NonParametric Methods

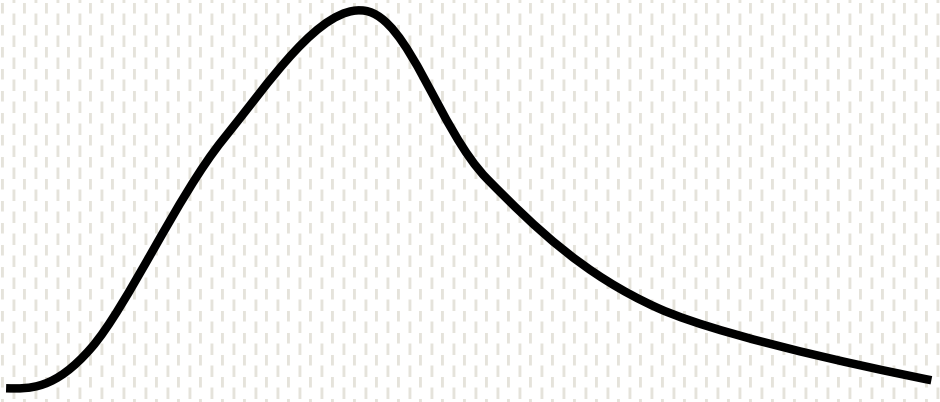
- Introduction
- Ranks & Median
- Paired Wilcoxon Signed Rank
- Mann-Whitney test (or Wilcoxon Rank Sum test)
- Others….

Parametric vs. non-parametric (distribution free) tests

- Non parametric tests:
  - No normality requirement
  - Do require that the underlying distributions being compared have the same basic shape
  - Ranks are less sensitive to outliers and to measurement error
- If the underlying distributions are approximately normal, then the parametric tests are more powerful

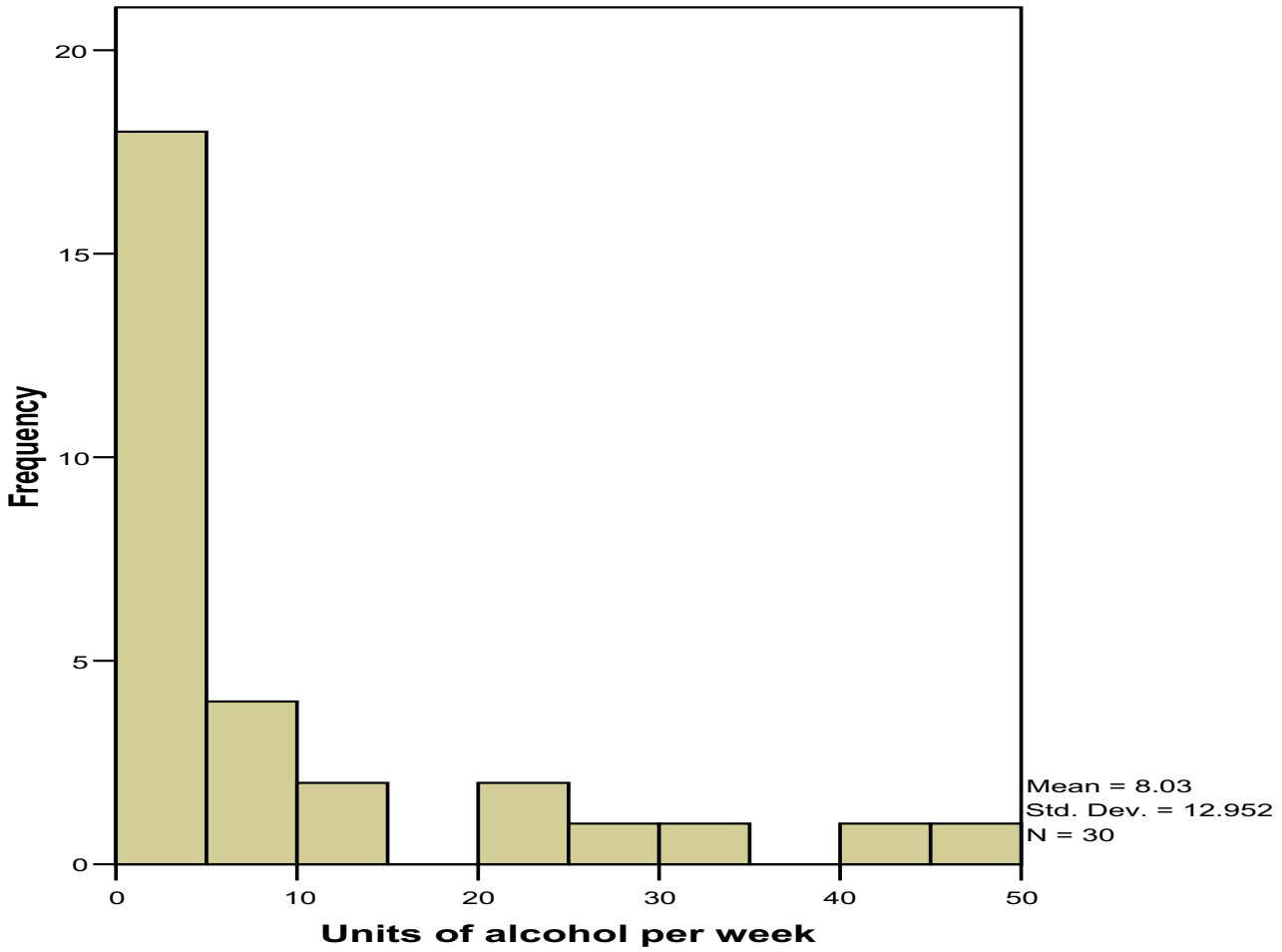
What are nonparametric tests?

- ‘Parametric’ tests involve estimating parameters such as the mean, and assume that distribution of sample means are ‘normally’ distributed
- Often data does not follow a Normal distribution e.g.; number of cigarettes smoked, health costs, etc.
- Positively skewed distributions



What are nonparametric tests?

A positively skewed distribution



What are nonparametric tests?

- ‘Non-parametric’ tests were developed for these situations where fewer assumptions have to be made
- Sometimes called Distribution-free tests
- NP tests STILL have assumptions but are less stringent and less powerful
- NP tests can be applied to Normal data but parametric tests have greater power IF the assumptions are met

Ranks

- Practical differences between parametric and NP are that NP methods use the ranks of values rather than the actual values
- e.g.

  1,2,3,4,5,7,13,22,38,45 - actual

  1,2,3,4,5,6, 7, 8, 9,10 - rank

Median

- The median is the value above and below which 50% of the data lie.
- If the data is ranked in order, it is the middle value
- In symmetric distributions the mean and median are the same
- In skewed distributions, median more appropriate

T-test

- T-test used to test whether the mean of a sample is sig different from a hypothesised sample mean
- T-test relies on the sample being drawn from a normally distributed population
- If sample not Normal then use the Wilcoxon Signed Rank Test as an alternative

Wilcoxon tests

- Frank Wilcoxon was Chemist at American Cyanamid who wanted to develop a test similar to t-test but without requirement of Normal distribution
- Presented paper in 1945
- Wilcoxon Signed Rank Ξ nonparametric paired t-test
- Wilcoxon Rank Sum Ξ nonparametric independent t-test

Wilcoxon Signed Rank Test

- NP test relating to the median as measure of central tendency
- The ranks of the absolute differences between the data and the hypothesised median calculated
- The ranks for the negative and the positive differences are then summed separately (W_- and W_+ resp.)
- The minimum of these is the test statistic, W

Wilcoxon Signed Rank Test Normal Approximation

- As the number of ranks (n) becomes larger, the distribution of W becomes approximately Normal
- Generally, if n>20
- Mean W=n(n+1)/4
- Variance W=n(n+1)(2n+1)/24
- Z=(W-mean W)/SD(W)

Wilcoxon Signed Rank Test Assumptions

- Population should be approximately symmetrical but need not be Normal
- Results must be classified as either being greater than or less than the median ie exclude results=median
- Can be used for small or large samples
- Similar to the paired t-test
- In Stata $signrank\ x_1 = x_2$

Paired samples t-test

- Disadvantage: Assumes data are a random sample from a population which is Normally distributed

- Advantage: Uses all detail of the available data, and if the data are normally distributed it is the most powerful test

The Wilcoxon Signed Rank Test for Paired Comparisons

- Disadvantage: Throws away the real values and only the sign (+ or -) of any change is analysed

- Advantage: Easy to carry out and data can be analysed from any distribution or population

Paired And Unpaired Comparisons

- If you have the same sample measured on two separate occasions then this is a paired comparison
- Two independent samples is not a paired comparison
- Different samples which are ‘matched’ by age and gender are paired

The Wilcoxon Signed Rank Test for Paired Comparisons

- Similar calculation to the Wilcoxon Signed Rank test, only the differences in the paired results are ranked
- A group of 10 patients with chronic anxiety receive sessions of cognitive therapy. Quality of Life scores are measured before and after therapy.

Wilcoxon Signed Rank Test example

QoL Score				
x1	x2	Diff	Rank	-/+
6	9	3	5.5	+
5	12	7	10	+
3	9	6	9	+
4	9	5	8	+
2	3	1	4	+
1	1	0	3	tied
3	2	-1	2	-
8	12	4	7	+
6	9	3	5.5	+
12	10	-2	1	-

$W_- = 2$
$W_+ = 7$
1 tied

Stata output: Wilcoxon vs. t-test

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	2	6.5	27
negative	7	47.5	27
zero	1	1	1
all	10	55	55

unadjusted variance	96.25
adjustment for ties	-0.25
adjustment for zeros	-0.25
adjusted variance	95.75

Ho: var1 = var2

z = -2.095

Prob > |z| = 0.0362

```
. ttest var1= var2
```

Paired t test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
var1	10	5	1.021981	3.231787	2.688119	7.311881
var2	10	7.6	1.284091	4.060651	4.695185	10.50481
diff	10	-2.6	.9568467	3.025815	-4.764538	-.4354624

mean(diff) = mean(var1 - var2) t = -2.7173

Ho: mean(diff) = 0 degrees of freedom = 9

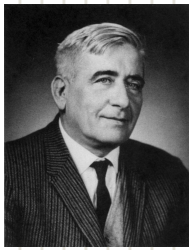
Ha: mean(diff) < 0 Pr(T < t) = 0.0119

Ha: mean(diff) != 0 Pr(|T| > |t|) = 0.0237

Ha: mean(diff) > 0 Pr(T > t) = 0.9881

Mann-Whitney test Ξ Wilcoxon Rank Sum

- Henry Mann, professor of mathematics @ Ohio State & Don Whitney was his student
- Used when we want to compare two unrelated or INDEPENDENT groups
- For parametric data you would use the unpaired (independent) samples t-test
- The assumptions of the t-test were:
  1. The distribution of the measure in each group is approx Normally distributed
  2. The variances are similar



HB Mann

Wilcoxon (Mann-Whitney) Rank Sum Example

- The following data shows the number of alcohol units per week collected in a survey:
- Men (n=13): 0,0,1,5,10,30,45,5,5,1,0,0,0
- Women (n=14): 0,0,0,0,1,5,4,1,0,0,3,20,0,0
- Is the amount greater in men compared to women?
- STATA: ranksum Alcohol, by(Gender)

Stata output: Wilcoxon vs. t-test

```
. ranksum Alcohol, by(Gender)

Two-sample Wilcoxon rank-sum (Mann-Whitney) test
```

Gender	obs	rank sum	expected
0	12	180.5	162
1	14	170.5	189
combined	26	351	351

```
unadjusted variance      378.00
adjustment for ties      -49.62
                         ------
adjusted variance        328.38

Ho: Alcohol(Gender==0) = Alcohol(Gender==1)
             z =   1.021
    Prob > |z| =   0.3073
```

```
. ttest Alcohol, by(Gender)

Two-sample t test with equal variances
```

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
0	12	4.75	2.471428	8.561276	-.6895755	10.18958
1	14	2.428571	1.42472	5.330814	-.6493492	5.506492
combined	26	3.5	1.364664	6.958448	.689422	6.310578
diff		2.321429	2.753409		-3.361328	8.004185

```
    diff = mean(0) - mean(1)                                      t =   0.8431
Ho: diff = 0                     degrees of freedom =       24

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.7963         Pr(|T| > |t|) = 0.4075          Pr(T > t) = 0.2037
```

Other NonParametric Methods

- Sign Test
- Median Test
- Chi-squared test – next week!!
- Spearman's Rank Correlation Coefficient – in 2 weeks when we tackle regression!