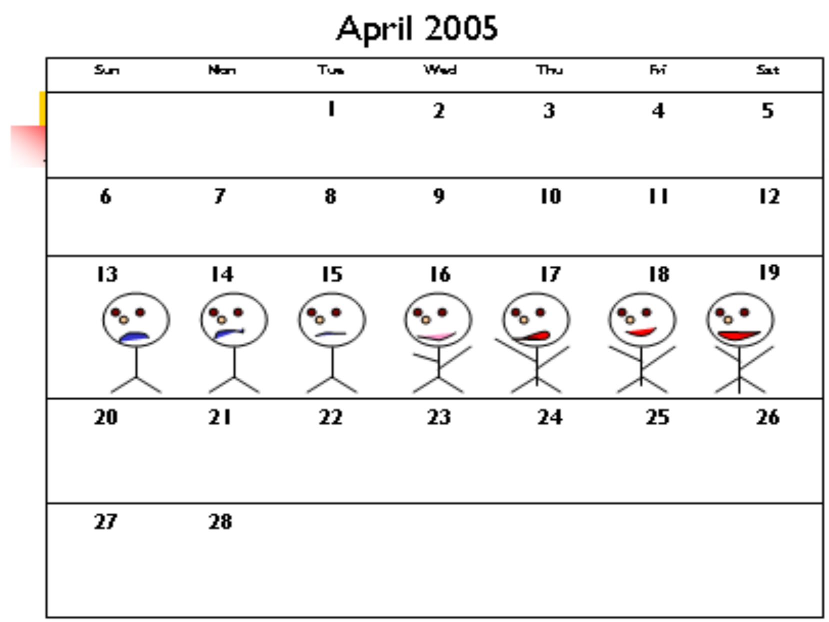
Type 1: Individuals nested within groups



Unit of Analysis = Individuals + Classes + Schools

Type 2: Repeated measures nested within individuals

- Focus
  - Change or Growth
  - Relationships between variables within an individual



Advantages of Multilevel Modeling Longitudinal Data

- Missing occasion data are no problem
  - Manova = listwise deletion, which wastes data
  - Manova = Missing Completely At Random (MCAR)
  - Multilevel model = Missing At Random (MAR)
- Can be used for panel & growth models
- Rate of change may differ across persons, and predicted by person characteristics
- Easy to extend to more levels (groups)

Multilevel model

Random intercept model

Level 1 (*i* students) $$Y_{ij} = \beta_{0j} + e_{ij}$$ random residual for level 1

fixed intercept for level 2
(grand intercept)

Level 2 (*j* classrooms) $$\beta_{0j} = \gamma_{00} + u_{0j}$$ random residual for level 2
(deviation from grand intercept)

By substitution, we get the full equation:

$$Y_{ij} = \underbrace{\gamma_{00}}_{\text{fixed}} + \underbrace{u_{0j}}_{\text{random}} + \underbrace{e_{ij}}_{\text{random}}$$

Two-level model example

- The combined level 1 and level 2 models form:

$$
\begin{aligned}
y_{ij} &= (\beta_0 + \alpha_{0i}) + (\beta_1 + \alpha_{1i})\, z_{ij} + \varepsilon_{ij} \\
&= \alpha_{0i} + \alpha_{1i}\, z_{ij} + \beta_0 + \beta_1\, z_{ij} + \varepsilon_{ij}\,.
\end{aligned}
$$

- The two-level model may be written as a single linear mixed effects model
- Specifically, we define $y_{ij} = \mathbf{z}_{ij}'\,\boldsymbol{\alpha}_i + \mathbf{x}_{ij}'\,\boldsymbol{\beta} + \varepsilon_{ij}\,.$
- We model and interpret outcome through the succession of level models
- We estimate the combined levels through a single (mixed effects) model

Two-level model example-cont'

- The combined level 1 and level 2 models form:

$$y_{ij} = \alpha_{0i} + \alpha_{1i} z_{ij} + \beta_0 + \beta_{01} x_i + \beta_1 z_{ij} + \beta_{11} x_i z_{ij} + \varepsilon_{ij}$$

- That can again be write as:

$$y_{ij} = \mathbf{z}_{ij}{}' \boldsymbol{\alpha}_i + \mathbf{x}_{ij}{}' \boldsymbol{\beta} + \varepsilon_{ij}$$

- The term β_{11} χ_i Z_{ij} interacting between level-1 variable x and level-2 variable x, is a cross-level interaction

Multi-level Models: Idea